PHYSICAL MEASUREMENT.

A SHORT COURSE

OF

EXPERIMENTS

IN

PHYSICAL MEASUREMENT.

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In Four Parts.

PART III.

PRINCIPLES AND METHODS.

NOTES AND EXPLANATIONS FOR THE USE OF STUDENTS.

MATHEMATICAL AND PHYSICAL TABLES.

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TABLE OF CONTENTS.

PRINCIPLES AND METHODS.

MATHEMATICAL AND PHYSICAL TABLES.

MATHEMATICAL TABLES.

PHYSICAL TABLES.

General Properties of Solids, Liquids, and Gases.

HYPSOMETRIC, HYGROMETRIC, AND BAROMETRIC TABLES.

REDUCTION OF WEIGHINGS TO VACUO.

SPECIFIC VOLUMES AND DENSITIES.

PROPERTIES OF SOLUTIONS DEPENDING UPON THEIR STRENGTH.

MISCELLANEOUS DATA.

ELECTROMOTIVE FORCE AND RESISTANCE.

ARBITRARY SCALES.

ASTRONOMICAL AND GEOGRAPHICAL TABLES.

REDUCTION OF MEASURES TO AND FROM THE C. G. S. SYSTEM.

PHYSICAL MEASUREMENT.

Part Third.

PRINCIPLES AND METHODS.

INTRODUCTION.

THE first step in all scientific progress consists in a classification of different objects based upon similarities and differences. The distinguishing characteristics of solids and liquids, minerals, metals, crystals, &c., were undoubtedly observed long before history began. The necessity for shelter and clothing must have drawn attention to the difference between insulating substances and conductors of heat; and in the same way all physical properties of importance to mankind cannot have failed to receive early recognition. The manner in which different branches of science have been developed is perhaps best illustrated in the case of electricity, the phenomena of which were virtually unknown[1] before the

[1] The development of electricity from amber was known to Thales several years before Christ. It would appear, however, that at this time little or nothing else was known about electricity. Ganot's Physics, § 723.

end of the sixteenth century. We find in very early writings tables like the following: —

CONDUCTORS OF ELECTRICITY.

Metals.	Animal Substances.	Sea Water.
Charcoal.	Vegetable Substances.	Vinegar, &c.

NON-CONDUCTORS.

Resins.	Glass.	Wax.
Sulphur.	Silk.	Oils, &c.

A division of substances into two classes may in certain cases be exceedingly useful. The reactions which take place in chemical solutions are, for instance, frequently determined by the solubility or insolubility of the compounds which may be formed. It is rarely necessary to make fine distinctions in the statement of chemical solubilities.[1] The term "sparingly soluble" must occasionally be employed; and, again, comparisons must be made between different solubilities. Most substances, however, are either very soluble, or else very insoluble, in a given liquid; and a single word, "soluble" or "insoluble," conveys to the chemist a valuable piece of information.

In the construction of electrical instruments, on the other hand, it became important to distinguish both good conductors and good non-conductors from a large class of substances called "semi-conductors" (Ganot's Physics, § 725); and with the growing importance of electricity came the necessity of still further distinctions. Substances were finally ar-

[1] See Storer's Dictionary of Solubilities.

ranged in a list in the order of their power to conduct or to insulate electricity (Deschanel's Natural Philosophy, § 409). In the same way certain bodies, at first classed simply as positive or negative with respect to the charges of electricity which they receive when rubbed together, are in later works arranged as follows (Deschanel, § 411): —

Fur of Cat.	Feathers.	Silk.
Polished Glass.	Wood	Shellac.
Wooden Stuffs.	Paper.	Rough Glass.

If any of the substances in this list be rubbed with one following it, it will generally become "positively electrified;" but if rubbed with one preceding it, it will be "negatively electrified." Such an arrangement is evidently more useful than a simple division into two classes.

Mohs' scale of hardness consists of 10 substances:[1]

1. Talc.	3. Calc Spar	5. Apatite.	7. Quartz.	9. Sapphire.
2. Gypsum.	4. Fluor Spar	6. Feldspar.	8. Topaz.	10. Diamond.

Each substance contained in this list will scratch the one above it. If, accordingly, a piece of steel which will scratch feldspar is scratched by quartz, its hardness must be represented by a number between 6 and 7 (let us say 6.5) on this arbitrary scale.

The distinction between any two substances in such a list is purely qualitative; that is, we know only that each possesses a certain quality or property *more than* the one below it. We do not know whether the

[1] Cooke's Chemical Physics, p 209.

gaps in the list are great or small, equal or unequal. We have no idea even of the relative values which the numbers (1–10) represent. Still, the assignment of numbers to the different substances may be considered as a first attempt to obtain precise results; and in the case of physical quantities which admit of no more exact estimation, the value of an arbitrary scale like that of Mohs must not be overlooked.

The next step in the accurate representation of results is to make the intervals between different scale-numbers equal, — or, at least, to make them follow in regular progression. Among the earliest applications of this principle may be mentioned the arbitrary hydrometer scales of Beaumé, Beck, Cartier and Twaddell. A mark was made upon a hydrometer (see Fig. *a*) to show how deep it sank in water; and this mark was numbered 0 or 10, as the case might be. Then the hydrometer was floated in some other liquid of known composition, and another mark was made to show how deep it sank in that liquid. The second mark was also numbered arbitrarily — 60 or 80, for instance (see Table 40). The distance between the two marks was then subdivided. The scale of an ordinary thermometer (see Fig. *b*) is constructed in a similar way. A mark is made to show where the mercury stands when surrounded with melting ice, and another

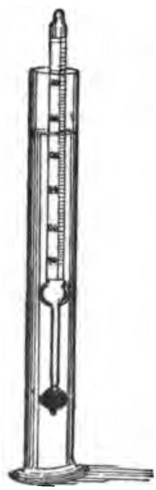


Fig. *a*.

mark is made to show where it stands in steam (see Exp. 25). The distance between the two marks is divided by Fahrenheit into 180 parts; by Celsius, into 100 parts; by Réaumur, into 80 parts. Fahrenheit called the freezing-point of water 32°, without any scientific reason; Celsius and Réaumur called it 0°. Their scales are accordingly simpler than Fahrenheit's, but none the less arbitrary. The Celsius scale is still in use in the ordinary centigrade thermometer (§ 4); the other scales, together with the hydrometer scales of Baumé, Beck, Cartier, and Twaddell, are going out of use. The gradual disap-

FIG. b.

pearance of arbitrary scales is in general an indication of scientific progress.

It is obviously desirable that the numbers in a scale should be proportional to the quantities which they represent. With the advance of science in the early part of the present century, we find an abundance of physical tables showing the relative values of different quantities (§ 3). Specific gravities of solids and liquids compared with water, specific gravities of gases and vapors compared with air or with hydrogen, specific heats compared with water, &c., were all more or less accurately determined.

At the same time that the physical properties of

different bodies were compared together, the changes which take place in a given substance under varying conditions were carefully studied. The expansion of solids, liquids, and gases due to heat were, for instance, observed and tabulated. We find in Biot's "Physique ' (1821, vol. i., page 320) a table showing the relative densities of water at different temperatures, some of which are compared below with the best results of modern observers, as given by Everett in § 34 of his "Units and Physical Constants." Calling the density of water at 4° equal to 1, these results become [1] —

	Biot.	Everett.	Difference.		Biot.	Everett.	Difference.
0°	.99993	.99987	+ 6	50°	.98778	.98820	− 42
4°	1.00000	1.00000		60°	.98251	.98338	− 87
10°	.99973	.99975	− 2	70°	.97652	.97794	− 142
20°	.99832	.99826	+ 6	80°	.96998	.97194	− 196
30°	.99579	.99577	+ 2	90°	.96285	.96556	− 271
40°	.99225	.99235	− 10	100°	.95537	.95865	− 328

This is but one of the many fairly accurate determinations dating back even into the last century. Most of our modern physical laws and principles were known in the early part of the nineteenth century, and a great number of physical properties had been investigated. The results of this early period are, however, characterized by the absence of all data by which it is possible to find anything more than the relative values of different quantities. The powers

[1] The results quoted by Biot, though creditable for his time, were generally inaccurate in the fourth and sometimes even in the third place of decimals. They were, nevertheless, carried out, according to the custom of early observers, to 7 and 8 decimal places.

of different metals to conduct heat were, for instance, given by Despretz as follows, counting gold as 1,000 (Ganot's Physics, § 404): —

	Despretz.	Wiedemann and Franz.
Platinum	981	158
Silver	973	1880
Copper	897	1384
Iron	374	202
Zinc	363	874
Tin	304	273
Lead	179	160

That these results were not particularly accurate may be inferred by comparing them with those of Wiedemann and Franz (1853), reduced in the right-hand column to the same system.[1] Thus platinum, which is the best conductor of heat according to Despretz, is the worst according to Wiedemann and Franz. Even, however, if we assume the accuracy of either set of results, it is still impossible to apply them unless we know, in a single case, how much heat flows from one place to another through a bar or plate of given length, breadth, thickness, and material, and the difference of temperature to which this flow of heat corresponds.

The determination of relative values (such as are contained in the table above) is in general a much easier task than the determination of absolute values (see Table 8, *et seq.*); and has the advantage that gross errors are not so likely to be made.

Relative measurements are, however, to a certain

[1] Wiedemann and Franz counted silver as 100 See Deschanel's Natural Philosophy, § 338.

extent non-committal, and hence justly unpopular with scientific men. The highest end of physical measurement is not attained unless every quantity with which it has to deal is compared directly or indirectly with the so-called *absolute units* (§ 8) which lie at the base of the system. Quantities subjected to such comparisons are said to be *determined in absolute measure.*

We have seen that, historically, in various branches of science, the absolute system of physical measurement has been approached by a series of stages. The first stage may be called classification; the second, ordination; the third, numbering; the fourth, graduation; the fifth, comparison; the sixth and last, determination. The first two stages deal with qualities, and involve only qualitative experiments. Physical measurement is properly confined to the last two stages. It deals exclusively with the numerical relations between different physical quantities. Measurements are, accordingly, quantitative in their nature.

It is unnecessary to distinguish physical measurement from measurement in general, as the term is usually employed. It is only physical quantities which are capable of being measured. Measurement implies observation; exact measurement implies accurate observation. The observation required in physical measurement is, it is true, exceedingly limited in its character (see § 23). In the natural sciences, the powers of observation have their widest application. In physical measurement the *sharpest*

use of this faculty is required. The student is apt to imagine that an increase of precision in the instruments at his disposal would relieve the continual tax which he feels upon his power of observation. Quite the reverse is generally true. The better the instrument, the harder it is to do justice to it. One must learn to obtain the best possible results with rough instruments before one is fitted to use instruments of precision. The habit of accurate observation is an important object to be gained by a course of physical measurement.

The most accurate results in physical measurement often require practice, not only in observation, but also in manipulation. The skill acquired in a course of quantitative determinations is an advantage by no means to be overlooked.

The principal benefit to be expected from a course of laboratory instruction is, however, familiarity with the *experimental method* and the processes of inductive reasoning which it involves. Certain of these processes belong especially to quantitative determinations. The results of physical measurement frequently depend, not only upon a long series of observations, but also upon a more or less complicated chain of reasoning, including the mathematical calculations by which the observations are reduced. A single error in any one of the data, or in any step in the process of reduction, will in most cases entirely change the result. The student is not, however, in physics as in philosophy, necessarily misled by such an error. Physical measurement abounds in what

are called "check methods" (§ 45), by which errors either in observation or in reasoning may generally be detected. Having once discovered the sources of error into which he has fallen, the student is less likely to commit the same errors in the future. The result of a course of physical measurement should be to give him a just confidence in what he has seen with his own eyes, and in what he has reasoned out in his own mind.

The student should learn, as early as possible, to distinguish between real and apparent accuracy. A kilogram of wood may, for instance, be weighed to a milligram on a good balance. Such a weighing would be called *precise*. The true weight would, however, be very inaccurately determined, if no account were taken of the buoyancy of the atmosphere, which may amount to several thousand milligrams.

A given degree of accuracy implies an equal degree of precision; but precision does not necessarily imply accuracy. Exact results are those which are both accurate and precise.

When a measurement, however inaccurate, is repeated several times *in exactly the same manner*, more or less concordant results are usually obtained. The object of the scientific observer is not to make his determinations *look* more accurate than they really are, but, on the contrary, to bring to light the errors by which they are affected. He seeks accordingly every possible variation of the conditions under which an experiment is tried, in order *to bring out discordances*, if possible, between methods which ought (as far as

he knows) to give exactly the same result. The simplest changes — the manner, for instance, of supporting an instrument — have frequently a most unexpected effect, and lead to the disclosure of unknown sources of error.

The student must not be discouraged by the discovery that his results are less accurate than he expected. He will find by comparing together the determinations of distinguished scientific men, that great discrepancies frequently exist between them. He must not be deceived by the number of decimal places to which their work is carried out. According to a custom prevalent, especially in the early part of this century, 3, 4, and even 5 figures, having little or no significance (§ 55) are often appended to results (see footnote, page 590). Within the last twenty years, the physical constants have acquired certain *conventional values*. There is an undoubted tendency to publish determinations by which these values are confirmed, and to suppress others equally good, leading to different results. The concordance of modern determinations is therefore, to a certain extent, apparent rather than real.

From time to time (as every one knows who follows scientific proceedings) inaccuracies in the accepted values of the physical constants force themselves upon our attention. In view of these facts, the student should return with increased confidence to his own determinations. When an investigation has been completed, and all sources of error, in so far as possible, allowed for, the facts should be made

known, no matter who has arrived at a different result.

The student should learn to value different determinations for what they are worth. It is a very rough weighing that is not accurate within one part in a thousand; but some of the best electrical measurements are subject to much greater errors.

The results of some observers in determining the conductivity of different substances for heat are twice as great as the results of others; these results are however, useful. They show, for instance, that it would be impracticable to heat a house by a system of conducting rods radiating from a common centre; but that the thin metallic coatings of a furnace offer a comparatively slight resistance to the passage of heat. A knowledge even of the *number of ciphers* necessary to express the magnitude of certain quantities,—as, for instance, the weight of molecules,—may be useful in certain calculations. The fact that some measurements are necessarily inexact should not prevent the student from doing his best where accurate work is possible.

The results of physical measurement can, from their nature, never be, like those of mathematics, perfectly exact. Errors of greater or less magnitude are not only possible, but we may say almost certain to occur. Herein lies an important distinction between mathematical and physical problems. A mathematical solution is either right or wrong. In regard to the results of physical investigations, we have to consider *how far* each is likely to be in error. The

quantitative methods which characterize physical measurement are extended even to the errors committed in these measurements. The treatment of such problems forms an important branch of the mathematical theory of probability, upon which all inductive methods are founded. It is not easy, from a philosophical standpoint, to regard the probable accuracy of results obtained by observation in *exactly the right attitude*. One cannot strictly affirm the accuracy of any figure in a result; but, as concerns some figures, it is difficult if not impossible to formulate the slightest doubt without enormously exaggerating the real uncertainty. Discussions of "probable error" (§§ 50–52) are characteristic of physical measurement, and teach a species of reasoning which, in problems of insurance, has assumed great practical importance.

One of the principal advantages derived from a course of physical measurement is, as has been said, the acquisition of habits of accurate thinking. When two quantities have been compared together, it is evident that, if the magnitude of one is known, that of the other must be determined. It is not, however, always clear *what* is determined by a given observation. It must be borne in mind that a physical determination consists, essentially, in the comparison of a quantity with one *better known than itself*. At the beginning of this century, the density of water at high temperatures was known only within a few tenths of 1 %. To-day, the density of water is one of the best known physical constants. The same experiment

(Exp. 19) which one hundred years ago constituted a determination of the density of water, now furnishes data only for calculating the volume of a solid, or the rate of expansion of the material of which it is composed. Great care must be taken to make a proper use of the results of physical measurement. One may, for instance, measure the circumference and radius of a circle, and from the results calculate the ratio which one bears to the other. It would, however, be incorrect to speak of this experiment as a determination of the ratio in question; since this ratio, being capable of exact mathematical calculation, is better known than the scale readings upon which the result depends. Physical measurement may be occasionally employed as a check upon mathematical calculations, particularly when (as in certain applications to physics) there is any doubt as to the validity of the assumptions upon which the calculations depend. Any attempt, however, to establish mathematical principles by data obtained from observation is an obvious abuse of the experimental method.

The so-called "proofs" of well-known physical laws and principles founded upon rough and insufficient data are hardly less objectionable.[1] The use of the experimental method as an illustration of such laws is not denied. One of the objects, however, of a course of physical measurement is to teach a stu-

[1] It may be remarked that the Law of Boyle and Mariotte (§ 79) was thus taught and implicitly believed in for more than a century, before more exact observation showed that this law is only approximately fulfilled.

dent how to make the best use of the tools at his command. The laws and principles which have been most carefully studied by scientific men should be made the instruments, not the objects of elementary research. The teacher should avoid, in so far as possible, experiments whose ostensible object is to establish well-known facts, — like the conservation of energy,— the truth of which is not really in question.

Among the habits of accurate thinking which it is the object of physical measurement to teach, may be mentioned those involved in a diligent and methodical search after the errors which are likely to be committed in one's work. It is hoped that the classification of errors in Chapter II. may be of assistance to the student who is thrown more or less upon his own responsibility. It is of course impossible to anticipate in any such classification all errors which may arise; but there are certain kinds of errors of such frequent occurrence that one must always be on one's guard against them. The student should ask himself, for instance, in respect to every scale reading, Have errors of parallax been guarded against (§ 25)? Have errors been committed in the estimation of tenths (§ 26)? Are there mechanical devices by which such errors could be diminished (§ 27)? Has the zero of the scale been carefully adjusted (§ 32)? Has the scale been carefully tested (§§ 31, 37)?

In addition to these considerations, by which errors may be frequently avoided, there are certain general methods, considered in Chapter III., by which (when

they can be applied) the accuracy of a result is *always increased.* The student who is planning for himself the details of a physical measurement should consider these general methods one by one. He should ask himself, for instance, Is the method proposed the most direct (§ 36)? Could not more accurate results be obtained by dealing with larger quantities (§§ 38, 39)? or quantities which happen to be more nearly coincident (§ 40)? Could not precision be gained by the use of differential instruments (§§ 41, 42)? or accuracy by the check methods (§§ 43-45)? Would it be possible to reverse or interchange the quantities compared (§ 44)? or to obtain and average results from several determinations (§ 46)? These and similar questions must occur habitually to every successful observer.

A course in physical measurement is not especially suited to students who wish to become acquainted with a wide range of physical phenomena. Dealing, however, with quantities of nearly every description, and with the numerical relations which exist between them, it affords numerous examples of the application of physical laws and principles. It is only through the aid of definite examples that most persons can arrive at an understanding of physics. It has been assumed in the experimental course described in Parts I. and II. of this book, that the student is already familiar with the *statements* of physical phenomena contained in ordinary text-books. If this is the case, he must expect to gain definiteness rather than scope in his conceptions from a course of quantitative determinations.

It would be impossible, in the limited space which can be devoted to the subject in the present volume to describe or explain in full more than a very small part of the principles which underlie physical measurement. The brief notes contained in Chapters V.-X. are intended simply to recall to the student (who has already taken a course in general physics) the laws and principles which he has to employ, and the proofs upon which they rest. They may also be useful to the instructor as a basis for his lectures, or to the student who is just beginning the study of physics as a "syllabus" of what he should read in order to follow intelligently the course of physical measurement described in Parts I. and II. For a full explanation of the physical principles involved in this course, the student is referred to the standard works of Daniell, Deschanel, and Ganot.

The advantages of a course in physical measurement have been considered chiefly from an educational standpoint. It is hardly necessary to point out that Physical Measurement is a science of great practical importance. The nice adjustments of the different parts of a machine would, for instance, be impossible without accurate measurements. Success in Chemistry, in Astronomy, in Surveying, in fact in all branches of Civil and Electrical Engineering, depends to a great extent upon a thorough understanding of the Principles and Methods of Physical Measurement.

CHAPTER I.

GENERAL DEFINITIONS.

§ 1. **Nature of Measurement.** — Measurement consists in finding out by observation how many things of one sort correspond in magnitude to a given number of another sort. When 10 spaces on a measure divided into inches are found to reach through the same distance as 254 spaces on a millimetre scale, the length of the inch is said to be measured in millimetres, and conversely the millimetre may be said to be measured in inches. Either the millimetre or the inch may be used as a standard of comparison. When a quantity of known magnitude is compared with one of unknown magnitude, the latter is said to be measured in terms of the former. Thus, if a load is found to be equal in weight to a given number ot grams, its weight in grams is said to be measured. It is obviously impossible to compare, in general, magnitudes of different sorts, — as, for instance, length and volume; but under certain circumstances, correspondences or relations exist between such quantities. When a stream of water, for instance, striking an obstacle with a velocity between 2 and 3 miles per minute is found to warm itself 1 Fahrenheit degree, a certain relation between temperature and velocity is said to be established. Such relations are properly objects of physical measure-

ment. Measurements are either relative or absolute (§ 8), and may be classed, accordingly, as comparisons or determinations.[1]

§ 2. **The Metric System.** — The metric system is now generally adopted in scientific work. It is so called from the metre, or standard of length upon which it is founded (§ 5). The metre is equal to about 39.37 English inches. A cubic metre of ice-water weighs 1 "tonne" (1,000,000 grams) or 2205 lbs. nearly. There are, accordingly, 15.432 grains, or about 15 drops of water in one gram (§ 6). In the metric, as in other systems, the unit of time is the second (§ 7). The chief advantage of the metric system consists in the simplicity of the relations which exist between the standards of length and mass, and in the use of units each of which is some decimal multiple or sub-multiple of the others in the same series.

These units are distinguished, in the metric system by the aid of prefixes, which have the following significations: *mega*, one million; *kilo*, one thousand; *hecto*, one hundred; *deka*, ten; *deci*, one tenth; *centi*, one hundredth; *milli*, one thousandth, and *micro*

[1] The word "absolute" must not be confounded with the word "exact." Measurements are said to be "absolute" only when *fundamental* standards or units are employed (see § 8). We speak of the measurement rather than the determination of *variable* quantities, as for instance the strength of an electric current. We speak also of the measurement of *accidental* quantities, like the length or weight of a body, especially when, as in measurements of length, *direct* methods can be employed. (See Chap. III.) On the other hand, a magnitude is said to be "determined" rather than "measured" by an *arbitrary* scale, and measurements of *invariable* quantities, like the physical constants. are customarily called "determinations"

one millionth. Thus a kilometre means a thousand metres; a microvolt a millionth part of a volt. When the unit begins with a vowel, the last vowel of the prefix is generally omitted; thus a million ohms is called a megohm.

§ 3. **Relative magnitudes.** — There are certain quantities which can be defined without reference to any particular system of measurement, such for instance as include simply a ratio between two things. Thus specific gravity is the proportion which the weight of a substance bears to that of an equal bulk of water; specific heat the proportion of heat it absorbs as compared to that absorbed by an equal weight of water; and specific electrical resistance is sometimes, though not generally, used in a similar sense.[1] Again, strains are defined as the proportion of the distortion which is produced to the whole quantity acted upon. Thus if a body has been stretched or sheared by an amount equal to $\frac{1}{100}$ of its length, or compressed by $\frac{1}{100}$ of its volume, it is said to have suffered a strain of $\frac{1}{100}$. Angles too are determined[2] by the ratio of the arc which they subtend to the radius; and the sine, cosine, or tangent of any angle[3] is simply the ratio between two of the three sides of a right-angled triangle in which the given angle occurs. Another instance is the index of refraction, or ratio of the velocity of a wave outside of a medium to its velocity in it. It

[1] See Experiment 88; also Trowbridge, New Physics, Experiment 120.

[2] See Table 3, columns *a* and *c*.

[3] See Table 3, columns *b*, *e*, and *f*.

is clear that when only a ratio is concerned, the results from all systems must agree.

§ 4. **Scale of Temperature.** Our present scale of temperature, though recently introduced, is equally independent of any particular system of units by which other physical quantities are measured.

The temperature of melting ice is defined as 0° on the centigrade scale; that of condensing steam as 100° under a standard atmospheric pressure, or that which sustains at Paris a column of mercury 76 *cm.* long, and at 0°.[1] At other points temperature is measured provisionally by the indications of a mercurial thermometer made of ordinary glass, the tube being divided into 100 parts of equal capacity between 0° and 100.°

It is assumed that a thermometer reaches, after a time, the same temperature as the bodies with which it is in contact.[2]

§ 5. **Unit of Length.** — The unit of length adopted in nearly all scientific work is the centimetre, or hundredth part of the length, at 0° centigrade, of a standard metre still preserved in the French Archives. This metre was intended to be the ten-millionth part of the distance along a meridian from the equator to the poles, but it was made about $\frac{1}{4}$ of a millimetre too short, the earth's quadrant being now supposed to lie between 10,007 and 10,008 kilometres; being, moreover, subject to shrinkage, though the amount has never been measured. The only absolute determination of the centimetre which we possess is in

[1] See § 5 below; also Table 14.

[2] For a further discussion of temperature see § 74.

wave-lengths of light. It contains, for instance, 16,972 waves of sodium light in air.

§ 6. **Unit of Mass.** — Our unit of mass is the gram, or thousandth part of the standard kilogram of the French Archives, which was intended to be equal to the weight in a vacuum of a cubic decimetre of distilled water at its temperature of maximum density (very near 4° centigrade). In addition to the error in the metre already noticed, the standard kilogram was made about 13 milligrams too light; but if this is taken into account, the gram can easily be reproduced from a given standard of length which has been compared either with the original metre or with wave-lengths of light. (See § 152.)

§ 7. **Unit of Time.** — The unit of time which we use is the second, of which there are 86,400 in a mean solar day. The second depends therefore on the rotation of the earth with respect to the sun. As no change has been detected in the rotation of the earth by comparing it with other astronomical motions, the second would seem to be practically constant. In one second, sound passes through 33,220 centimetres of dry air at 0° centigrade; light through 30 thousand million centimetres of empty space, as nearly as we can tell. From any of these data the second could be reproduced independently of the rotation of the earth.

§ 8. **Absolute System.** — The system followed in this work is that recommended by the British Association, and is known from its fundamental units as the centimetre-gram-second system, often abbreviated C. G. S.

The three units of length, mass, and time are called fundamental, because all other units of this system are derived from them; and they may be called absolute, because they can be reproduced (without the use of any standard) from the general properties of such universal substances as salt, water, and air. It is in this sense only that any system of measurement may be called absolute.

§ 9. **Surface, Volume, and Density.** — Surface or area is measured in square centimetres; volume or capacity in cubic centimetres; density in grams per cubic centimetre. Density in general is defined as the ratio of mass to volume. (See § 154.)

§ 10. **Velocity.** — Velocity is expressed in centimetres per second. It is well to remember that a velocity of one hundred centimetres per second or one metre per second corresponds to a very slow walk, only a little over two miles per hour. It is incorrect to speak of a velocity of so many centimetres, or of so many miles. A railway train may move at the rate of one mile per minute, while a steam roller makes only one mile per hour. Both the distance traversed and the time occupied in so doing are necessary to specify a velocity.

§ 11. **Acceleration.** — Acceleration is defined as the rate of change of velocity,[1] or the change of velocity per unit of time. If a steamer starting from a wharf acquires in one minute a velocity of three miles per hour, in two minutes a velocity of six miles per hour,

[1] For a discussion of what is meant by a change of velocity, see § 105.

in three minutes a velocity of nine miles per hour, etc., increasing its velocity every minute by three miles per hour, we should say that its acceleration amounts to three miles per hour per minute. It would be incorrect to speak of its acceleration as three miles per hour, for a horse and carriage might acquire the same velocity in one second.

It is necessary to state not only the magnitude of the velocity acquired but also the time it takes to acquire it. Since velocity is measured in centimetres per second, and time in seconds, acceleration is expressed in centimetres per second per second. The repetition of the words "per second" in scientific works is not therefore, as is commonly supposed, simply a printer's favorite mistake.

§ 12. **Force.** — The dyne or unit of force is defined as that force which acting on a gram for a second would give it a velocity of one centimetre per second.

A dyne is almost too small a force to be felt. It may be thought of as the weight of a piece of very thin tissue-paper a centimetre square; meaning by weight the force with which, for instance, it presses against the hand. In the same sense a drop of water weighs from 50 to 100 dynes; a man from 50 to 100 millions of dynes.

The dyne can be best represented by means of a delicate spring-balance. The weight of a gram in latitude 40°–45° is shown by such an instrument to be about 980 dynes; at the equator, however, it is only 973 dynes, and at the poles nearly 984. The weight at the centre of the earth would be nothing.

On the other hand a given number of dynes as above defined always stretches the balance to a given mark, whether at the equator or at the poles. Hence we say that the weight of a gram varies,[1] but the dyne, in terms of which we measure it, remains always the same. Force in general is measured as the product of mass and acceleration. (See § 106 and § 153.)

§ 13. **Couple.** — The unit couple is a force of 1 dyne acting on an arm 1 centimetre long, at right angles to it, with an equal and opposite force at the other end of the arm. A couple consists in general of two equal forces acting in opposite directions, not in the same straight line but in two parallel lines, and is measured by multiplying together *either* force in dynes by the arm, or perpendicular distance between the two lines of action. Anything which can twist a body or make it spin contains a couple; anything which can push it or pull it or shove it to one side contains a force. All motions originate either in forces or in couples or in combinations of forces and couples. (See § 113.)

§ 14. **Work.** — The unit of work is the erg, defined as the amount of work done in moving through a distance of one centimetre against a resistance of one dyne. It makes no difference how long it takes to complete the motion; but we assume that there has been no gain or loss of velocity on the part of the

[1] By the weight of a gram is here meant the varying force with which gravity attracts it. This is the proper signification of weight. Some writers, however, use weight in the sense of mass, or quantity of matter. The mass of a gram is by definition constant. See "Elementary Ideas, etc.," by E. H. Hall (published by Sever, Cambridge).

moving body, since that would also have to be taken into account. (See § 121.) Work in general is measured as the product of the force in dynes, and the motion in centimetres; considering of course only the effect or component of the force in the direction of the motion. (See § 119.) When the force acts on a body in the direction in which it is moving, it is said to do work upon the body; when the force opposes the motion, the body is said to do work against the force.

Those who have been accustomed to measure work in foot-pounds (multiplying the motion in feet by the number of pounds which have been raised), may notice that the erg or dyne-centimetre naturally replaces the foot-pound in a system in which all forces are measured in dynes and all distances in centimetres.

While three hundred foot-pounds in England are the same thing as three hundred and one foot-pounds in Brazil, the erg has one great advantage in that it is the same all the world over. Ten million ergs are sometimes called a joule.

§ 15. **Power.** — The practical unit of power is the watt, or ten million ergs per second. A man can easily do the work of 100 watts. One horse-power is rated at 746 watts. It takes about 4.166 watts to generate, through friction, one unit of heat per second. (See below.) A common paraffine candle is equivalent in heating power to 60 or 70 watts; 10 or 12 candles represent a horse-power.

§ 16. **Unit of Heat.** — The unit of heat is the quantity required to raise a gram of water from 0° to 1°

centigrade. It takes about forty-two million ergs to bring this about; more exactly, 41,660,000; hence this number is said to represent the mechanical equivalent of heat. Other substances take more or less (generally less) heat than water to raise 1 gram of them 1° in temperature, and more or less work in proportion. This proportion determines the specific heat of the substance in question. (See also § 86.) Specific heat is strictly defined as the number of units of heat necessary to raise 1 gram of a given substance 1° in temperature.

§ 17. **Unit of Magnetism.** — A unit quantity of magnetism is one which attracts or repels an equal quantity at a centimetre's distance with the force of 1 dyne. There are two kinds of magnetism, positive and negative. Two positives or two negatives repel each other, while positives and negatives attract.

§ 18. **Unit of Electrical Current.** — The absolute C. G. S. unit of electrical current is one which in flowing through a centimétre of wire acts with a force of 1 dyne upon a unit of magnetism, distant 1 *cm.* from every point of the wire.

§ 19. **The Ampère.** The practical unit of current is the ampère or tenth of an absolute unit. A common quart Daniell cell will give a current of about 1 ampère under favorable conditions.

§ 20. **The Ohm.** — The practical unit of resistance is the ohm. It was intended to be the electrical resistance of a wire in which a current of 1 ampère would generate in one second an amount of heat equivalent to 10,000,000 ergs. That is, an engine of 1 watt

power would keep up a current of 1 ampère through such a resistance. In point of fact the standard ohm prepared by the British Association is a little more than 1% too small, and as this error has been kept in our copies, we have to allow for it in our calculations.

The ohm may be remembered as the resistance of about fifty metres of copper wire 1 *mm.* in diameter, or as that of a column of mercury 106 *cm.* long and 1 *sq. mm.* in cross section. The value of the latter resistance at 0° is adopted in France and elsewhere as the legal definition of the ohm. The liquids of a quart Daniell cell usually offer a resistance of about 1 ohm.

The resistance of a conductor in general is numerically equal to the power necessary to maintain a unit of current through it.

§ 21. **The Volt.** — The practical unit of electromotive force is the volt, or that which is required to maintain a current of 1 ampère through a resistance of 1 ohm. A Daniell cell has an electromotive force of about 1 volt.

Electromotive force in general is defined as the ratio of the power (§ 15) to the current. We have seen that it takes one watt to maintain a current of 1 ampère through a resistance of 1 ohm; and that it takes 1 volt to do the same. It will not do to conclude that one volt is the same thing as one watt; two volts will keep up a current of two ampères through one ohm, but four watts will be required. Electromotive force corresponds not to power but to hydrostatic pressure. (See §§ 137–139.)

§ 22. **Intensity.** — There are various other terms a definition of which might be useful here, but it has been thought better to explain each as the necessity arises. The use of the word "intensity" in the sense of concentration is, however, important. By intensity is meant the proportion of one quantity per unit of some different quantity. The force in dynes (about 980) with which gravity attracts each gram of matter is sometimes called the intensity of gravity. Intensity of pressure, generally called simply *pressure*, is expressed in dynes per square centimetre, corresponding to the ordinary use of pounds per square inch. The pressure of the atmosphere is, for instance, about one megadyne per *sq. cm.*, averaging in this latitude about 1.3% more than this. Intensity of stress, or simply *stress* is measured in the same units; as when we say that steel bars break under a stress of eight thousand megadynes per *sq. cm.* In the same way intensity of illumination ought to be expressed, not as it often is, in candle power, but in candle power *per square centimetre* of surface illuminated. Intensity should always be distinguished from quantity in this way. Like *rate* with respect to time, or the word *per*[1] with respect to quantities in general, intensity signifies a ratio or proportion.

[1] Everett's Units and Physical Constants, page 10.

CHAPTER II.

OBSERVATION AND ERROR.

§ 23. **Coincidence.** — Almost every physical measurement involves the reading of a scale of some sort, by means of what may be called an index or pointer. Temperature, for instance, is measured by a thermometer, consisting of a tube of glass with a scale marked upon it, let us say in degrees, and an index of mercury or some other liquid moving up and down the tube. Aneroid barometers, pressure-gauges, clocks, compasses, and galvanometers are read by a hand or pointer of some sort moving over a dial. An ordinary balance has an index, and a small scale behind it to show, when the weights are nearly adjusted, which pan is the heavier, and how much. Spring balances are read by the position of a small index. When the length of a body is measured by the scale on a metre rod, one end of the body is used as the index; or, again, a mark on a sliding scale is used as an index with respect to a fixed scale, and conversely. The above list contains a small part of the various instruments used in physical measurement; but a great part of those from which numerical results are actually obtained. Most observations therefore consist in reading scales of various

sorts, by noticing the point with which the index apparently coincides.

The coincidence of two objects *in position* may be determined with great delicacy by the touch, or the coincidence of two sounds *in time* by the ear; but most observations relate to the coincidence or agreement of two phenomena *both in space and in time*, and can be made conveniently only by the eye.

§ 24. **Classification of Errors.** — It is obvious that mistakes are likely to arise in observation, as when we take a figure 3 for a figure 8; but mistakes of this sort should be distinguished from errors proper. A reasonably small error is more likely than a large one; but a mistake in the thousands is as probable as in the units. (See § 156.)

Errors may be divided into two classes: constant errors, or those which always tend to increase or to diminish a result by a definite amount; and accidental errors, or those which tend sometimes to increase it and sometimes to diminish it. Constant errors can be allowed for if we have sufficient information about them; but no correction can be applied for accidental errors.

For instance, in measuring length, the temperature of a tape, the moisture which it may have absorbed, the strain upon it, and the curvature of the surface measured, all affect the result. It is impossible to predict whether the temperature will be higher or lower, the dampness greater or less, the strain more or less intense than when the tape was graduated. We study accidental errors as we would combinations

of "heads and tails" in tossing coins. No result is entirely free from them. Their influence may be indefinitely reduced (§ 46), but never completely eliminated.

Errors may further be distinguished into three classes: first, errors of observation (§§ 25–30); second, instrumental errors (§§ 31, 32); and third, errors of inference (§§ 33, 34). The various methods of avoiding errors of observation are considered below in connection with the sources from which they arise, the commonest of which are as follows: uncertainty in a point of view (§ 25), the coarseness of a scale (§ 26), the minuteness of the object observed (§ 27), the necessity of observing two different things at the same time (§ 28), the unequal rates at which different sensations are transmitted (§ 29), and the effect of mental impressions (§ 30).

§ 25. **Parallax.** — In many scales where the index is between the graduation and the eye, the apparent position of the pointer is affected by the point of view. The index seems to *slide along* the scale as the eye moves from one end to the other. This phenomenon is called *parallax* (from παρά, along, and ἀλλάσσω, to alter). Clearly to avoid errors from parallax, the eye must be held in a fixed position so as, for instance, to look perpendicularly upon the scale. To this end one of the simplest devices is to use a mirror parallel to the scale and behind it if possible. The eye is placed so as to see its own reflection in the mirror in the direction of the pointer; in this case the line of sight must be perpendicular to the scale.

§ 26. **Estimation of Tenths.** — One may readily distinguish in most cases whether the pointer apparently coincides with a certain mark on a scale, or with the space between two marks; but this is by no means the limit of the eye's accuracy. If the pointer falls between two marks, it is generally possible to decide whether it is half-way between them, or nearer to one than to the other. In other words, the eye is accurate to fourths. It is, in fact, possible to imagine the space between two marks in an ordinary scale divided into at least ten parts, and to decide correctly in the majority of cases in which of these parts the pointer lies.

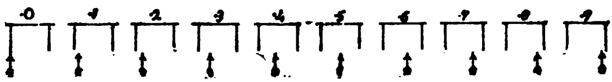


Fig. 1.

The ten diagrams in Fig. 1 show the relative positions of a pointer dividing the space between two marks into various proportions, the figures indicating the number of tenths to the left of the pointer in each case. A close study of such diagrams will in a short time justify the division of spaces into tenths by the eye. It is assumed henceforth that in the case of any index and scale under favorable conditions, the reading is expressed in tenths of the smallest divisions. The estimation of tenths is not confined to the eye. It will be found that the ear is equally reliable. Thus the time between two ticks of a clock can be divided into tenths, so that the occurrence of a sound can be determined with practice to a tenth of a second.

§ 27. **Mechanical Devices.** — When a space or line is too small to be seen we generally resort to a lens or microscope, as in Experiment 19; but there are various other devices to measure small distances. One of the most delicate tests of the adjustment of the four points of a spherometer to the same plane is the noise made by rocking the instrument from side to side, (see Experiment 20), and an electrical contact is sensitive to a change of distance which the eye fails to see (see Experiment 65). The motion of the top of a vacuum chamber in an aneroid barometer is magnified by a system of levers, and finally by a chain passing round a small axle so as to render the smallest motion perceptible. When a motion is too rapid to be seen by the naked eye, we may still often observe it through some optical device. An instantaneous view, for instance, will show the body as if at rest, and in the case of periodic motion a series of instantaneous views may give it an apparent motion so slow that it is easily observed (see Experiment 51). Again motion may be made to record itself by marking on a moving surface. The vertical motion of a barometer is thus recorded by means of a pen on a piece of paper moving by clockwork horizontally beneath it. This method is called graphical. Any instrument which moves uniformly so that time can be accurately recorded in this way is called a chronograph, literally a *time-writer* (from χρόνος, *time*, and γράφω, *to write*). A chronograph can be used to record the vibrations of a tuning-fork, even one which emits the highest or fastest audible note.

Similar results can be obtained when the pen is not moved directly by the tuning-fork or moving body, (see Trowbridge, New Physics, Experiment 155), but indirectly through the aid of electricity, and various electrical devices may be employed to magnify the effects of small intervals of time, and thus detect the smallest variation from coincidence (see ¶ 147). Optical, Graphical, and Electrical Devices include the principal methods of aiding observation.

§ 28. **Use of Two Senses.** — When we wish to observe two things in different places at the same time we often resort to the use of two senses. The Eye and Ear method[1] consists, for instance, in the use of the eye to watch one moving body while the ear listens for the occurrence of a sound defining the motion of another.

This is the method by which one ordinarily compares his watch with a striking clock or with a noon gong. The sense of touch is used by the engineer to help him count correctly the revolutions of a wheel without looking off his watch, and a variety of methods can be devised by which two or more senses bring together from different sources a knowledge of what is taking place at different places at a given time. The use of two senses often obviates the necessity of employing complicated mechanical devices.

§ 29. **Personal Equation.** — It is generally found that the eye is quicker than the ear to report what is taking place, but the difference is greater in some persons than in others. Thus if two persons were

[1] See Pickering's Physical Manipulation, § 15.

to estimate at what time the report of a cannon is heard, one would tend always to return figures greater than the other, let us say by several hundredths of a second. Such a difference, however small it may seem, might seriously affect a determination like that of the velocity of sound, and is a perpetual source of annoyance in astronomy. The allowance which each *person* must make to produce results *equal* to the true or average result is called his *personal equation.* It is not specially considered in this course of measurement, being eliminated together with what is called "zero error," as explained in § 32.

§ 30. **Effects of Anticipation.** — One of the most dangerous sources of error in observation lies in the habit of anticipating results. Experience shows that under the influence of a strong expectation, the eye is not only incapable of estimating fractions correctly, but that it becomes blinded to gross errors, — pronounces weights, for instance, equal when the balance-beam is not free to move; reads sixty-odd centimetres instead of seventy-odd, several times in succession. It is sometimes necessary to prepare one's self by calculating beforehand — particularly in astronomy — the values which one expects to observe; but independence of observation is obtainable only in ignorance of the meaning of the indications which one records, and particularly in ignorance of the fact whether the values obtained are likely to be too great or too small.[1]

[1] The teacher may amuse himself at the expense of his class by determining the effects of "gravitation" towards various values which he may choose to suggest.

For these reasons the following rule will be found useful: Take your observations first; second, give a copy to some one else; third, reduce them; fourth, report the result; and fifth, inquire what values others have found.[1]

§ 31. **Instrumental Errors.** — Without any fault on the part of the observer, errors often arise through the imperfections of the instruments which he employs. These may be divided into two classes: first, errors of adjustment, as when two parts are not exactly parallel or perpendicular; and second, scale errors, for instance, irregularities in a graduated rod or in a set of weights.

The various tests which have been devised to correct errors of adjustment will be described in connection with the several instruments to which they belong. Scale errors may arise either from a change in, or from the original misplacement of, certain fixed points; like the "freezing" and "boiling" points of a thermometer, or from inaccurate calibration. They are avoided in general as explained in § 36. The commonest error of this sort is a misplacement of the zero of a scale.

§ 32. **Zero Error.** — When the greatest care has been taken to read one end of a scale correctly, an error often arises because the other end is out of adjustment. The graduation of a tape measure seldom begins at the ring, and yet it is common to see

[1] The examination of substances whose composition is known only to the teacher — or to the apothecary — will afford a sufficient opportunity to test the application of this rule.

distances measured by professional mechanics as if this were the case. It is always well, even when no error of this sort is suspected, to confirm an observation by taking two others, the difference between which should agree with a previous result. Thus the length of a pencil might be found by laying it along the middle portion of a metre-rod instead of making one end of it even with the rod, and in this manner, even if the end of the rod were worn away or broken off, the true length of the pencil would be discovered. This is called the method of difference.

The error due to the inaccuracy of the beginning or zero of a scale is called zero error, and it is necessary to guard against such errors in general. It should be borne in mind that every measurement, like that of length, depends upon at least *two observations*, or their equivalent; and that the accuracy of one is just as important as that of the other. However evident it may seem to be that if the quantity which is being measured were taken away, the index would point to zero, it is continually necessary to test the truth of this fact. The balance when both pans are empty, from a slight dislocation of one of the knife-edges, often tends to one side; springs do not always return to their original length after stretching, owing to a permanent set; galvanometer-needles do not always point north and south when the current is cut off, — a bunch of keys may perhaps account for the variation.

§ 33. **Errors of Inference.** — One must distinguish carefully between what he sees and what he infers.

It would be impossible to state any general principle by which errors of inference may be avoided; but in order to correct them, it is often necessary to refer to the original observations from which the inferences have been drawn. Hence the necessity of preserving the records, however rough in form, made at the instant when a given phenomenon occurs. The turning-points of an index should for instance be recorded, and not simply the position where it is *inferred* that the pointer will come to rest; or, if at rest, its actual position should be noted, not the weight which one *infers* would produce an exact adjustment. Again, the reading of a standard English barometer should be written down first in inches, and afterwards reduced to centimetres.

In addition to the observations necessary to a given measurement, every circumstance should be noted which may have a possible influence on the result. The appearance of air-bubbles, in hydrostatics, may, for instance, determine the relative accuracy of different weighings. The *time* of an experiment enables us to supply the barometric pressure, roughly, at a later date, by consulting a weather report. An exact description of *place* may furnish a subsequent clue to the magnetic deviation. We must also be able to *identify the instruments* which we have used, if we would confirm the inferences drawn from their indications. In fact, the severest test of a laboratory note-book must occasionally be applied, namely, one's ability to repeat with it a measurement from beginning to end.

It is important to the clearness of one's notes to enter actual observations in one place and calculations in another. Errors in reasoning are almost always due to confusion in regard to the nature of the quantities dealt with. The student should learn from the first to *write opposite each number what that number represents.* Every figure necessary to the calculation of a result should be preserved for future reference,—even those which enter, for instance, into ordinary multiplication or division. In calculation, as in observation, corrections are most easily made in those records which are most complete.

§ 34. **Logical Analysis.** — The use of logical analysis for the purpose of discovering unknown sources of error is seldom dwelt upon by writers on physical measurement. It is, however, obvious that the reduction of results may be thrown into the form of a demonstration; and after errors of observation have been allowed for, if the reasoning is correct, unknown errors must lie in the assumptions. It is, therefore, important to determine what these assumptions are.

Thus in the case of a Nicholson's hydrometer we reason that since the weight required to sink it to a given mark is, let us say, 30 grams at 10 o'clock without a load, and 10 grams at 11 o'clock with a load, *assuming that a given weight always produces a given result*, the apparent weight of the load must have been equivalent to that of 20 grams, according to the set of weights.

Both theory and experiment show that the assump-

tion is true only when the temperature of the water is constant and when various other conditions are fulfilled. Changes in quantities which we unconsciously assume to be constant are a frequent source of error in physical measurement.

CHAPTER III.

GENERAL METHODS.

§ 35. **Methods of Trial and Approximation.** — The ordinary method used in the arts for testing the diameter of a wire is to fit it into a series of slits, each narrower than the one before it, until one is found which the wire cannot be made to enter. A series of trials, *systematically arranged*, leads very quickly to the desired result. The trials are of course limited in practice to a set of slits of *about* the same width as the wire. The first trial should be made with one near the *middle* of such a set; for if this slit be two small, little time is lost, while, if it be too great, only half of the set remains to be tried. In any case, we find out which half contains the slit fitting the wire. The second trial should be made about the middle of this half. A quarter of the original set then remains to be tried. A third trial is made near the middle of this quarter, &c.[1] By thus continually halving the limits between which an unknown quantity has been found to lie, its precise value may be determined with the smallest possible number of trials.

In certain cases, we have no clew whatever to the magnitude of the quantity which we desire to meas-

[1] 10 halvings reduce a quantity in the proportion 1024 : 1; 20 halvings reduce it in the proportion 1,048,576 to 1.

ure.[1] A bad electrical connection may, for instance, amount to a small fraction of an ohm (§ 20), or to several million ohms. We begin, therefore, by comparing it with a standard which comes in the *order* of its magnitude, as expressed in the decimal system, about half-way between the extreme limits within which measurement is possible. With an apparatus capable of measuring resistances from 1 to 1,000,000 ohms, we should first try, for instance, 1000 ohms. If 1,000 were too great, we should next try 10 ohms; and if this were too small, 100 ohms. Very few trials are usually required to determine the order of magnitude to which any measurable quantity belongs.

When the result of a given trial can be anticipated, this trial is needless, and should be omitted from the series which would otherwise be made. We begin, for instance, by comparing an unknown weight with a standard as nearly equal to it as possible. Then a second standard or combination of standards is tried. A good practical rule is to try weights in their *order of magnitude*,[2] each weight in a set being generally about half or twice as great as the one next above or below it. If the first estimate be reasonably close, the result of following this rule will be *probably* to turn the balance. It is evidently useless to make

[1] If there is any doubt whether the apparatus which we employ is capable of measuring the unknown quantity, it is well to compare this quantity at the start (1) with the smallest and (2) with the largest available standard. A reversal of the indication of an instrument obtained in this way is valuable, because it shows that the instrument is in working order and that a measurement can probably be made.

[2] See Pickering's Physical Manipulation, vol. i., page 48.

changes in weight which are *certain* to turn the scales. If, accordingly, two weights appear by any chance to be nearly balanced, a much smaller change should be made.

The method of trial employed in weighing is essentially the same as that used in finding the diameter of a wire. When an unknown weight has been found to lie between two limits, in the absence of any indication which limit is the nearer, we try a weight as nearly half-way between these limits as convenience will allow. To avoid, however, complicated combinations of a set of weights, we follow this rule only in so far as may be possible by the addition of one weight at one time or by the substitution of one weight for another (see Exp. 1, ¶ 2). A similar method is employed with a set of electrical resistances (Exp. 86).

A great many physical instruments show only which of two quantities is the greater, without indicating how great the difference is between them. The best results are obtained with such instruments by the methods of trial described above. When, however, it is possible to calculate approximately the magnitude of an unknown quantity from the results of one or more trials, this method may be greatly shortened. Thus, by observing how much the temperature of a mixture is lowered by cooling one of the ingredients a certain number of degrees, we may calculate roughly how many degrees this ingredient must be warmed or cooled to bring about any desired temperature in the mixture (see ¶ 99, I.) A series

of trials may be arranged in this way so that each is much closer than the one before it. This is called the "method of trial and error," or the "method of successive approximations" (Pickering, Physical Maniplation, vol. i., page 10).

§ 36. **Methods of Graduation and Calibration.** — (1) PRODUCTION OF A SET OF STANDARDS. The purposes of physical measurement frequently require the production of a set of standards, each of which must be an accurate multiple of a given unit. Let us first suppose that a suitable standard unit can be obtained. The first step is to make an accurate copy of this unit. This requires the aid of some instrument capable of detecting the slightest difference between two quantities (§ 42). With such an instrument, the copy is made as nearly as possible like the original by the method of trial and error (§ 35). Let us call the original *A*, and the copy *B*. The two are then combined, and by the aid of the same instrument two standards, *C* and *D*, are prepared, each equal to the sum of the standards *A* and *B*, — that is, 2 *A*, nearly. There are then two ways of producing a standard *E* equal to 5 *A*. We may combine *C*, *D*, and *A*; or *C*, *D*, and *B*. The former is preferred because, in employing the original standard *A*, instead of a copy of it, there is one less chance of error; see (4). By combining *A*, *C*, *D*, and *E*, two standards, *F* and *G*, may be produced, each equal to 10 *A*, nearly. There are, then, two ways of making a standard, *H*, equal to 20 *A*. One way is to combine *F* and *G*, the other is to combine one of these — *F*, for instance — with *A*,

C, D, and E. The latter is preferred because it makes use of the sum of the standards (A, C, D, and E) instead of a copy of this sum ; see (4). In a similar manner, we may prepare standards of the magnitudes 50 A, 100 A, &c.

Let us now suppose that a suitable standard unit cannot be obtained, and that the only available standard is some multiple of this unit, as for instance 1000 A. We then assume a provisional unit of any magnitude, x, and construct a series of provisional standards, of the magnitudes $2\,x$, $5\,x$, $10\,x$, &c., until we reach a value as great as the given standard. Then by the method of trial (§ 35) we find how many provisional units are equal to this standard. The values in the provisional series are now known; and by making and copying the proper combinations of this series, we may construct a series of standards which are more or less accurate multiples of the standard unit which we desire to represent.

It would be out of place to consider here the mechanical operations by which graduated scales and circles are produced. Standards must in general be subjected to a series of tests, as will be explained in (2) and (3).

(2) TESTING A SET OF STANDARDS. The construction of a set of standards may be considered as a first step toward the accuracy of results; but no matter how carefully such a set may be prepared, it is almost always possible to detect a difference between any two combinations of nominally the same value. It is generally easier to measure and allow

for such differences than it is to avoid them. A set of standards may accordingly be tested by a series of comparisons involving essentially the same combinations as those employed in processes of construction; see (1). Instead, however, of comparing H with $A + C + D + E + F$, we should in practice compare it with $F + G$, since the latter combination $(F + G)$, being more frequently employed,—see (4),—needs to be known with greater precision. We prefer, in fact, tests involving the use of the smallest possible number of standards.

In addition to a series of comparisons by which we may determine the relative values of different standards in a set (see Exp. 7), either the sum of the set or one or more of the larger standards which it contains should be compared with some standard of known value.

(3). CALIBRATION. Variations in the bore or "calibre" of a tube may evidently give rise to errors in the estimation of its contents by means of a scale attached to the tube. Any process by which such errors may be eliminated is properly called "calibration" (see ¶¶ 68 and 71, Exps. 25 and 26). This term has, however, been extended to the correction of a scale of any sort.

To obtain accurate results with an ordinary scale of length, it is obviously necessary that all the intervals of a given nominal value should be equal, or at least that they should not differ from one another by a perceptible amount. A simple way to test the accuracy of a scale is to lay beside it another scale

graduated in exactly the same manner. Let a, b, c, &c., represent the spaces on one scale, and a', b', c', &c.,those on the other scale, and let us suppose that the division lines between these spaces are opposite one another. Then $a = a'$, $b = b'$, $c = c'$, &c. The first scale is then to be moved along so that a may come opposite to b'. If the division lines again come opposite, $a = b'$, $b = c'$, &c. Since in the first case $b' = b$, and in the second case $b' = a$, it follows that $a = b$, and in the same way all the intervals, a, b, c, &c., must be equal.

To test, accordingly, the uniformity of the millimetre divisions on a metre rod, we place two such rods side by side, then we move one of them along 1 *mm.* The equality of the centimetre spaces may be similarly established by moving one of the rods 1 *cm.*, and the decimetres may be tested by moving the rod 10 *cm.* It must not be imagined, because there is no perceptible irregularity in the millimetre divisions, that there can be none in the centimetre or in the decimetre divisions. If for instance, the first 100 *mm.* spaces on each rod were longer than the next 100 *mm.* spaces by $\frac{1}{100}$ *mm.* in each case, we should hardly notice the difference between them; but the first decimetre would be longer than the second by a whole millimetre. For a similar reason it is important to compare the two halves of a scale,—see (4),—the two quarters into which each half may be divided, &c. (see Exp. 24).

The relations between the magnitudes compared in testing a graduated scale or circle are, to a certain

extent, the same as in the case of a set of standards; see (2).

When there is no other way of testing the relative values of different scale indications, we do so by measuring with the scale different quantities bearing known ratios to one another (Exp. 96); the scale may then be used for relative indications. Every scale which is to be depended upon for absolute results must be compared in one case at least with a standard of known absolute value.

(4) Direct and indirect processes. The correction of a scale or of a set of standards usually depends, as we have seen, upon a series of comparisons, each of which must introduce a certain chance for error in the result. Standards should evidently be compared *directly* with the originals which they are intended to represent, whenever it is possible to do so, rather than with copies of these originals. Again, the two halves of a scale should be compared *directly* with one another, not indirectly, by means of the spaces into which they are subdivided; see (3). Short and direct methods of comparison are always preferable, other things being equal, to long and indirect processes.

It will be seen from (1) that in certain cases the sum of several weights is more reliable than a single weight of the same nominal value. In general, however, each weight in a set is subject to a certain error, especially when the set has been copied from another set, or when the weights are worn or corroded. In such cases the chances for error in weighing increase

in proportion to the number of weights which we employ. For this reason, as well as for convenience in manipulation, we make it a general rule to use as few weights as possible. Further illustrations of the principles which underlie this rule will be found in § 38.

§ 37. **Methods of Subdivision.**[1] — The subdivision of a scale or of a set of standards may be carried theoretically to almost any extent by ordinary methods of graduation (§ 36); but there is always a practical limit to the process. The smallest quantity actually indicated by a given instrument is called the "least count" of that instrument. Errors due to "least count" may easily arise. Their influence on a result may be lessened by methods of multiplication or repetition (§ 39) or by methods of "least error" in general (§ 38). It is, nevertheless, desirable that the "least count" of an instrument should be reduced to the smallest practicable amount.

Even with the best analytical balances, weights smaller than 1 milligram are seldom employed. The fractions of a milligram are usually estimated by means of a "rider" or small weight sliding along a graduated scale on the beam of a balance. It has been found similarly impracticable to make use of standards of electrical resistance less than one tenth of an ohm. Fractions of the smallest available standards are estimated in general by methods of interpolation (§ 41).

[1] References in this edition to the Method of Multiplication or Repetition should read § 39, not § 37.

In the measurement of length, there are certain methods of subdivision by which the least count of a scale may be greatly diminished without a proportionate increase in the number of divisions. Thus a centimetre scale 1 metre long, requires for its production 100 lines besides the zero; but if the first centimetre be divided into 100 parts, we may with 200 lines measure any length less than a metre to a tenth of a millimetre.

When this method of subdivision is employed, the application of corrections for errors in graduation (§ 36) is comparatively simple, since a given measurement can be made in only one way. We lose, however, the advantage which is sometimes gained by making measurements in different parts of the scale, and averaging the results (see § 46). For this reason there would be an obvious advantage in using a short movable scale very finely divided, in connection with a scale of centimetres. This principle has been applied in the construction of various sliding-scales or gauges. It is found, however, impracticable to read any scale with the naked eye unless the divisions are at least $\frac{1}{5}$ of a millimetre apart. The use of sliding scales was therefore very limited until Vernier showed how, by a slight modification in these scales, comparatively accurate results could be obtained. The divisions of a Vernier scale are made nearly but not exactly equal to one or more main-scale divisions.

A common form of Vernier gauge consists of a fixed scale in millimetres and a sliding piece with ten or eleven marks, each nine tenths of a millimetre

from the next (see Fig. 2). The first of these, numbered 0, points out the reading of the instrument, in millimetres, upon the main scale, just as if there were no "vernier." It comes opposite a millimetre mark only when the reading is a whole number of millimetres. In this case the next mark on the vernier (No. 1), being $\frac{9}{10}$ *mm.* further on, falls $\frac{1}{10}$ *mm.* short of the nearest main-scale division; No. 2 falls $\frac{2}{10}$ *mm.* short, and so on. Hence if the sliding scale be moved along $\frac{1}{10}$ *mm.*, the mark No. 1 will come opposite a mark on the main scale (not the one nearest the zero of the wire), and if the vernier is moved $\frac{2}{10}$ *mm.* along, mark No. 2 will be exactly opposite still

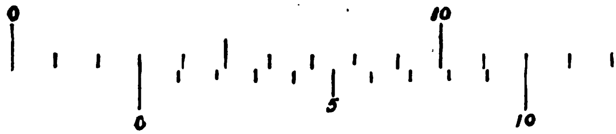


FIG. 2.

another mark on the main scale. In the same way Nos. 3, 4, 5, &c., will come opposite various marks in the main scale, when the vernier is respectively $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, &c., *mm.* beyond the original position. Obviously we have only to find the number of the vernier line which is opposite a line on the main scale (no matter which) to determine the number of tenths of a millimetre between the zero of the vernier and the line just below it on the main scale.

The same principle holds in the case of any vernier. By a series of steps, easily counted, the spaces on the vernier gain or lose one space with respect to the main scale. The reading of the main scale is

thus practically divided into as many parts as there are steps in the gain or loss of one space.

It often happens that in comparing the vernier and the main scale, no two lines are found to be exactly opposite, so as to form a single continuous line; instead, two lines are found, which, though nearly continuous, show, when closely examined, more or less dislocation. We then estimate by the eye the relative amount of dislocation in each case, and reduce the result as accurately as possible to decimals. Thus if in a vernier the third and fourth lines are equally dislocated, the reading is .35; if the third line is only

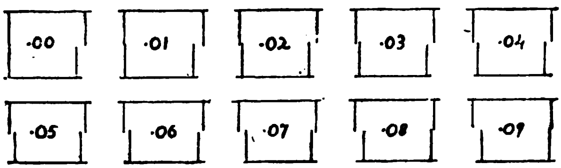


FIG. 3.

one fourth as much dislocated as the fourth, then the reading is .32. By reference to the diagrams in Fig. 3, it will generally be possible to express the reading of the gauge to hundredths of a millimetre, and with almost as much accuracy as if the vernier contained a hundred lines.

The use of a vernier for the subdivision of a scale is closely related to the method of coincidences (§ 40), and may be considered also as one of the various methods of interpolation (§ 41) by which fractions of the smallest available standards are customarily estimated.

§ 38. **Methods of Least Error.** — It is desirable in physical measurement that observations should be accurate; it is equally desirable that the conditions under which they are made should be favorable for the exact determination of results. There are certain general principles by which experiments are, when possible, arranged so that a given error in the observations may cause the least possible error in the result. Any method in which these principles are applied may be called a method of least error.

The advantages of direct methods of comparison have been already pointed out (§ 36). We prefer, in general, determinations which depend upon the fewest data, assume the fewest laws, and make use of the fewest and best-known physical constants. The present section is devoted especially to the relations which should exist between physical instruments and the quantities which they are used to measure.

The delicacy of most instruments is somewhat diminished by an increase in the magnitude of the quantities measured, but *not in proportion* to this increase. The best results are accordingly obtained with quantities nearly as great as the capacity of the instrument will admit. We employ, for instance, large quantities of a substance in determinations of specific gravity by means of a balance. On the other hand, it would be impracticable to measure accurately the weight of copper deposited (Exp. 81) on an electrode weighing several thousand times as much as the deposit in question; for a balance capable of weighing the electrode would not be sensitive

enough for the deposit. While, therefore, it is desirable to increase the deposit of copper, the weight of the electrode should obviously be diminished. We avoid, in general, determinations of the difference between two nearly equal quantities depending upon observations of the quantities themselves. Such differences should be measured *directly* if possible (§§ 41, 42).

Some instruments are particularly adapted to measuring quantities of a given magnitude. A tangent galvanometer, for instance, gives the best results with electrical currents which deflect it 45°. Let us suppose that when three turns of wire are used, the needle points to 26°; with six turns, to 45°; with 12 turns, to 63°. An error of observation equal to + 1° would give 27° instead of 26°, 46° instead of 45°, and 64° instead of 63°. Now the results depend upon the *tangents* of the observed angles (see Exp. 78). The tangents of 26° and 27° differ (see Table 5) by about 4.4 %, and the tangents of 63° and 64° differ in the same proportion; but the tangents of 45° and 46° agree within 3.6 %. We should obviously employ 6 turns of wire in preference to 3 or 12.

In making selections or modifications of the instruments which we employ, we must consider, in general, the nature of the formulæ by which the results are to be reduced. It will be found, for instance, that a 1 % error in a quantity causes an error of about 2 % in estimating the square of that quantity but only about ½ of 1 % in the estimation of its square root

(see § 57). We prefer, accordingly, determinations depending on roots rather than on powers of the quantities directly observed. The relative value of different determinations must be judged, not by the accuracy of the observations, but by that of the results.

The principles of "least error" may require, under certain circumstances, the use of the method of multiplication or repetition (see § 39), the method of coincidences (see § 40), or the method of reversal or interchange (see § 44).

§ 39. **Methods of Multiplication and Repetition.**[1] — It would be impossible to weigh a single drop of water very accurately on a coarse balance; but if we knew under what circumstances the drop was formed it might be possible to produce a thousand drops of almost exactly the same size, and by finding their combined weight to arrive at that of a single drop.

The error in measuring 1000 drops may not be perceptibly greater than in the case of a single drop, and since in the process of reduction this error is divided by 1000, we may obtain at least a comparatively accurate result. The use of any means for increasing the magnitude of a quantity in a given proportion for the purpose of finding a more accurate measure of that quantity constitutes in general a "method of multiplication." The value of such methods evidently depends on the accuracy with which a quantity may be reproduced as compared with the accuracy of a direct measurement.

[1] References in this edition to the Method of Graduation or Calibration should read § 87, not § 39.

We may find, for instance, the weight of mercury required to fill a capillary tube by emptying the contents of the tube several times in succession into a vessel, in which the mercury is collected and weighed. The same method could not, however, be employed with water, on account of the considerable portion which sometimes adheres to the tube.

The method of multiplication is often used in the determination of times of vibration; for it may be proved mathematically (see § 111) that successive vibrations executed under certain conditions do not differ by a perceptible amount. The rate of a pendulum should accordingly be determined by a long series of observations. Such a series may be extended, by a system of mechanical counting, for days or even for months. There must evidently be no break in the series. The method of multiplication is applicable only to *consecutive intervals* in the measurement of time.

The method of multiplication is sometimes used for the estimation or detection of a series of small impulses given to a pendulum or to a vibrating needle at the middle point of a swing, so that the effects may be added together. A large allowance must sometimes be made for the effects of friction, or other causes tending to destroy the motion. For the "method of multiplication and recoil" see Kohlrausch, Physical Measurement, Art. 76.

The method of multiplication is applied in the construction and use of an ordinary galvanometer or "multiplier," the object of which is to increase the

effect of an electrical current in a known or measurable proportion. Methods of multiplication are also applied in the measurement of length.

There are various mechanical devices by which a body may be moved in a straight line through successive distances, each equal (or nearly equal) to its own length. We have an example in the ordinary method of measuring distances with a rod or chain. This is, however, more or less inaccurate on account of the uncertainty of the marks which show where the ends of the measure are placed. One method by which greater precision may be obtained is to place a block end to end in front of a measuring rod, then to remove the rod, to place a second block behind the first, just touching it, then to remove the first block and to put the rod in front of the second block. This process is then repeated over and over until the length of the rod has been multiplied, or, as we say technically, "repeated," a sufficient number of times. By this means very long distances may be quite accurately measured even with a short millimetre scale. This and similar methods are properly called "methods of repetition."

Methods of repetition are frequently used in the measurement of angles. Let us suppose that a given angle, cut out of thin metal, reaches from the zero of a circle, graduated in degrees, to a point between 40° and 41°; and that by some method of repetition similar to that just described, the angle is found to reach from the last point (between 40° and 41°) to one between 80° and 81°, &c. We should obtain in this

way a series of observations like the following: — 0°, 40° +, 80° +, 120° +, 160° +, 200° +, 240° +, 280° +, 320° +, 360° +, 401° — 441° —, &c. We see from any two successive observations that the angle must lie between 40° and 41°, but we have no means of estimating the fraction of a degree over 40. If however, we consider the first and last observations, we see that the angle must be less then $\frac{1}{11}$ of 441°, which gives $40\frac{1}{11}$ as the superior limit of the angle. In other words, the angle becomes known within $\frac{1}{11}$ of a degree. By considering two observations which differ by 360° (or any multiple of 360°) we escape from a great variety of errors by which the results obtained with graduated circles are apt to be affected. A method by which we may utilize, not simply the first and last, but nearly all of a series of consecutive observations will be considered in § 61.

§ 40. **Method of Coincidences.** — We have seen (§ 39) that some lines on a vernier come almost exactly opposite the lines nearest them on the main scale, while others do not. In the same way, when any two scales are compared together, cases of more or less approximate "coincidence" usually occur. Every fifth inch on an English scale coincides, for instance, as nearly as the eye can judge, with every 127th division on a millimetre scale. We should evidently prefer to calculate the length of the inch in millimetres from a case of perfect coincidence than from one where a given number of inches was found to be greater or less than a given number of millimetres by a fraction which could only be estimated by the eye.

The method of coincidences may be used with advantage to avoid errors due to "least count" (§ 37) in the comparison of any two sets of standards of the same sort, no matter what kind of physical quantity they represent. 11 Troy ounces happen, for instance, to balance 342 grams within a few milligrams. With two ordinary sets of weights, the smallest of which are 1 ounce and 1 gram respectively, it is possible accordingly, to find the value of the Troy ounce in grams within a small fraction of a milligram.

The most important application of the method of coincidences is, however, in the comparison of intervals of time. Let us suppose that two pendula differ slightly in their rates of oscillation, so that one gains slowly upon the other, and that they start together at a given point of time. After a certain number of oscillations have been executed by one of the pendula, the two will be swinging in opposite ways, and again after a given number of oscillations, they will be swinging the same way. The relative rate of oscillation may be accurately determined by counting the number of oscillations in question. If, for instance, the faster pendulum makes n vibrations between two successive coincidences, the slower pendulum must make $n - 1$; hence the relative rate is $n \div n - 1$. Let us suppose that through an error in observation $n + 1$ oscillations were counted instead of n; the relative rate would then be estimated as $n + 1 \div n$. The error committed would therefore be,

$$\frac{n+1}{n} - \frac{n}{n-1} = \frac{n^2-1}{n^2-n} - \frac{n^2}{n^2-n} = \frac{-1}{n^2-n}.$$

If n is moderately large such an error would be inappreciable.

§ 41. **Methods of Interpolation.** — We have seen that errors due to the "least count" of an instrument may be almost indefinitely reduced by the methods of multiplication, repetition, and coincidences (§§ 39, 40). Such methods cannot, however, always be applied. The value of an observed quantity, q, is usually found to lie between two limits, one A, the other $A + a$, where a represents the "least count" or smallest change which can be produced in a set of standards. That is, we have —

$$A + a > q > A.$$

If more precise results are required, we seek some instrument or indicator by which we may estimate, relatively at least, the differences between the quantity q and the two nearest values of the standards, A and $A + a$, with which we are able to compare it.

The sensitiveness of any instrument used as an indicator may be defined as the number of scale divisions by which its reading changes when the smallest possible change (a) is made in the standards. We will first suppose the sensitiveness to be known. Let the quantity q be compared with the combination of standards (A) just below it in magnitude, and let the indicator show a motion of x scale divisions. Then since s divisions correspond to the quantity a, we may infer that x divisions must correspond to x s^{ths} of a, hence the true magnitude of q is —

$$q = A + \frac{xa}{s}.$$

In the same way, if the indicator shows a motion of y scale divisions when the quantity q is compared with the combination of standards $(A + a)$ just above it, we have —

$$q = A + a - \frac{ya}{s} = A + \frac{(s - y)\,a}{s}.$$

By comparing this equation with the last, we see that x must be equal to $s - y$, or —

$$x + y = s.$$

The last equation enables us to calculate the sensitiveness of any indicator from two deflections, obtained as stated above. The value of s may vary according to circumstances. The special value here determined is the sensitiveness of the indicator to a change of the magnitude a in the quantity q. The process of estimating a quantity (q) from the relative differences (x and y) separating it from two magnitudes (A and $A + a$) between which it lies is called "interpolation" ("putting in between").

We have instances of the method of interpolation when, in the use of a Nicholson's Hydrometer (Exps. 2, 3, 4), the distances of a certain mark above or below the surface of the water are used to estimate fractions of a centigram, or when in the use of a vernier (§ 38), the relative dislocations of two lines are used to estimate hundredths of a millimetre. The vernier itself may be considered as one means of interpolation. The use of a "rider" (¶ 259) enables us to determine weights exactly by interpolation even if the weight of the rider be unknown. The

indications of the pointer of a balance afford another means of interpolation in weighing (see ¶ 20). The deflections of a galvanometer are similarly used (see Exp. 93) to estimate small differences between two opposing electromotive forces which we seek to bring into equilibrium.

§ 42. **Null Methods.** — Most physical quantities cannot, like scales of length, be directly compared with one another, but are measurable only through the effects which they produce upon some instrument. Electrical currents, for instance, are usually determined by their action upon the needle of a galvanometer. When two effects lie in the same direction, they are generally compared by the method of substitution (§ 43). It is, however, frequently desirable to *oppose* two effects, especially when they are nearly equal, in order that the difference between them may be directly measured (see § 38). In weighing with a balance, the effects of two nearly equal weights upon the instrument are thus opposed. Any method by which two effects may be made to neutralize or *annul* each other may be called a *null method.*

In electrical measurements, the term "null method" is usually applied to cases where two equal electromotive forces are opposed to one another so as to produce no current through a delicate galvanometer. Null methods are characterized by the fact that the conditions of perfect adjustment between the different parts of an apparatus is shown by the *absence of any indication* on the part of some delicate instrument.

Null methods do not require the use of instruments which indicate the magnitude of the difference between two nearly equal quantities, although it is often convenient to employ such instruments for purposes of interpolation (see § 41). It is only necessary that an instrument should show whether two quantities are equal or unequal. Being used solely to detect differences, such instruments are sometimes called "detectors." They take the place of sight, touch, or hearing (§ 23) with quantities which do not affect these senses.

There are two principal precautions to be observed in the use of null methods. One is to make sure that the instrument employed responds to the slightest variation in either of the two quantities which are compared; the other is to test the zero of the instrument (§ 32). Errors may occur, for instance, from a break or from a cross-connection in the circuit of a galvanometer; for in this case there will be no perceptible deflection, no matter how great may be the difference between the electromotive forces which are compared together. Again, if the needle of a galvanometer does not naturally point to zero, it may require a current to make it do so (see Exps. 89, 90). We should infer wrongly in such a case that the current had been reduced to zero.

Null methods usually depend upon the use of very sensitive instruments; but the conclusions which we draw from them, being founded upon purely negative indications, must be examined with great care. Null methods are considered highly desirable on account

of their precision, but they need in general some kind of confirmation.

§ 43. **Method of Substitution.** — The "method of substitution" is the fundamental method for testing any result the accuracy of which is questioned. It is so called because a known quantity is *substituted* for an unknown. Thus if the resistance of a wire has been found by means of any electrical combination sensitive to variations in resistance (Exps. 86, 87) to be equivalent to 10 ohms, we have only to substitute for it a resistance *known* to be 10 ohms to find whether there is or is not any error in our work.

The scale of a densimeter (Exp. 15) may be tested by substituting a liquid of known, for one of unknown density, or the indications of a volt-meter (Exp. 96) by substituting known for unknown electromotive forces. The method of substitution is often used where no other is possible, as in Experiments 2, 3, and 4. It depends upon the principle that two quantities must be equal if they can be substituted one for the other without affecting a combination sensitive to variations in the magnitude of the quantities in question. Evidently the known and unknown quantities thus compared should be as nearly equal as possible.

In the method of substitution, as in null methods (§ 42), we must make sure that the instrument which we employ is free to move, since otherwise very unequal quantities might apparently produce the same effect upon it. The "zero-error" of an instrument (§ 32), and instrumental errors in general (§ 31), are

usually eliminated by the method of substitution. Borda's method of weighing is to counterpoise accurately an unknown weight in one pan of a balance with material of any sort in the opposite pan, then to substitute known weights for the unknown until an exact balance is again established. In a similar manner, when, in electrical measurements, null methods (§ 42) are employed, it is well to test the accuracy of the results by substituting known for unknown quantities. The use of the method of substitution in combination with null methods is the most general way of obtaining both accuracy and precision in physical measurement.

§ 44. **Methods of Interchange and Reversal.** — In the ordinary method of double weighing (see Exp. 8) an unknown weight is first placed in the left-hand pan of a balance, and a known weight in the right-hand pan. Let us suppose that the former is greater than the latter by a small amount, which is sufficient to send the pointer of the balance x divisions to the right of its natural resting-point. The unknown weight is next placed in the right-hand pan, and the known weight in the left-hand pan. The pointer will evidently move about x scale divisions to the left of its natural resting-point. The total movement produced by interchanging the weights will therefore be about 2 x scale-divisions. If, however, the unknown weight were exactly counterpoised, the substitution of the known weight for it would cause a motion of the pointer through only x scale divisions. It is easier, accordingly, to detect

a difference between two weights by the method of interchange than by the method of substitution (§ 43).

The method of interchange is generally used in connection with null methods of comparison (§ 42) when *reversible instruments* are employed. Whatever may be the difference between the two nearly equal quantities thus compared, its effect upon a reversible instrument is doubled by interchanging these quantities. For this reason the method of interchange, when applicable, is always preferred to the method of substitution.

A similar method is employed in case of reversible instruments in general. Thus an electrical current which deflects a galvanometer needle $x°$ to the east of north, should if reversed deflect it $x°$ to the west of north. The needle is thus moved, by a reversal of the current, through $2\,x°$. Since an angle of $2\,x°$ can be measured as accurately as an angle of $x°$, the method of reversal has to a certain extent the advantage of a method of multiplication (§ 39). In the methods of interchange and reversal "zero-errors" are eliminated (§ 32), for the increase of one reading due to an error in the zero will be nearly offset by a decrease in the reversed reading. Methods of reversal are always, when practicable, employed.

§ 45. **Check Methods.** The methods of substitution and of reversal are instances of check methods. In physical measurement, as in arithmetic, an indefinite number of such methods may be devised. The use of check methods is not, however, limited to such

as yield accurate measurements. We often find an advantage in checking results which we believe to be precise, with others obtained by different methods, which we consider comparatively unreliable. It is in this way, principally, that gross mistakes are discovered, such as are otherwise likely to be repeated over and over. But the use of check methods is also important in the detection of smaller errors. Even if a method is uncertain, there is probably some limit to its inaccuracy, and if the results fail to agree with those of a different method by an amount greater than this limit, we are led immediately to suspect an unknown source of error in one of these methods. The densimeter, for instance (Exp. 15), though not nearly so exact as the specific gravity bottle (Exp. 14) should be accurate at least within 1%; hence if the results differ by more than 1% we at once repeat the determination with the specific gravity bottle. On the other hand an agreement of the two results within 1 % indicates the absence of gross mistakes in either determination.

Whenever the results of check methods, however rough, agree with previous results as closely as may be expected, there is always a certain degree of mutual confirmation. It should be remembered, however, that a check method is such only in so far as it makes use of different data, different constants, different instruments, and different laws or principles from those already employed. Accuracy in physical measurement is generally obtained only when every possible variation has been made in the conditions of

an experiment, the results compared, and the differences between them explained.

§ 46. **Method of Averages.** — When finally all possible care has been taken to avoid sources of constant error, and to increase the accuracy of determinations, there remains one general method of escaping from what are known as accidental errors (§ 24), or those which tend sometimes to increase, and at other times to diminish, the result. This method is simply to take a great number of measurements, and to find the average. It is not likely, for instance, that in ten observations all should by accident be greater, or all less, than in the long run; in fact, the chances are more than one thousand to one against it. It is much more likely that three or four should be affected one way, and the rest the other way. In fact, we must expect that the errors due to chance shall to a certain extent offset one another. The consequence is that the average of several observations is more reliable than any one alone. For a discussion of the advantages gained by taking the average of several observations, see § 51.

§ 47. **Allowance for Errors.** — We have considered, so far, the principal methods by which errors may be eliminated from physical measurement. There are, however, certain errors which cannot thus be avoided. The effect of some of these may be submitted to calculation. The buoyancy of air, for instance, is computed and allowed for in all accurate weighings (§ 67). There is another class of errors which cannot be calculated in this way from data already in our posses-

sion. The causes from which such errors arise may require separate investigation. Thus the heat lost in transferring a hot body from one place to another can be estimated only by comparing results of different experiments (see Part I. ¶¶ 93, 94).

No single observer can expect to discover all the sources of error which are likely to arise in measurements. Our knowledge of the corrections which are to be applied in the determination of a given physical quantity is one of slow historical growth. It is necessary to refer continually to examples which have stood the test of long criticism. At the same time, each observer must be on the alert against new sources of error. The slightest alteration in the conditions of an experiment may entirely change the nature of the corrections to be applied.

Errors of greater or less magnitude are sure to creep into our work notwithstanding every possible effort to avoid them. The student is advised not to pay too close attention to fine corrections, lest in so doing he may overlook others of much greater importance. It is a well-known fact that the accuracy of results is apt to be grossly overestimated (see Introduction). Sufficient allowance for errors is seldom if ever made.

The application of corrections to the results of physical measurement must be considered separately in connection with each experiment or class of experiments. The discussion of errors and corrections belongs perhaps to the "Reduction of Results" (Chap. IV.), rather than to "General Methods" of

measurement. The student must not, however, forget that a just allowance for errors constitutes one of the most important parts of an accurate physical measurement.

§ 48. **Standard of Accuracy.** — The distinction between accuracy and precision has been pointed out in the Introduction. One generally knows by experience, roughly at least, what degree of accuracy is attainable with a given instrument. Thus a weighing with ordinary prescription scales will doubtless be accurate to centigrams, but not to milligrams; temperatures taken with a common laboratory thermometer are reliable to degrees, but not generally to tenths of degrees; lengths may be true to hundredths, but not perhaps to thousandths of a centimetre. From such data we may generally estimate roughly the degree of accuracy attainable in the final result. All parts of a measurement should be made with a corresponding degree of accuracy.

Let us suppose, for instance, that it is desired to determine the density of alcohol at a given temperature (*e. g.* 20°) within a few hundredths of 1 % by means of a specific gravity bottle (see Exp. 14) of about 100 *cu. cm.* capacity. To do this, the weight of water and the weight of alcohol required to fill the bottle must be determined within a few centigrams; the temperature of the water must be known within about 1° (see Table 25), and that of the alcohol within a few tenths of 1° (see Table 27). The real difficulty in this experiment consists ac

cordingly in the accurate determination of the *temperature of the alcohol*, — a point to which the student's attention needs generally to be directed. An accurate reading of the barometer would be wholly out of place in such a determination, since an error of several centimetres (see Table 22) would scarcely affect the last significant figure (§ 55) in the result.

§ 49. **Distribution of Time.** — Time is often misspent in the exact determination of quantities which have comparatively little influence in the result. Thus the correction for atmospheric pressure seldom affects the decigrams in a weighing, and ordinary variations make only a few milligrams' difference in the result. It is therefore unnecessary, in many experiments, to read a mercurial barometer closer than to millimetres, much less to correct it for variations of temperature, for capillarity, or for the tension of mercurial vapor. A double weighing, with a rough allowance for the buoyancy of air, takes about the same time as a single weighing with the exact correction, and is, with rough balances, decidedly to be preferred.

When a measurement depends on several determinations of about the same degree of precision, we generally devote an equal amount of time to each; but if we can see that the result will be affected by the errors in one case more than in another, the number of observations is increased *in proportion*. Thus in the determination of the volume of a cylinder from its length and diameter we take twice as many ob-

servations of the latter as of the former, because the diameter occurs twice as a factor, while the length occurs only once in the calculation of the result. A fuller discussion of this principle will be found in Part IV.

CHAPTER IV.

REDUCTION OF RESULTS.

§ 50. **Probable Error.** — When several observations of a given quantity have been made, their "probable error" may be found roughly by the following rule: throw out alternately the highest and lowest values until only a majority remains; take half the range of that majority as the probable error of a single observation.

Thus from the ten following observations of the boiling-point of alcohol —

78°.79	78°.33	78° 02	78°.93	78°.46
78°.67	78°.00	78°.81	78°.43	78°.56

we have, throwing out 78°.93, 78°.00, 78°.81 and 78°.02, a majority of six, ranging from 78°.33 to 78°.79, that is, through 0°.46. The probable error of a single observation is therefore about 0°.23.

In saying that the probable error is 0°.23, we do not mean that this error is more probable than any other, 0°.20 for instance. We mean simply that in the long run more than half the errors will probably be less than 0°.23 (see Table 7), and hence, as some errors are positive and others negative, that a majority of the observations will be scattered through a range not

exceeding 0°.46. This is evidently the case if the observations above are a fair sample of those which would be obtained in an extended series.

§ 51. **Probable Error of an Average.** — To find the probable error of the average of several observations, we divide that of a single observation by the square root of the number of observations.

Thus if the probable error of a single observation of temperature is, as in the last section, 0°.23, that of the mean of ten observations is $0°.23 \div \sqrt{10}$, or less than 0°.08.

The relation between the probable error of an average and that of a single observation is established by the theory of the combination of errors as explained in Part IV.

§ 52. **Probable Error of a Result.** — The probable error of a result can be calculated if we know that of each datum upon which it depends, as will be explained in Part IV. It is often, however, less laborious to work out several independent results, the probable error of which can be found by inspection, as shown at the beginning of this chapter. Thus instead of calculating the density of a block (in Experiment 1) from its *average* weight, length, breadth, and thickness, we may use each measurement of length, breadth, and thickness for a separate calculation, and average the results. In all such cases the probable error should be determined.

§ 53. **Representation of Probable Error.** — The average of the ten observations of the boiling-point of alcohol mentioned in § 50 is 78°.50; the probable

error of this average as found in § 51 is 0°.08. We say, accordingly, that alcohol boils (probably) at 78°.50 ± 0°.08.

In the same way the probable error of any result is often written after it with the "plus - or - minus" sign.

§ 54. **Notation.** — It is convenient for many reasons to express results in units of such magnitude that the probable error may lie below the decimal point. When no such units exist, we introduce as a factor 10 raised to the necessary power. Thus the mechanical equivalent of the unit of heat is not written 41,660,000 ergs, but 41.66 megergs, or 4.166×10^7 ergs.

In this notation we escape any possible confusion between ciphers which are the result of actual measurement and those which we are obliged to use from the necessity of the case.

Ciphers are used in physical measurement at the end of a decimal as freely as any other figure. Thus the average of ten observations in the last section was written 78°.50. The cipher informs us that the average was between 78°.495 and 78°.505. Without the cipher we should infer simply that the average was between 78°.45 and 78°.55. The existence of a cipher in the last decimal place has therefore as much significance as that of any other figure. The question how many figures it is advisable to retain is discussed in the next section.

§ 55. **Significant Figures.** — In arithmetic any number of figures may be significant. In physical meas-

urement those figures only are significant to the left of which the probable error does not extend.

Thus, in the observations at the beginning of this chapter, the degrees and tenths are significant, but the hundredths are not, because the probable error is 0°.23. In the average of the ten observations, the hundredths, also, are significant, since the probable error is 0°.08. One figure is generally enough to describe the probable error. The place which this figure occupies is the same as that of the last significant figure.

It is customary to retain only significant figures either in an observation or in a result. Some authorities use two or more places affected by probable error. When the probable error is stated, there is no objection to this practice. Otherwise it is equivalent to a false pretension to accuracy.[1]

§ 56. **Use of Significant Figures.** — Labor is saved in physical reductions by using only significant figures. The rejection of subsequent figures is not found in practice to impair the accuracy of the result. In deciding how many places to retain, the following approximate rules may be of assistance: —

1st. In addition or subtraction, retain the same number of *decimal places* throughout, — as many as are significant in the least accurate of all the terms.

2d. In multiplication or division, retain the same number of *figures* throughout, — as many as are sig-

[1] The student is cautioned in particular against cases where the result of some mathematical process is to generate an indefinite number of figures. It is true that a metre is about 3⅓ feet; but it would be misleading to state that it is about 3.33333, etc., feet.

nificant in the least accurate of the factors, — not counting, of course, initial ciphers.

3d. In logarithmic work, use as many decimal places as there are significant figures in the least accurate of the arguments.

Thus in weighings with a balance accurate only to a fraction of a centigram, we carry out corrections only as far as the milligrams. Again, in calorimetry, where results are often proportional to differences of temperature less than 10° and accurate only to tenths, these results seldom contain more than three significant figures, and corrections not affecting the third figure may be disregarded.

§ 57. **Rules for Approximation.** — A great deal of time is often saved by applying rules which give approximate but not rigorously accurate results. Thus to add 1% or 2% to any quantity corresponds nearly to adding twice that per cent to the square of that quantity, three times that per cent to its cube, half that per cent to its square root, or to subtracting the original per cent from its reciprocal. The truth of these assertions will be seen by reference to Table 2.

It is obviously the same thing to add a certain per cent to a quantity as to add it to a product in which that quantity occurs as a factor; and nearly the same thing, if the per cent is small, as to subtract it from a quotient obtained with the quantity as a divisor.

One of the most valuable rules for approximation is that used in finding the product of several quantities, each nearly equal to unity. Instead of multiplying, we *add them together*. The resulting decimal is ap-

proximately the same. Since the product cannot be far from unity, the figure in the unit's place is easily supplied.

Thus if the ratio of the arms of a balance is 0.99996, the correction for the use of brass weights in air 0.99984, for the buoyancy of air on water 1.00122, and the space occupied by 1 gram of water is 1.00175, the volume of water is found by multiplying its apparent weight by the factors $0.99996 \times 0.99984 \times 1.00122 \times 1.00175$. The product found by the ordinary laborious process is 1.0027715+, or, to five places of decimals, 1.00277. The same decimal is found by adding the four numbers together.

The arithmetic mean (or half-sum) of two quantities differing by less than 2% may usually be substituted for their geometric mean (or square root of their product) which is harder to calculate.

It will be noticed in Table 3, *b*, *c*, *d*, and *e*, that the sine, tangent, arc, and chord of small angles are approximately equal. It is frequently useful to substitute one for the other. It is also seen that the cosine of a small angle is nearly equal to unity, so that the difference may often be disregarded.

The above rules for approximation may be applied without injury to all results which are not expected to contain more than four significant figures, provided that the corrections do not exceed 2% nor the angles 2°.

§ 58. **Use of Tables.** — The reductions in physical measurement are often facilitated by the use of tables. There are two kinds of these: one in which the quan-

tity sought is given in terms of a single argument; the other where it is given in terms of two arguments. The first kind is readily understood by any one who has used logarithms. In one column, generally at the left of the page, we find the argument; in the next column, the corresponding values of the quantity sought. Generally, however, there are ten such columns on the same page. The argument is not printed at the left of each column, but, to save space, the last figure of it is at the head of the column and the rest at its left in the first column on the page. The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 at the head of different columns usually indicate a table of the first kind.

When the argument lies between two values in the table, we cannot directly find the quantity which we seek. We have to make use of interpolation, the rules for which need hardly be explained.

Interpolation depends upon the principle that slight differences in any quantity are nearly proportional to the corresponding differences in its argument, and upon the application of the rules of simple proportion to the differences in question.

The second kind of table is similar to the first, only that at the head of the different columns is contained a second and independent argument upon which the quantities in the body of the table also depend.

Thus the density of air at different pressures and temperatures is contained in Table 19. We follow the line corresponding to a given pressure until we reach the column corresponding to the given temperature, and there find the density in question.

Interpolation in such a table is more difficult than in one of the first kind, because the variation due to both arguments must be taken into account, as explained in ¶ 153. Interpolation is, however, unnecessary when the quantities are, as in Table 20, close enough together, or where only a rough value is required.

§ 59. **Graphical Method.** — Co-ordinate paper (that is, paper ruled in small squares) is useful in many experiments, both for representing results so that any gross error is visible to the eye, and for purposes of interpolation. At the left of the paper there is usually

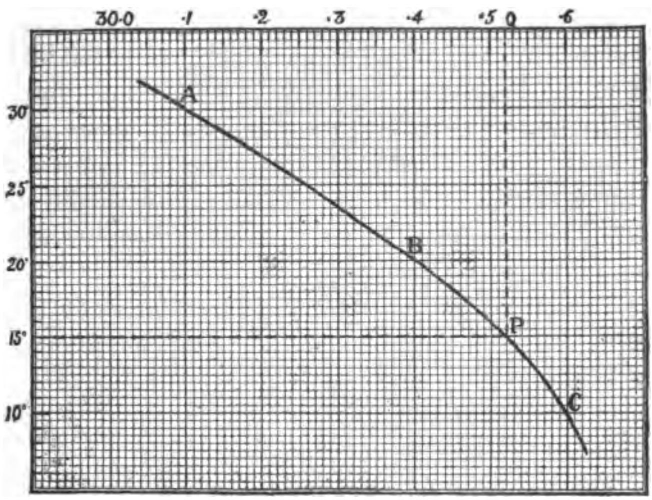


Fig. 4.

ally constructed a vertical scale, like the scale of degrees in the diagram. At the top there is a horizontal scale, like that in the diagram representing the

weights floated by a Nicholson's hydrometer. The correspondence of two values is represented by a point opposite the two values in question. Thus in Fig. 4, A represents that at 30° the hydrometer floats 30.1 grams; B, that at 20° it floats 30.4 grams; C, that at 10° it floats 30.6 grams. The dotted line ABC drawn with a bent ruler thus supplies an indefinite number of approximate values. To find the weight floated at 15°, we find a point P opposite 15°, and then a point Q opposite P. The answer is 30.52 grams. In the same way the relation between any two quantities can be represented by points, and intermediate values found.

§ 60. **Use of Rough Methods.** — It is always prudent to revise any reduction involving much numerical work, applying the various tests which arithmetics contain. It is, however, easier to reason clearly about small quantities than about large ones, since the former only can be carried in the head. Mistakes in reasoning can often be discovered by rough mental processes when no error can be detected in the figuring.

Thus, if the buoyancy of air relieves water of a little more than a thousandth part of its weight, 50 grams will lose a little over 5 centigrams. If we find that we have introduced a correction of 6 decigrams or 6 milligrams, we at once detect the mistake.

The use even of rough tables, when they can be found, is a very convenient check upon numerical work. When a multiplication runs into the millions, logarithms will be useful, — not always, however, five places. Gross errors are most easily detected by loga-

rithms carried out only to a single place of decimals, the whole attention being placed upon the characteristic. It is thought advisable in physics to use negative characteristics in preference to subtracting from 10. The student may be reminded that a most serious and at the same time a most common mistake in calculation is the misplacement of the decimal point.

§ 61. **Reduction of Consecutive Observations.** — In § 38 we obtained the following series of angles: 0°, 40°+, 80°+, 120°+, 160°+, 200°+, 240°+, 280°+, 320°+, 360°+, 401°—, and 441°—; the first and last give us a difference of 441° —, indicating less than $40\frac{1}{11}°$ for the angle; the second and next to the last give less than $40\frac{1}{9}°$, but the 3d and 3d from the last as well as the 4th and 4th from the last give each 40°. The average of these four results is $40\frac{5}{99}°$, or 40°.05 nearly.

Again, the 1st and 9th, the 2d and 10th, the 3d and 11th, and the 4th and 12th give respectively 40°+, 40°, $40\frac{1}{8}°$ —, and $40\frac{1}{8}°$ —; the average of these four values is $40\frac{1}{16}°$, or 40°.06 nearly. Either of these methods of reduction is accurate enough for the measurements in question. In each case the 5th, 6th, 7th, and 8th observations were omitted. By using them we could have obtained two more pairs of observations; but the shortness of the interval between them takes off from their value. The probable error of the result would actually be increased by treating them as we have the others. It is generally advisable to omit in this way the middle third of a series of consecutive observations.

There is a third way of reducing consecutive intervals against which the student must be cautioned. The differences between the 1st and 2d, the 2d and 3d, etc., are in 10 cases 40°, in one 41°. There is a common fallacy to the effect that the average of these, $40\frac{1}{11}°$, makes use of all the observations. It is easy, however, to see that in taking the average we must first add the intervals together, and that we shall obtain as a result the interval between the 1st and 12th observations, since the whole is equal to the sum of all its parts. We subsequently divide by 11, but the result depends solely upon the 1st and 12th, and not in any way upon the intermediate observations, the value of which is therefore completely lost.

This method of averaging consecutive intervals should be accounted a serious error, not simply because it is unnecessarily laborious, but because of the self-deception which it involves.

CHAPTER V.

HYDROSTATICS.

§ 62. **Pascal's Principle.** — From experiments in weighing liquids we might infer that their weight exerted simply a downward action. By immersing a pressure-gauge[1] in any liquid we find, however, that at a given depth the liquid exerts an equal force upon it in all directions, whether horizontal, vertical, or oblique, whether up or down. The same instrument shows that when a fluid is at rest the pressure is the same at all points on the same level. If this were not so, a perfect fluid would evidently be unable to remain at rest. Conversely, all points in a stationary liquid which are subject to a given pressure are found on a given level.[2]

§ 63. **Hydrostatic Pressure.** — If we have a column of liquid in a tube with vertical sides which it cannot cling to, the whole weight of the column must rest upon the bottom of the tube. Let the tube be 1 *sq. cm.* in section; then the weight of the whole column

[1] For the construction of such a gauge see Descriptive list of Experiments in Elementary Physics, 1889, Exercise 5. This experiment is due to Professor Hall.

[2] When (see Fig. 60, page 127) the air-pressure is greater on one part of a liquid surface (c) than on another (b), the liquid stands at unequal heights in two parts of the apparatus, but if the air-pressure is the same it stands at the same level in both places (Fig. 61). That part of a liquid in a U-tube which lies below a given level transmits or communicates pressure along this level without increasing or diminishing it.

rests upon a surface 1 *sq. cm.* in area, and the pressure in dynes per *sq. cm.* is numerically equal to this weight reduced to dynes. The weight of the column is evidently the product of its volume in *cu. cm.*, the density (or weight of 1 *cu. cm.* in grams), and the intensity of gravity (or weight of 1 gram in dynes); and as the tube has a unit cross section, the volume is numerically equal to its height. The hydrostatic pressure (that is, the pressure of the liquid per unit of area) at the bottom of a tube is therefore the product of the depth and density of the fluid and the intensity of the earth's gravitation. It is clear that the size of the tube makes no difference, for in a tube of twice the cross-section we should have twice the weight distributed over twice the area, and the pressure per *sq. cm.* would be the same. Since pressure is the same in all directions, we may therefore state as a general principle that pressure increases with the depth.

§ 64. **Principle of Archimedes.** — Suppose we suspend a solid in a fluid. The pressure on the solid will of course be greater the more we lower it into the fluid, but the pressure on the bottom of the solid will always be greater than on the top; hence the fluid will buoy up the solid more or less. One can calculate the amount of this buoyancy by the principles which have already been stated if the shape of the solid is not too complex, but there is a much simpler way of arriving at the result. Imagine the solid out of the fluid, and its place filled by a separate portion of that fluid, having the same shape and

bounding surfaces as the solid. The pressures on this new portion of the fluid must be the same as on the actual solid, because the surfaces and their depths are the same; but the forces produced result simply in holding the fluid in place, hence their resultant is equal and opposite to the weight of a portion of the fluid equal to the solid in bulk. This principle is known by the name of its discoverer, Archimedes, (287 to 212 B.C.), and may be thus stated: a solid immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced. The difference between the weight of a body and the buoyant force of a fluid in which it is submerged may be called the *effective weight* of the body in that fluid.

§ 65. **Buoyancy of Air.** — According to the principle of Archimedes just explained, a body loses weight in air just as it would in any other fluid. Seven grams of brass displace, for instance, about five-sixths of a cubic centimetre of air; that is, about one milligram, or one 7000th of their nominal value. Bodies weighed against them also lose in weight according to the amount of air displaced. Ordinary weighing consists, therefore, in a comparison of effective weights. The number of grams which balance a body in air is called its apparent weight in air. If, however, the body is in water (the weights being as before in air), we find what is called the apparent weight in water. The effective weights in air or in water can always be found roughly from the corresponding apparent weights by subtracting, for reasons above explained, one part in 7000 from the nominal values of the brass

weights. The exact correction is given in § 67. Only apparent weights are obtained by Nicholson's hydrometer, by the hydrostatic balance, or by the specific-gravity bottle.

§ 66. **Apparent Specific Gravities.** — It is obvious that in weighing a body first in air, then in water, as in Experiments 2, 3, and 4, or 8 and 9, we find first the apparent difference between the weight of the body and that of an equal bulk of air, and second, the apparent difference between the weight of the body and that of an equal bulk of water. Subtracting the latter from the former we have the apparent difference of weight between the water and air displaced, or what is the same thing,[1] the apparent weight in air of an equal bulk of water. The ratio between the apparent weight of a body (in air) and that of an equal bulk of water (in air) is called the apparent specific gravity of the body. Without corrections for the buoyancy of air, we can obviously find only apparent specific gravities.

§ 67. **Correction of Apparent Weights.** — Given the apparent weight of a body in air and in water, we usually proceed as follows: First calculate by subtraction the weight of an equal bulk of water, as explained in § 66. Multiply this by the space apparently occupied by 1 gram (see Table 22) to find the volume in question. This is obviously equal to the number of *cu. cm.* of air displaced by the substance. Multiply it, therefore, by the weight of

[1] This holds strictly for *effective* weights from the principle of Archimedes; hence also for apparent weights, to which the former are proportional. See § 65.

1 *cu. cm.* of air (see Tables 19 and 20) to find the weight of air displaced. Next multiply the weight in grams of the body in air by the weight of air displaced by 1 gram of brass (Table 20, A) to find the weight of air displaced by the brass weights. Subtract the latter from the apparent weight of the body in air to find its effective weight in air (§§ 64, 65). Add to this the weight of air displaced by the body to find its true weight *in vacuo.*

When the density of a substance is approximately known, either by reference to Tables 8–11, or from an actual determination of its apparent specific gravity, we may at once reduce its apparent weight to *vacuo* by applying the appropriate coefficient from Table 21.

The apparent weight of a liquid, obtained either by methods of displacement or by the specific gravity bottle, must be reduced to *vacuo*, like any other apparent weight, starting with either (1) the volume, or (2) the density of the liquid, or (3) with the weight of an equal bulk of water. The apparent weight of a *body in a liquid* needs, however, to be corrected only, as has been explained above, for the buoyancy of air on the brass weights by which the body is counterpoised.

§ 68. **Correction of apparent Specific Gravities.** — To find the density of a body, we first find, as explained in § 67, the *volume* of the body from the apparent weight of water displaced, and second the weight of the body *in vacuo.* The weight *in vacuo* is then simply divided by the volume to find the true

density of the substance at the given temperature and pressure.

In case we have given, as in Experiment 13, not the apparent weight of water displaced by a solid, but that of some other fluid of known density, we may divide the corrected weight of the fluid, *in vacuo*, obtained as above, by the density of the fluid, to find the space occupied; or we may divide its apparent weight by its apparent specific gravity, if we know it, to find the apparent weight of an equivalent bulk of water, and work out the result as before.

We notice that, in reducing apparent specific gravity to density, we apply to the numerator of a fraction a factor from one table, and to the denominator a factor from another table. The same result, essentially,[1] may be obtained (see § 57), by a single process. Subtract the factor in Table 22 from that in Table 21, multiply the apparent specific gravity by the algebraic difference, and apply the correction thus found. The difference between density and specific gravity is usually less than one per cent.

§ 69. **Density and Specific Gravity distinguished.** — Specific gravity is defined as relative density. Hence density bears to specific gravity (referred to water) the same ratio that the density of water bears to unity. (See Table 25.) By the specific gravity of a substance at a given temperature, we understand, in the absence of any statement to the contrary, the proportion between its weight and that of an equal bulk

[1] **Results thus reduced show a slight error, usually confined to the sixth place of decimals.**

of water at the same temperature. It is understood also, unless otherwise stated, that both bodies are under atmospheric pressure (76 *cm.*). Specific gravities of gases, however, are often stated with respect to hydrogen or air at the same temperature and pressure. Specific gravities are also referred to water at its temperature of maximum density. Having accepted the value 1.000013 for the maximum density of water, we see that such specific gravities are less than densities by an amount (13 parts in a million) which is small compared with the probable error of observation.

§ 70. **Calculation of Difference of Density.** — Since density, D, is the quotient of mass, M, by volume, V, or

$$D = \frac{M}{V},$$

two bodies having the same volume, V, densities D_1, D_2, and masses M_1, M_2, have a difference of density equal to the difference in their masses divided by the volume, that is,

$$D_2 - D_1 = \frac{M_2}{V} - \frac{M_1}{V} = \frac{M_2 - M_1}{V}.$$

Hence we may find the difference in density between two liquids or two gases (as in Experiment 18) from the difference in weight of a flask of known capacity filled first with one, then with the other. It is obvious that in weighing a flask filled first with air, then with a liquid (as in Experiments 11 and 14), we might determine in this way the difference of density between the liquid and air, and that by adding to this result the density of air, D_1 (from Tables 19 and 20), we

should find the density, D_2, of the liquid in question; that is,

$$D_2 = D_1 + \frac{M_2 - M_1}{V}.$$

When the substance weighed is (as in Experiment 18) lighter than air, the difference of density may be considered negative, and must be subtracted numerically from the density of air as indicated by the formula identical with the above,

$$D_2 = D_1 - \frac{M_1 - M_2}{V}.$$

§ 71. **Accuracy of Meteorological Instruments.** The density of the atmosphere is found to affect all delicate weighings. For many purposes it is sufficiently accurate to assume a mean density of 1.2 *mgr.* to the cubic centimetre;[1] but for the most accurate determinations we need to correct it for temperature, pressure, and humidity. The corrections are so slight that a rough estimate is sufficient for this course of measurements, and hence we may accept provisionally the indications of such weather instruments as may be found in the laboratory. We shall learn, later on, the means of detecting errors in these indications, and shall expect to prove that these errors have not perceptibly affected our results.

In place of the ordinary weather instruments, we may employ a sensitive baroscope, or *barodeik*, consisting of a hollow cylinder which has been counterpoised *in vacuo* against a weight occupying say 1000

[1] The probable error under this assumption may be estimated as between 1 part in 10,000 and 1 part in 100,000.

cu. cm. less space than itself. The apparent difference of weight between the hollow cylinder and its counterpoise indicates at once the actual density of the atmosphere.

§ 72. **Accuracy of Gram-Weights.** We must choose between accepting such copies of the gram as are attainable, and determining independently the weight of a cubic centimetre of water. Experience shows that weights can be copied (and that they generally are copied) with a very great degree of precision, while it is comparatively difficult to copy standards of length, and still more difficult to reproduce them.[1] There is also more or less uncertainty as to the temperature at which a cubic centimetre of water may be assumed to weigh one gram (see § 6 and Table 25), and it is by no means easy to find the weight of a cubic centimetre of water with any degree of precision. It is, moreover, important to express our results in *conventional* units. For these reasons we prefer to accept a set of gram-weights, provided, however, that we are not able to detect any gross error in them by such means as are in our power.

§ 73. **The Density of Water.** — On account of the inaccuracy of our standards of length we are unable to determine the volume of a body very accurately from its length, breadth, and thickness; and hence we cannot find its density absolutely, as in Experiment 1, with any degree of precision. The same inaccuracy affects the volume of water which such a body dis-

[1] The error in the original determinations was nearly a tenth of one per cent. (See § 5.)

places, and hence also the density of water, which is found by comparing the weight and volume displaced. We prefer, therefore, to accept the results of a great number of determinations (see Table 25) rather than any rough measurements of our own, and we make use of this table of density for testing or correcting our standards of length, and not of our standards of length for the determination of a new table of densities. It is thought that measurements of length corrected in this way will be nearer the conventional standard than those depending directly on such rough copies as are found in the market. The approximate agreement of our actual standards of length and mass is the first of a series of tests to which these standards must be subjected, and through which, finally, any gross error in either is sure of detection.

CHAPTER VI.

HEAT.

§ 74. **Temperature.** — Temperature is believed to depend upon the vibration of the molecules of which a body is composed, and hence be akin to what we call heat. Temperature is not, however, heat, but the state of saturation with heat which determines, under certain conditions, whether heat will be imparted or absorbed. Bodies which can communicate heat to others are said to have a higher temperature. Two bodies in contact are said to have the same temperature when no heat flows from one to the other. It is found that two bodies at the same temperature as a third are themselves in thermal equilibrium. Heat corresponds in a certain sense to quantity, temperature to intensity of vibration (see § 84). The temperature of a gas is seen from its nature to be intimately connected with pressure; for pressure is explained as the effect of the perpetual bombardment of the molecules against the sides of a vessel which contains them.

§ 75. **Absolute Zero.** — We must distinguish the absolute zero of temperature from that which we have provisionally adopted. At the absolute zero, the par-

ticles of a body are supposed to be at rest. Gases therefore exert no pressure at this temperature, and occupy no space, save that which their molecules take up when closely packed together.[1] The absolute zero must be the same for all bodies, since when their heat is wholly taken away they cannot communicate any from one to another, and hence have, by definition, the same temperature. There is reason to believe that the absolute zero of temperature is, on our provisional scale, about 273° centigrade below the freezing-point of water.

§ 76. **Absolute Temperatures.** — We have seen that the temperature and pressure of gases are intimately connected. The absolute scale of temperature is founded upon this fact. By definition, *absolute temperature is proportional to the pressure of a perfect gas confined to a constant volume.* All permanent gases are found to be essentially perfect in this sense.

To compare absolute temperatures, we may seal up a mercurial barometer in a tube, or an aneroid barometer in a preserving jar. The corrected indication of the pressure of the air enclosed will be proportional to the absolute temperature.

We are still at liberty to adopt any length of degree which we please, and for convenience we will choose that of the centigrade scale. Let us suppose that the barometer rises ten inches when we heat the air from the freezing to the boiling point of water. Then a tenth of an inch will represent a degree. The abso-

[1] The molecules are thought to occupy at least one half as much space as the liquid formed by the condensation of a gas.

lute temperature of freezing or boiling can now be found from the corresponding pressure of the barometer in tenths of an inch. We discover in this way that water freezes at 273°, and boils at 373° on this absolute scale.

Whatsoever means we adopt for estimating the pressure of a confined gas, the same result is obtained, since the pressure at boiling is to that at freezing as 373 is to 273.

It is found that all temperatures on the mercurial thermometer may be converted approximately to the absolute scale by adding 273°.

§ 77. **Velocity of Molecules.** — From the definition of force (§ 12) depending on mass, time, and change of velocity, it is clear that the pressure of a gas must depend both upon the number and upon the velocity of the molecules which strike a given surface in a given time. If we double the velocity of the molecules without changing the distance they must travel before hitting the sides of the vessel, the blows will be twice as frequent and twice as strong; hence the pressure will be quadrupled, — also, by definition, the absolute temperature, as the volume remains the same. So, in general, temperature may be shown to vary as the square of the molecular velocity.

We do not know the mass of a single molecule, except within wide limits; but we can find the weight of a cubic centimetre of a gas, and thus independently of the number of molecules in the given space, we can calculate the average velocity which will account for a given pressure. Molecu-

lar velocity is not therefore a matter simply of conjecture.[1]

§ 78. **Pressure and Density of Gases.** — The density of a gas is evidently proportional, other things being equal, to the number of molecules in a given space. In the case of exceedingly rarefied gases, the molecules are so far apart as not practically to interfere with one another; hence each will hit the sides of the vessel as often as if the others were not present.[2] It follows from the principles explained in the last section that in such a case pressure and density are proportional when the average velocity, or temperature, remains the same. Hence at a constant temperature, *the pressure of a perfect gas varies with the density.* Experiment confirms this assumption in the case of exceedingly rarefied gases.

As a gas becomes more and more condensed, there is less and less space between the molecules free for vibration, and cohesion may come into play, particularly in the case of a vapor near its point of condensation. In such cases the law connecting density and pressure cannot be applied. Even the most permanent gases are more or less compressible than theory would indicate (see Table 12), though in most experiments the variation is barely perceptible.

§ 79. **Law of Boyle and Mariotte.** — As the volume of a gas increases, the density obviously diminishes,

[1] The average velocity of a hydrogen molecule at 0° is found to be not far from a mile per second; that of oxygen is one fourth as great. For a further discussion of this subject, see Maxwell's Theory of Heat, chapter 22.

[2] See Daniell's Principles of Physics, page 224.

and the pressure, as we have seen, diminishes in proportion. Hence *the volume of a perfect gas at a given temperature varies inversely as its pressure.*

§ 80. **Law of Charles.** — As the volume of a gas increases, the pressure diminishes; but as the absolute temperature increases, the pressure increases. It follows that if both the volume and the absolute temperature increase in the same proportion, the pressure will remain the same. Hence *the volume of a perfect gas at a constant pressure is proportional to its absolute temperature.*

By this principle absolute temperature can be estimated from the volume of a gas at a constant pressure as in Experiment 26, as well as from the pressure of a gas at a constant volume, as in Experiment 27 (see § 76).

§ 81. **Reduction of Density to Standard Temperature and Pressure.** — If D is the density of a gas, P its pressure, and T its absolute temperature, then the pressure, P_1, at the standard temperature, T_0, will be given by the proportion, $P_1 : P :: T_0 : T$, or $P_1 = PT_0 \div T$; the density, D_0, at the standard pressure, P_0, is given by the proportion, $D_0 : D :: P_0 : P_1$; whence $D_0 = D\ P_0 \div P_1 = D\ P_0 \div (P\ T_0 \div T) = D\ P_0\ T \div P\ T_0$.

If the pressure, p, is expressed in centimetres of mercury, and the temperature, t, is on the ordinary centigrade scale, we have

$$D_0 = D \times \frac{76}{p} \times \frac{273 + t}{273}.$$

§ 82. **Expansion of Solids and Liquids.** — In the case of solids and liquids, the effects of temperature in causing expansion are slight in comparison with those in the case of gases. It is probable that the cohesive forces which bind their particles together leave very little available space for their vibration, and it is quite possible that this available space obeys the same laws in general as in the case of gases. We have, however, several cases where bodies contract with heat, the most notable of which is water below 4°. Such cases may be explained as the result of the gradual re-arrangement of the particles consequent on a rise of temperature, — that is, to the same cause which makes water occupy about ten per cent less space than the same weight of ice.

§ 83. **Linear and Cubical Co-efficients of Expansion.** — A co-efficient of expansion is a number which always occurs as a factor or *co-efficient* in calculating expansion produced by heat. The increase of the volume of one cubic centimetre caused by a rise of 1° in temperature is called the cubical co-efficient of expansion of a substance. The increase of the length of 1 *cm.* is called the linear co-efficient of expansion. Unless otherwise stated, the co-efficient of expansion of gases and liquids is assumed to be cubical; that of solids, linear, affecting length, breadth, and thickness alike, and hence only one-third as great as the corresponding cubical co-efficient.

§ 84. **Relation between Heat and Temperature.** — The relation which temperature bears to heat is analogous to that which hydrostatic pressure bears to

water. Heat flows from high temperature to low temperature, water from high level to low level. When we pour water into a vessel, the level rises; so heat increases the temperature of a body. It takes more water to fill a large jar to a given depth than a small one, more heat to warm a heavy body to a given temperature than a light one. Heat, like water, is indestructible, though it can be transformed into many shapes. We usually estimate quantities of heat relatively to a certain unit, which has been defined (§ 16), or, in the absolute system, by the quantity of work to which it is equivalent.

§ 85. **Thermal Capacity.**—The thermal capacity of a substance may be defined as the total amount of heat necessary to raise its temperature one degree. It corresponds to the cross-section of a vessel. A common measuring-glass, flaring a little at the top, requires more and more water to raise the level by a given amount. So most substances require more heat to raise their temperature one degree as the temperature increases. The variation is, however, frequently imperceptible.

§ 86. **Specific Heat.**—If we put pebbles into a vessel it will take less water to fill it than before; still less if the spaces between the pebbles are filled with sand.

Specific heat corresponds to the material which a vessel contains before water is added. It is something irrespective of the weight or bulk of a body which gives it a greater or less capacity for heat. From experiments in mechanics we infer that the

fineness of subdivision of the particles of a body is what fits them to be set in vibration, that is, to absorb heat. Specific heats accordingly increase as what we call the "molecular" weight diminishes. In the case of elementary substances this can almost be called a law.[1]

§ 87. **Latent Heat.** — If a small vessel is put inside a large one, and water poured into the space between, the level rises up to the edge of the small vessel, then is constant until the small vessel is filled, after which it rises again. So when ice is heated it rises in temperature until it begins to melt, then the temperature is constant until the ice is all converted into water, then it rises again.

A certain quantity of heat disappears in melting the ice, without raising the temperature, just as a certain quantity of water disappears in filling the inner vessel. The quantity which is thus absorbed in melting a gram of a substance is called its latent heat of liquefaction. In the same way heat disappears when a liquid is changed into a vapor. The amount of heat necessary to convert a gram of a liquid into a vapor is called its latent heat of vaporization.

Thus it takes about 80 units of heat (or 3,300 megergs) to change a gram of ice at 0° into a gram of water at 0°. The water is not any warmer than the ice, because water and ice may remain indefinitely in contact and yet perfectly distinct. In the same way

[1] The products of the atomic weights and the corresponding specific heats (see Table 8, *a*) will be found in most cases to be nearly equal to the number 6.

it takes about 536 units[1] of heat (or 22,000 megergs) to change a gram of water at 100° into a gram of steam at 100° when the atmospheric pressure has to be overcome.

§ 88. **Explanation of Latent Heat.** — When the particles of a body are separated in such a way as to overcome certain forces called cohesive, because they tend to hold particles together, it is clear that work must be done. If a particle of ether escaping from a drop of that fluid is held back by the attraction of that drop, it will evidently lose a part of its velocity; and as only the swiftest particles can escape at all, the slowest must remain, and the drop will grow cooler and cooler. The work done in evaporation is at the expense of temperature. When finally the liquid has been all converted into vapor, heat must be communicated to the latter to restore to it the *same* temperature that it had in the liquid state. The boiling of a liquid depends upon the *continuous* communication of heat necessary to maintain a *constant temperature*. This heat is said to be latent, because it does not affect the thermometer. It can, however, be recovered; for the heat absorbed in vaporization is given back in the act of condensation. The process is in fact reversed. A particle of vapor is accelerated by the attraction of the liquid mass into which it falls, and gains in velocity what before it lost.

§ 89. **Law of Cooling.** — There are three ways in which heat is likely to escape from a calorimeter:

[1] Of this, about 40 units are consumed in overcoming the pressure of the atmosphere.

first by conduction, or passing from one particle to another; second by convection, or being carried bodily by currents of air; and third by radiation, or directly passing from one place to another as the sun's heat does in waves or rays. When all these causes have been guarded against, there is apt to be a very slight loss of heat, which has to be allowed for. In all three ways in which heat can escape the amount is found to be proportional, nearly, to the difference of temperature between the contents of the calorimeter and the surrounding air. Hence we have Newton's law of cooling: *Loss of heat per unit of time is proportional to difference of temperature.*

If, for instance, the temperature within the calorimeter is 40° and that outside of it 20° and the rate of cooling 1° in 5 minutes, we should infer that if the calorimeter were at 30° the temperature would fall only about 1° in 10 minutes. We are thus able to estimate the temperature at a point of time when observation would be impracticable. (See Experiment 31.)

§ 90. **Principle of Calorimetry.** — When substances at different temperatures are mechanically mixed in a calorimeter so that no chemical or physical reaction takes place, with the exception of a small quantity of heat which escapes as has just been explained, the total amount remains constant. What is lost by one body is therefore taken up by another.

If m_1 is the mass of one body, s_1 its specific heat, t_1 its temperature before mixture, and t its temperature after mixture, then the number of units it has ab-

sorbed is $m_1 s_1 \times (t-t_1)$. If it has lost heat instead of gaining it, the expression will be negative. Denoting by subscripts 1, 2, 3, &c. in the same way the properties of the several substances contained in the calorimeter, we have

$$m_1\, s_1\, (t-t_1) + m_2\, s_2\, (t-t_2) + m_3\, s_3\, (t-t_3) + etc. = 0.$$

The temperature of the mixture, t, is the same for all. The products $m_1\, s_1$, $m_2\, s_2$, $m_3\, s_3$, *etc.*, are evidently the thermal capacities of the bodies in question. For if s is the heat required to raise 1 gram 1°, $m\, s$ will be that required to raise m grams 1°.

To calculate the thermal capacity of a calorimeter, we multiply the weight of the *inner vessel* in grams by the specific heat (from Table 8, *a*) of the material, usually brass, of which it is composed. The thermal capacity of a stirrer attached to the bulb of a thermometer is calculated in the same way. The thermal capacity of a thermometer is about one-half of the number of cubic centimetres immersed, whether of mercury or of glass, — more exactly, $\frac{4}{9}$ in the case of mercury. The various methods of calculating specific heat by the above principles will be explained in Experiments 32, 33 and 34.

§ 91. **Heat Developed in a Calorimeter.** — When a substance contained in a calorimeter undergoes a change of state, whether physical or chemical, heat is usually developed or absorbed. The fact is recognized by the departure of the temperature of the mixture from that which it would be expected to have if the mixture were purely mechanical. The

heat developed or absorbed when a gram of a solid is dissolved is called the (latent) heat of solution; when it unites chemically with another substance, it is the heat of combination; or if it *burns* in the process, the heat of combustion. The calculation of these heats is explained in Experiments 35–38.

CHAPTER VII.

SOUND AND LIGHT.

§ 92. **Wave Motion.** — When a row of marbles is set in a crack of the floor, and one at the end of the row is hit, it strikes the one next to it and comes to rest after giving up nearly all its motion, the second marble gives up its motion to the third, and so on, until finally the last marble is set in motion. In the same way a string can transmit a pulse. The string however, has generally a lateral motion and each portion pulls the next one side instead of pushing it forward. A wave of sound in air is transmitted like a pulse through a row of marbles, a wave of light like a pulse through a string. In both cases, however, the pulse, if not obstructed, is carried from the origin not simply in one direction but in all. The different paths by which light spreads out are illustrated by a system of strings radiating in all directions from a given point. These strings represent also what are called rays of light. To explain the distribution of sound we may imagine a space filled with solid bodies having springs of some sort between them so as to keep them apart and yet allow any one to transmit a blow to its neighbor, as in the case of the marbles.

§ 93. **The Air and the Ether.** — A pulse of sound in air is in reality transmitted by the impact of the molecules of air, which are perfectly elastic, whereas marbles are not. The velocity of sound in air is a little over 33 thousand *cm. per sec.* While sound is intercepted by what we call a vacuum (there being no molecules to transmit it), light passes more easily through a vacuum than through air. What carries light we do not know. We call it *the ether*. The ether, like air, is perfectly elastic; but it has no weight, and no perceptible resistance to motion through it; it seems to pass between the particles of the densest solids "as freely as the wind passes through a grove of trees."[1] And yet it transmits, as we have seen, transverse vibrations, after the manner of a string.

In some respects the ether reminds us of magnetism, which, though perfectly immaterial, can hold a piece of iron firmly through a piece of glass. Electricity, however, affords the only true analogy to light. It is well known that telephone messages are carried from one wire to another, either through a vacuum or through almost any medium which we can interpose. The fact is certainly significant that electrical vibrations may pass in this way with the velocity of light (30 thousand million *cm. per sec.*), and the belief is gaining ground that light is carried by what is called electromagnetic induction from one particle to another.

§ 94. **Law of Inverse Squares.** — Since both sound and light spread out equally in every direction, a pulse

[1] Lloyd's Undulatory Theory of Light, § 21.

naturally takes the form of a hollow shell, perfectly spherical, and growing larger as the wave passes farther from the source. The area of such a shell is proportional to the square of its radius; hence the intensity of sound or light per square centimetre varies inversely as the square of the distance, — the same amount of energy being distributed over a greater amount of surface. The transmission of sound and light without any perceptible loss affords another illustration of the principle of the conservation of energy.

§ 95. **Relation of Wave-Front and Rays.** — The surface of a shell such as is formed by a pulse spreading out in all directions, or any portion of such a surface, is called a wave front. It is clear that a wave-front is perpendicular at every point to the ray of light passing through that point, as the radius of a sphere is perpendicular to its surface. When a portion of a wave passes through an orifice, the rest being interrupted, most of it still continues to advance very much as if the whole wave were present. It is found, indeed, that waves tend to move in straight lines, and in all cases in a direction at right angles to their front. It follows that any cause which can change the direction of the wave-front will also cause a bending of the rays. In the absence of any such cause, the general direction will remain constant.

This tendency of waves to move in straight lines is much more marked when a great number of pulses are sent one behind the other, as is always, practically, the case. The wave-fronts then find it impossible to bend much without interfering with one another.

A series of wave-fronts issuing from an orifice constitutes in the case of light what is called a beam. The middle part of a beam is perfectly straight; the bending is confined to an almost imperceptible portion at the edges. Sound shows also a tendency to move in straight lines; but, owing to the great distance between the pulses, not nearly to the same extent.

§ 96. **Frequency of Vibration.** — When a toothed wheel, by striking on a card, gives a regular series of pulses to the air, a musical note is often produced. The pitch of the note depends on the number of pulses per second. There are three classes of notes, one in which the pulses are too infrequent to produce a continuous effect upon the ear, the second audible (say from 30 to 30,000 pulses per second), and the third too rapid to be heard. In the same way there are three classes of vibration in light; one too slow to affect our organs of sight, a second visible (from 400 to 800 millions of millions per second), and a third more rapid still and in consequence invisible.

When sound is intercepted, it is usually changed into heat. All kinds of light when absorbed by an opaque body are generally transformed into heat. In all such cases the heat is equivalent, erg for erg, to the energy spent in producing the vibrations in question. All kinds of light act on a photographic plate, but principally those of the third class alluded to, often called actinic. In sunlight the principal source ot energy is from invisible vibrations of the first class, often called calorific for this reason.[1]

[1] See Tyndall's Fragments of Science, pages 182–184.

§ 97. **Reflection.** — All waves are reflected from a surface as an elastic ball is from the floor. That part of the motion which is perpendicular to the surface is reversed, and that parallel to it preserved; hence the path of the ball makes the same angle with the surface before and after reflection. One can see, without a special examination of the motion of separate particles, that a reversal of one component accounts for a similar change of direction in a wave.

§ 98. **Wave-length.** When sound is reflected back and forth between two walls, an echo is heard at intervals corresponding to the time it takes sound to traverse the distance back and forth between the walls. When the walls are only a few feet apart, the echo may become so frequent as to produce a musical note. Thus a tube closed at both ends exhibits this phenomenon. The distance which sound travels between two successive pulses is called in general a wave-length, and is clearly equal in this case to twice the length of the tube. When a particular color is produced in the same way by the reflection of light back and forth between two pieces of glass very close together, its wave-length is twice the thickness of the space between the glasses.

§ 99 **Resonance.** — The vibration of a tube closed at both ends may be described as a periodic rush of air from one half to the other and back again. When such a tube is cut in two in the middle, each half has the power of vibrating essentially as before. The atmosphere receives the rush of air out of the tube and supplies air to fill the vacuum thus caused, taking in

fact to each half the same place as the other half of the tube. Since the whole tube was equal to half a wave in length, the halves will be nearly quarter-wave-lengths; but as the vibration extends a little beyond the open ends,[1] a tube closed at one end only is not quite a quarter of the length of the wave to which it responds.

When a tuning-fork emitting the corresponding note is held near the mouth of the tube, the sound is greatly increased. The downward pulses from the fork are reflected from the bottom of the tube so as to reach it in the middle of its upward motion, which is therefore reinforced in its effect upon the air. The slightest variation in the length of the tube causes the phenomenon to disappear; but if the tube is made just one half a wave-length longer, or any number of half-wave-lengths, the reflected pulses, traversing the distance twice, are retarded a whole wave-length or several whole wave-lengths, meet the fork as before, and resonance reappears.

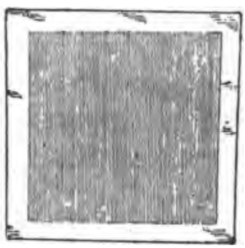
A tube open at one end therefore responds to a given note when its depth is equal to $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, etc., wave-lengths or thereabouts. The first quarter-wave-length is approximate; the other lengths are greater than the first by exactly $\frac{1}{2}$, 1, $1\frac{1}{2}$, etc., wave-lengths respectively.

§ 100. **Interference.** — When two series of pulses arrive at the same place at the same time the effect

[1] It has been estimated that the vibration virtually extends beyond the open end of a tube to a distance equal to a fourth or a fifth part of its diameter.

is greatly increased; but if they arrive at different times, each tends to fill up the gaps in the other, and thus often to diminish the effect. Hence if a musical sound enters a room by two windows, a person standing between the windows on the opposite side might receive the pulses from each at the same time, while one by his side, being nearer one window than the other, would receive the pulses at different times.

Again, a person still further to one side would receive pulse No. 1 from the further window at the same time as pulse No. 2 from the nearer window, and the sound would be reinforced. Evidently the difference of his distances from the two windows must be the same as that between two pulses, or in other words, a wave-length. There will be reinforcement again when one window is 2, 3, 4, etc., wave-lengths further off than the other; but whenever there is a fraction of a wave-length involved there will be more or less interference. The same holds for a series of windows, or when sound arrives by any two channels whatsoever. We can always find the wave-length of a given note if we know the smallest difference in the length of different channels producing reinforcement or interference.



DIFFRACTION-GRATING.

§ 101. **Diffraction-Grating.** — Precisely the same method is applied to light. An ordinary diffraction-grating (see illustration) consists of a series of lines with slits between them, through which light passes.

We find the difference in length of the paths followed by the light arriving at a given point by two successive slits, and this is the wave-length of the light which is reinforced at that point by the grating.

There is an obvious advantage in employing a grating with a large number of lines, let us say a thousand. If each line is exactly one wave-length further off than the next, a thousand pulses will arrive simultaneously at the eye; but if there is the least error in adjustment, let us say a thousandth of a wave-length, the pulses will all arrive at different times, and thus produce complete interference.

It is to be observed that waves of light and sound tend to move in straight lines only when the breadth of the waves is considerably greater than the distance between them; hence the phenomena of bending or diffraction in passing through narrow orifices. Sound-waves, being on the average a million times farther apart than waves of light, bend much more readily, and require a screen proportionally broad to produce a distinct "sound-shadow." The longest light-waves are, however, comparable with the shortest waves of sound. All waves bend round a small obstacle very much like the waves of the sea.

§ 102. **Refraction.** — If a line of soldiers should march obliquely into a swamp, those who met it first would be most retarded, and their front would change its direction. In the same way a wave changes its direction in entering a medium in which it moves more slowly. Let AB (Fig. 5) be the wave-front *in vacuo* advancing in the direction AC at right

angles to AB; and let CD be the wave-front advancing in the direction BD at right angles to CD, after passing through the surface BC of a refracting medium. Since the time in passing from A to C is the same as from B to D, AC is to BD as the velocity *in vacuo* is to the velocity in the refracting

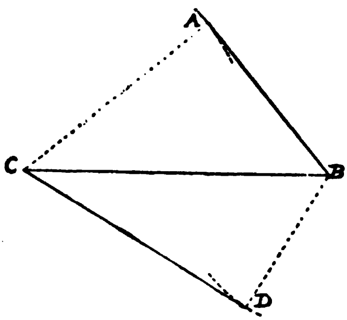


FIG. 5.

medium; but $\frac{AC}{BC}$ is the sine of ABC, which may be called the angle of incidence (i), and $\frac{BD}{BC}$ is the sine of BCD, the angle of refraction (r); hence

$$\frac{\sin i}{\sin r} = \frac{AC}{BC} \div \frac{BD}{BC} = \frac{AC}{BD}.$$

The ratio of AC to BD, or the velocity *in vacuo* to the velocity in a given medium, is called the index of refraction of that medium, μ, and hence is calculated by the formula

$$\mu = \frac{\sin i}{\sin r}.$$

The index of refraction of glass, for instance, is given as 1.5, nearly. This means that light travels half as fast again *in vacuo* as in glass.

§ 103. **Law of Lenses.** — When waves of light diverging from a point B (Fig. 6) pass through a lens $A\,I$, and converge to a point H, the central portions are clearly retarded by a constant amount $D\,F$ included between two spherical surfaces $A\,F\,I$ and $A\,D\,I$ with B and H respectively as centres. $D\,F$ may be divided by a plane $A\,E\,I$ into two portions, $D\,E$ and $E\,F$, which, by geometry, are inversely as the distances $B\,E$ and $H\,E$ (nearly), called conjugate focal lengths. As $D\,F$ must be constant, $D\,E + E\,F$ must be constant, — hence also the sum of the reciprocals of the conjugate focal lengths.

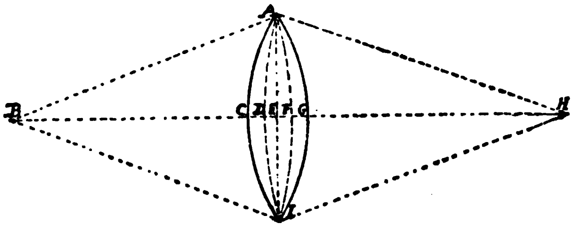


FIG. 6.

When rays emanate from a distant point, like a star, so as to be nearly parallel, they are focussed at the shortest possible distance by a given lens. This distance is called the principal focal length. As its conjugate is very large, the reciprocal of this conjugate may be neglected. Hence the law of lenses: *The reciprocal of the principal focal length* (F_0) *is equal to the sum of the reciprocals of any two conjugate focal lengths* (F_1 *and* F_2), *or*

$$\frac{1}{F_0} = \frac{1}{F_1} + \frac{1}{F_2}$$

The calculation of the index of refraction of a lens will be explained in Part X.

§ 104. **Images.** — If waves of light emanate, not from a single point, as *B* in Fig. 6, but from several such points, as *B*, *B'*, *B''* (Fig. 7), they will be focussed at several points, as *H*, *H'*, *H''*, so situated as to be in the straight lines *B E H*, *B' E H'*, *B'' E H''*, as the middle of a lens, having two parallel surfaces, does not bend the rays.

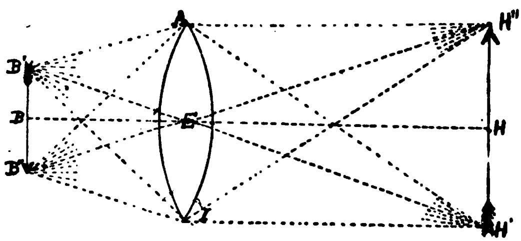


Fig. 7.

Since every point *B* is represented, we find at *H* a perfect image of an object at *B*, but completely inverted; and the separation between any two points is clearly proportional to the relative distance of the image and object from the lens.

We distinguish between real and virtual images. *H*, *H'*, *H''* is a real image of *B*, *B'*, *B''*, because the rays of light from *B*, *B'*, *B''* actually meet at *H H'*, *H''*, respectively, and again diverge from these points as from a real object. A photograph requires a real image for its production. On the other hand, an image in a looking-glass is virtual, because rays do not really meet in it or diverge from it.

A virtual image may be located as in experiment 43. When for instance an object is too near a convex lens to have a real image on the opposite side, we may still find a virtual image behind the object. That is, rays diverging from the object may, after passing through the lens, seem to diverge from a more distant point on the same side of the lens as the object.[1] Concave mirrors furnish similar examples of real and virtual images. Convex mirrors and concave lenses do not tend to bring rays to a focus, and give therefore only virtual images.

[1] By a construction similar to Fig. 6 it may be shown that in such cases the reciprocal of the principal focal length is equal to the *difference* of the reciprocals of two conjugate focal lengths.

CHAPTER VIII.

FORCE AND WORK.

§ 105. **Components and Resultants.** — When a body moves from A to B (Fig. 8), then from B to C, it passes of course from A to C; the two motions $A\,B$ and $B\,C$ may also be thought of as relative motions taking place at the same time. Let the points A, B, and C all start at A; let B move with respect to A, through the distance and in the direction $A\,B$, and at the same time let C move *with respect to* B through the distance and in the direction $B\,C$; then clearly C has moved with respect to A through the distance and in the direction $A\,C$.

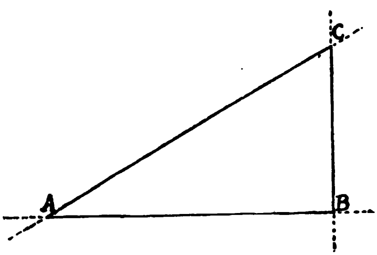


FIG. 8.

We express this fact by calling the motion $A\,C$ the *resultant* of the two motions $A\,B$ and $B\,C$, and by calling $A\,B$ and $B\,C$ *components* of $A\,C$, because when

compounded together they produce $A\ C$. We shall have occasion to consider only components which are at right-angles.

If $A\ B$ and $B\ C$ are motions which take place in the unit of time, they represent velocities; hence clearly the resultant of two velocities $A\ B$ and $B\ C$ is $A\ C$.

Again $A\ B$ and $B\ C$ may represent component velocities which a body acquires in the unit of time; in other words, component accelerations (§ 11); evidently the resultant of two accelerations $A\ B$ and $B\ C$ must be an acceleration $A\ C$.

Finally, we may multiply the accelerations $A\ B$, $B\ C$, and $A\ C$ by the mass of the body which they affect, without disturbing their relative values; but the products of mass and acceleration are forces (§ 12); hence two component forces, $A\ B$ and $B\ C$, must give a resultant force $A\ C$.

In fact it is evident that all quantities involving distance and direction, whether motions, velocities, accelerations, or forces, must be compounded by the same rules as lines in geometry.

Now since $A\ B$ and $B\ C$ are geometrically equivalent to $A\ C$, $B\ C$ must be the geometrical difference between $A\,B$ and $A\,C$. Hence a change of velocity from $A\,B$ to $A\ C$ means the acquisition of a new velocity, $B\ C$. We are thus able to represent the change of velocity consequent on a change of direction as well as from a change in magnitude.

Again, a motion $A\ C$ carries a body as far away from the line $A\ B$ as the motion $B\ C$, and a motion $A\,C$ carries it as much nearer to $B\ C$ as a motion $A\ B$.

Hence if the components, $A B$ and $B C$, are at right-angles, $A B$ and $B C$ measure respectively the effects of a motion $A C$, in the general directions $A B$ and $B C$.[1]

§ 106. **Absolute Measurement of Force.** — If a body is free to move in every way, the force acting upon it is always said to have the same direction as the velocity which the body acquires, as explained in the last section. It is also said to have a magnitude such that the product of the force f and the time t it acts is equal to the product of the mass m acted upon and the velocity v acquired. This *definition* of force is expressed also by the formula

$$ft = mv.$$

Experience shows that force defined as above corresponds to that which we ordinarily measure with a spring-balance.

The student should bear in mind that the fundamental law of motion contained in the formula applies only to bodies perfectly free to move, like masses in astronomy. It is a common fallacy to suppose that force is necessary to *maintain* motion. Our formula

[1] The relation between the components and resultants of forces may be illustrated by the strains which they produce. Let A be the head of a nail bent by one force from A to B, and by another force from B to C. As a result, it is bent from A to C. Now by Hooke's Law, as explained in § 114 below, forces are proportional (with certain limitations) to the strains produced; hence two forces $A B$ and $B C$ must have a resultant $A C$ when estimated in this way.

Again, a nail bent from A to C is bent in the general direction $A B$ by the same amount as if bent from A to B; and in the general direction $B C$ the same as if bent from B to C. Hence $A B$ and $B C$ are the components of $A C$ in their respective directions.

expresses the fact that, in the absence of friction or other interference, motion maintains itself; for if $f = 0$, $v = 0$, — that is, in the absence of force there is no change of velocity either in magnitude or in direction. This is essentially Newton's first law of motion. The force which one body exerts upon another is found to be equal and opposite to that with which the second body reacts upon the first. It is necessary, therefore, to measure only one of these forces.

§ 107. **Average Velocity.** — If we take any series of consecutive numbers beginning at 0, we shall find the average value to be half the last value. Thus the average of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is 5. So if we begin with a body at rest, and increase its velocity uniformly up to a given point, the average velocity will be half the final velocity.

The average velocity is also found if we divide the distance traversed by the time; or the distance a body moves is the product of the average velocity and the time.

§ 108. **Laws of Falling Bodies.** — The force in dynes which gravity exerts upon a body is the product of the mass m in grams and the intensity of gravity g, in dynes per gram. Substituting mg for f in the general formula of § 106, we have

$$mgt = mv, \quad \text{or}$$
$$gt = v.$$

The velocity acquired by a falling body is therefore proportional to the intensity of gravity and to the time it acts.

The final velocity is, by the last section, equal to $\frac{1}{2}v$; and the distance d traversed, being the product of the average velocity and the time, is

$$d = \tfrac{1}{2}\, vt.$$

Substituting the value of v above we have

$$d = \tfrac{1}{2}\, gt \times t = \tfrac{1}{2}\, gt^2.$$

In other words the distance a body falls is proportional to the intensity of gravity and to the square of the time.

Again, we find the value of t,

$$t = \frac{v}{g};$$

and substituting this in the last formula, we have

$$d = \tfrac{1}{2}\, g \times \frac{v^2}{g^2} = \tfrac{1}{2}\,\frac{v^2}{g}.$$

The square of the velocity which a body acquires is therefore proportional to the distance fallen.

The same formulæ express the relation between the velocity lost by a body projected vertically upwards, the time it takes it to reach its highest point, and the distance it rises in so doing.

§ 109. **Ballistic Pendulum.** — When a body A suspended by a vertical cord AC (Fig. 9) is given a horizontal velocity v along the arc AB, it continues until it reaches a point B at a vertical height AD above A the same as if it had been projected vertically upwards. The reason of this will be seen later on, when we have considered problems in the conservation of energy. We have from the last section

$$AD = \tfrac{1}{2}\,\frac{v^2}{g}.$$

Drawing the diameter AE, and the chords AB and BE, we have in the similar triangles ABE and ADB,

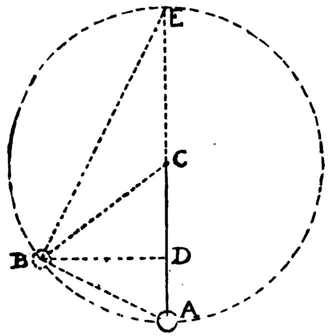


FIG. 9.

$\overline{AD} : \overline{AB} :: \overline{AB} : \overline{AE}$, or $AD = \overline{AB}^2 \div \overline{AE}$. Hence, substituting,

$$\overline{AB}^2 \div \overline{AE} = v^2 \div 2g;$$

and as $\overline{AE} = 2\overline{AC}$,

$$v^2 = \overline{AB}^2 \times g \div AC,$$

$$v = AB\sqrt{\frac{g}{AC}}.$$

The velocity of a pendulum at its middle point is therefore proportional to the distance AB of the point where it turns, measured in a straight line; that is the velocity is proportional to the chord of the arc AB. This is the principle used in comparing velocities by the ballistic pendulum.

We shall see that a suspended magnet differs from a pendulum chiefly in the nature of the force which causes it to return to its normal position. When a

needle, previously at rest, is given a sudden angular velocity, the arc through which it swings is called the *throw* of the needle. The velocity is therefore proportional to the chord of the throw.

§ 110. **Laws of Vibration.**—The square of the velocity of a pendulum at the middle point of its swing resulting from a given displacement is seen from the last section to vary as the intensity of gravity, and inversely as the length of the pendulum. We may infer that the length of a pendulum is proportional to the square of the time occupied by a single swing; and the force acting upon it is proportional to the square of its rapidity of oscillation.

The same principle applies to a magnetic needle, and is frequently used in comparing the strength of the forces which are exerted upon it. See Experiments 75 and 82.

§ 111. **Isochronism.**—It is well known that a pendulum vibrating in a very small arc keeps almost exactly the same time as in a comparatively large one. This shows that the average velocity of the pendulum (§ 107) must be proportional to the arc. The explanation is simply this, that the force urging the pendulum towards its middle point becomes greater as the arc increases. This force is proportional to $A\,F$ (Fig. 10), perpendicular to $B\,C$, drawn as in § 109 and hence approximately equal to the distance $A\,B$ which the pendulum must travel. We have already seen that the velocity acquired in reaching the middle point is proportional to the chord $A\,B$ and hence approximately to the arc.

From the fact, however, that the lines *A F* and *A B* are not quite equal to the arc *A B*, we infer that a common pendulum is not perfectly isochronous. The effect of different arcs on the rate of vibration will be

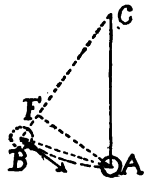


FIG. 10.

found in Table 3, column *g*. In all experiments with a pendulum or with a vibrating needle, we must limit the arc of oscillation according to the degree of accuracy required.

§ 112. **Point of Application of a System of Forces.** — It may be observed that the weight of a body acts as if a single force were applied to a certain point called the centre of gravity, and that it must be sustained by a single force, or its equivalent, applied in the same vertical line with the centre of gravity, equal and opposite to the weight of the body in question, in order that the body may remain at rest. In the case of a magnet the forces which it exerts act for most purposes as if they came from two points, represented in Fig. 13, § 126. We say therefore that the point of application of the forces exerted by gravity is at the centre of gravity, while the centres of magnetic forces are at two points called poles.

§ 113. **Couples.** — A pair of forces equal in magni-

tude but opposite in direction are said to constitute a *couple*. The perpendicular distance between the lines in which they act is called the arm of the couple; the product of the magnitude of either force and the arm of the couple is called the *magnitude* of the couple.

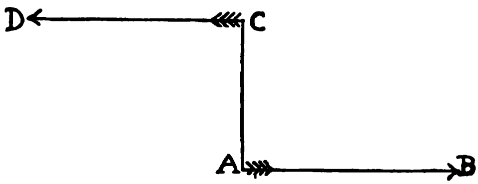


FIG. 11.

Thus AB and CD (Fig. 11) constitute a couple with an arm AC, and magnitude $AB \cdot AC$. The effect of a couple in a given plane ($ABCD$) does not depend upon the location or direction of the arm AC with respect to the (rigid) body acted upon, and it is indifferent at what points in the lines AB and CD the corresponding forces are applied. A left-handed couple ($AB \cdot AC$) can be balanced only by an equal and opposite right-handed couple ($A'B' \cdot A'C'$) such that

$$AB : A'B' :: A'C' : AC.$$

§ 114. **Hooke's Law.** — The effect of a force applied at the end of a rod is either to stretch or to bend it; the effect of a couple is to twist a rod. These effects are found to be proportional to the magnitude of the forces or couples in question. Hooke's law "ut tensio sic vis" may be translated, *strains are proportional to stresses.* (See § 22.) The ratio of a stress to a strain constitutes what is called a *modulus of elasticity*.

§ 115. **Laws of Flexure.** — The force required to bend a beam is evidently proportional to its breadth, but the thickness must be taken three times into account, first, because a greater strain or distortion necessarily accompanies a given amount of bending; second, because (as in the case of breadth) there is more material to be bent, and third, because the force has less purchase upon the material.

The force required is in fact proportional to the cube of the thickness. It can be shown in a similar way to be inversely as the cube of the length, for less force will be required, first, because it has a greater purchase; second, because the longer the beam is, the less sharply need it be bent to deflect it through a given angle; and third, because it takes a smaller angle to produce a given deflection.

§ 116. **Laws of Torsion.** — The couple required to twist a rod of a given shape increases with its breadth or thickness, first, because the average strain or distortion is greater — at the edges, for instance; second, because the purchase of the forces is less; third, because the material acted upon is proportional to the breadth; and fourth, because the material is also proportional to the thickness. In the case of a square or round rod the couple is therefore[1] proportional to the fourth power of the diameter. It is also inversely as the length, because the strain is less in proportion to the length of the rod for a given amount of twisting.

[1] It may be remarked that if there are N *independent* reasons why one quantity should increase in proportion to another quantity, the former always varies, other things being equal, as the N^{th} power of the latter.

§ 117. **Measurement of Work.** — Work is measured by multiplying together the distance through which a point has moved and the force which has been overcome. Thus the work transmitted through a belt can be found if we know the difference of tension between the two portions moving respectively to and from the driving-wheel, and the total distance traversed. If the belt is prevented from moving, as in Experiment 69, we can find the work done by the wheel in rubbing against the belt. We multiply together in this case the difference of tension in the belt and the distance traversed by the rim of the wheel. The work in question is transformed by friction into heat, but it could easily be utilized by allowing the belt to turn machinery. The measurement of work transmitted through a belt while in motion is more or less complicated.

§ 118. **Work of Water under Pressure.** — The work represented by a flow of water under pressure is easily calculated. Suppose the orifice to be 1 *sq. cm.* in section; then the force behind the stream is numerically equal to the pressure (see § 63). Let the stream advance 1 *cm.*; then the work done, being the force times the distance, or in this case the pressure times the distance, is also numerically equal to the pressure. The volume of water which escapes from the orifice is clearly 1 *cu. cm.* Hence the work done on 1 *cu. cm.* is numerically equal to the pressure. The same is also true, no matter what the size of the orifice may be; for with a given pressure per *sq. cm.* the force must vary with the cross section of the stream, and hence also

the work represented by an advance of 1 *cm.*; but the volume in *cu. cm.* delivered also increases in the same proportion, and therefore the work per *cu. cm.* remains the same.

Since pressure in dynes per square centimetre is numerically the same as work in ergs per cubic centimetre, we have the following rule: To find the work in ergs represented by a flow of water under pressure, multiply together the flow in cubic centimetres and the pressure in dynes per square centimetre.

§ 119. **Work done by Oblique Forces.** — When the direction of the force and the motion is not the same, we consider only the effect or component of the force in the direction of the motion (see § 105); or we may, on the other hand, take the component of the motion in the direction of the force, and multiply by the whole force in question; because in taking the component of either the force or the motion we reduce it in a given proportion determined by the angle between the two directions in question (see § 105). Evidently it makes no difference which of the two terms in a product is thus reduced.

§ 120. **Conservation of Work.** — It follows from the principle set down in the last section that moving from A to B (Fig. 12), then from B to C, against a force acting in any fixed direction, FF', requires the same amount of work as in moving directly from A to C. For if we drop perpendiculars AA', BB', CC', upon the line FF' representing the direction of the force, the components of the motions are $A'B'$, $B'C'$, and $A'C'$ respectively, and since these are in the same

straight line, $A'B' + B'C' = A'C'$. That is, the sum of the component motions is the same by a direct or by an indirect path, and hence also the work required,

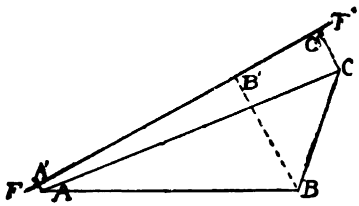


FIG. 12.

or the product of these components by the whole force in question. The fact that no work is gained or lost by choosing different paths is an illustration of the more general principle of the conservation of energy.

§ 121. **Energy of a Moving Body.** — A question which often arises is, how much work is stored up in a moving body, as for instance in a gram of matter with a velocity one *cm. per sec.* Suppose a dyne to act on a gram at rest, we know that it would give it, by definition (§ 12), in one second a velocity of one *cm. per sec.* We know (by § 107) that the average velocity for this second will be half a centimetre per second, or that the gram will have moved $\frac{1}{2}$ *cm.* The work done upon it is therefore $\frac{1}{2}$ *dyne-cm.* = $\frac{1}{2}$ *erg.*

To give a gram twice the velocity in the same time would require twice the force and double the average velocity; the distance would also be doubled. This would mean four times the work. In the same way three times the velocity would mean nine times the

work, or in general the work done upon a moving body is proportional to the square of its velocity. It is obviously also proportional to the mass; and as 1 gram with a velocity of 1 *cm. per sec.* has been found to contain $\frac{1}{2}$ erg, we have the following rule: Multiply the mass in grams by the square of the velocity in centimetres *per sec.* and divide by 2 to find the work in ergs which a moving body contains.

It is easily found by calculation that a moving body in coming to rest can do the same amount of work as was required to set it in motion. A gram, for instance, with a velocity of 1 *cm. per sec.* will be brought to rest by a force of 1 dyne in 1 second. The average velocity is therefore $\frac{1}{2}$ *cm. per sec.*; the distance traversed $\frac{1}{2}$ *cm*; the work done against 1 dyne through a distance of $\frac{1}{2}$ *cm.* is $\frac{1}{2}$ erg, — the same that was required to start it in motion.

§ 122. **Conservation of Energy in Mechanics.** — Work stored in a body is often called energy. Energy is again defined as the power of doing work. We distinguish between the energy of motion of a body (kinetic energy) and its energy of position (potential energy), due to the level, for instance, to which it has been raised. All kinds of energy are measured in ergs.

We have seen that it takes the same amount of work to raise a body from one level to another, no matter by what path it may be raised (§ 120). When it returns to the original level the work is given back. The energy spent in setting a body in motion is also restored when the body comes to rest (§ 121).

Energy of position may be changed into energy of motion and the reverse, as is particularly evident in the case of falling bodies or bodies projected into the air; but in mechanics *no energy is ever lost.* This statement is an illustration of a more general principle known as the "Conservation of Energy" (§ 149).

CHAPTER IX.

ELECTRICITY AND MAGNETISM.

§ 123. **Nature of Electricity and Magnetism.** — We do not know what electricity and magnetism are; that is, we are ignorant of their fundamental relations to matter and motion. Electricity circulating around the particles of steel is believed by many to be the sole cause of its magnetism. This hypothesis accounts for all the observed effects. It has been suggested by leading scientific men that the rapidity with which light is transmitted may be due to electrical action (see § 93), and it is suspected that chemical affinity is closely related to electricity. (See §§ 142–144.) We speak of electricity as if it were a fluid; but there are three reasons why neither electricity nor magnetism can be regarded as a fluid in the ordinary sense: first, they have no inertia (or resistance to being set in motion); second, they have no weight (or attraction for ordinary matter under the law of universal gravitation); and third, they repel, instead of attracting their own kind.

In the first two respects electricity and magnetism resemble heat more than a fluid. It has been suggested that they may be forms of energy; but there are more objections to this view than to the other,

and comparatively little help is to be derived from it. Even if electricity were proved to be a kind of motion, we should still think of it as a fluid, as we do of heat when it is said to *flow* from one point to another (§ 74).

§ 124. **Positive and Negative Electricity.** — As compressed air can be distinguished from rarefied air, so positive may be distinguished from negative electricity. When mixed together they neutralize one another; and in this neutral condition, electricity, like the atmosphere, seems to be everywhere present. Positive electricity can be separated from negative by various means; but we produce in all cases equal quantities of both. For instance, glass rubbed with a piece of silk receives a positive charge; an equal charge of negative electricity is found in the silk. Some writers maintain that there are really two distinct kinds of electricity which unite, somewhat as an acid does with a base to form a neutral compound; and mathematicians are apt to take this view, finding it convenient to treat electricity as incompressible. Positive electricity may, however, be thought of as under greater pressure than negative, whether it yields to that pressure or not. We imagine that it is this pressure which causes electricity to flow from one place to another. We consider only the flow of positive electricity; though it is maintained by some that half the effect is due to the flow of an equal quantity of negative electricity in the opposite direction.

§ 125. **Electrical Attractions and Repulsions.** — Two bodies charged with positive electricity repel each

other, or two charged with negative electricity repel each other; but a body charged with positive electricity attracts one with a negative charge. The force exerted is proportional to the charge, or quantity of electricity in each body. It is, in fact, equal to the product of the two charges, divided by the square of the distance between them. There is also a mutual repulsion between different portions of the same charge, which tend therefore to fly as far apart as possible. Hence electricity collects in the surfaces of bodies which conduct it, and (except while flowing through them) is never found at any appreciable depth.

§ 126. **Nature of a Magnet.** — In a similar way positive and negative charges of magnetism may be sepa-

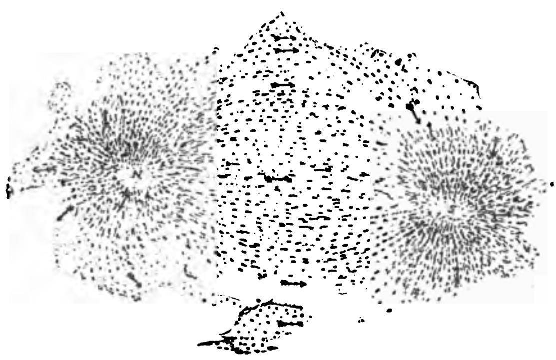


Fig. 13.

rated, but only in a few substances like steel. With magnetism, as with electricity, a positive charge implies an equal negative charge; but in the case of magnetism both charges are always found in *the same*

body. Such a body constitutes a magnet, and is said to have two poles, corresponding to the centres of positive and negative magnetism. The position of the poles *N* and *S* (Fig. 13) is shown by sprinkling iron-filings on a piece of glass over the magnet. The iron-filings arrange themselves in lines as in the diagram, radiating from the two poles *N* and *S*. One of these poles, *N*, is called north because, when the magnet is freely suspended, it tends to point approximately in that direction;[1] the other is called the south pole. The direction in which a magnet is said to point is always determined by its north pole.

§ 127. **Lines of Force.** — The iron-filings arrange themselves along what are called "lines of force." A small compass-needle placed close to the glass always points parallel to the lines of iron-filings, and gives the direction of the lines of force, as indicated by arrows in the diagram. The lines accordingly are said to come *from* the north pole, and go *to* the south pole. It is found that where the lines are closest, the magnetism is strongest. A strong horseshoe magnet can hold a solid mass of iron-filings between its poles.

§ 128. **Field of Force.** — The space around or between the poles of a magnet, wherever its action is felt, is called the *field* of force, or simply the *field* of that magnet. By the *intensity* of this field we mean the force exerted by the magnet on a unit quantity of magnetism (§ 17) placed at any point of the field.

[1] At Cambridge, Massachusetts, a magnet points very nearly north by west.

The intensity varies in different parts of the field. At a given point the intensity of the field due to a single magnetic pole is equal to the strength of the pole divided by the square of its distance from the point in question. Both poles of a magnet must, however, be taken into account in calculating the intensity of a field. The resultant (§ 105) of the forces upon a unit of positive or north[1] magnetism determines, by its direction and magnitude, both the direction of the lines of force, and the intensity of the field.

The earth, for example, is a great, though weak magnet. The intensity of its field at Cambridge, Massachusetts, is about 1 dyne per unit of magnetism; or more exactly, $\frac{3}{4}$ dyne. The lines of force are, however, more nearly vertical than horizontal, and only their horizontal component, or about one quarter of the whole effect, is felt by a compass. The angle between the lines of force and a horizontal plane (70°–80°) is called the magnetic dip.

The field of a dynamo machine may be several thousand times stronger than that of the earth.

§ 129. **Magnetic Attractions and Repulsions.** — Two north poles, or two south poles, repel each other; a north and a south attract; the force exerted is proportional to what we call the *strength* of each pole — in the case of two poles, it is equal to the product of their

[1] By "north magnetism" we mean the kind of magnetism contained in that end of a magnet which points north. This is evidently the opposite kind to that which we find in the north polar regions of the earth, since only dissimilars attract. The "magnetic north pole" of the earth is therefore technically a negative or south pole.

strengths divided by the square of the distance between them. Comparing this statement with that in § 128, we see that the force acting on a magnetic pole is equal to the product of its strength, and that of the *field of force* in which it is placed. The strengths of the north and south poles of a given magnet are always alike.

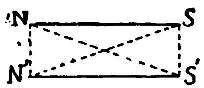


FIG. 14.

When two magnets with poles, N, S, N', S', of nearly equal strengths, $\pm s$, and $\pm s'$, are placed parallel and opposite to one another, as in Fig. 14, if the distance between them is d, there is a perpendicular repulsion between N and N' equal to $ss' \div d^2$; and one between S and S', of the same amount. There is furthermore an oblique attraction between N and S', also between N' and S; but if the distances NS' and $N'S$ are great in comparison with NN', or d, the oblique forces may be disregarded.[1] The resultant is therefore approximately equal to $2ss' \div d^2$.

By supposing one of the magnets reversed, we find in the same way a resultant attraction nearly equal to $2ss' \div d^2$. Counting attractive forces as negative, the

[1] The effective components of the oblique forces bear to the perpendicular forces a ratio equal to $(NN' \div NS')^3$. If NS' is 5 times as great as NN', the error committed by disregarding the oblique forces will be less than 1 per cent. The chief source of error in the application of the principles contained in this section lies in the fact that magnetic forces are only approximately centred in poles.

algebraic difference,[1] Δ, between the repulsion and the attraction will be

$$\Delta = 4\,\frac{s\,s'}{d^2}, \text{ nearly.}$$

We measure Δ by an ordinary balance in experiment 72, with a small distance, d, between two nearly equal magnets, and thus determine roughly the mean strength of the poles in question.

§ 130. **Action of Currents on Magnets.** — When an electric current flows through a wire, it affects all magnetic bodies in its vicinity. It creates, in fact, a magnetic field. When only a short portion of the wire is considered, the intensity of the field due to this portion is proportional to its length and to the strength of the current passing through it; the intensity also varies inversely as the square of the distance from the wire. The lines of force are perpendicular to the wire at every point. They are in fact circles with the wire at their centre, as shown by the arrangement of iron-filings about a vertical current, in Fig. 15. Hence, a magnet tends to point at right angles to an electric current, and to the line joining the two. To remember which way the magnet points, place the thumb across the forefinger of the right hand; if the

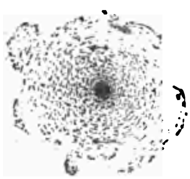


FIG. 15.

[1] Charges of magnetism which each magnet "induces" upon the other increase the mutual attraction of the magnets, but decrease their mutual repulsion by a nearly equal amount. The algebraic difference remains essentially the same.

finger represents the direction of the current, the thumb shows how the north pole of a magnet points.

§ 131. **Action of Magnets on Currents.**—Conversely, an electric current is acted upon by magnetic bodies in its neighborhood. It is, in short, affected by a magnetic field. The effect is equal, under the most favorable circumstances, to the product of the length of wire, the strength of the current, and the intensity of the field. In general, however, we consider only that portion or component of a current which is perpendicular to the lines of force. The direction in which a field acts upon a current is at right angles to the lines of force and to the current. To remember which way the field acts on the current, let the thumb represent a north pole as before, and the forefinger a current; then the thumb will point in the direction in which the pole is urged; hence as action and reaction are equal and opposite, the current must be urged towards the base of the thumb.

The lines of force due to the current are, as we have seen, parallel to the thumb; but those due to the pole are perpendicular both to the thumb and to the forefinger. They issue in fact from the north pole (see § 127) and follow, accordingly, the *line of pressure* between the thumb and forefinger. It is these lines alone which affect the current. Neither the pole nor the current is influenced by the field of force which it itself creates.

§ 132. **Magnetic Current Measure.** — From our definition of the unit of current (see § 18) and the laws stated in the last section, it is clear that the field of

force due to a current C flowing through a length of wire L at a distance D is equal to $CL \div D^2$, and that the action of a field of force F on the same current, if they are at right angles, is CLF. These expressions enable us to measure a current through its magnetic action, as will be explained further in §§ 133–135.

§ 133. **Constant of a Coil.** — The constant of a coil of wire is defined as the field at its centre due to a unit of current passing through the wire. If the radius of a circular coil is r, the number of turns of wire n, the length of wire is $2\ \pi\ r \times n$, every portion of which acts in the same direction on a magnet at the centre (see § 130); hence the constant is

$$K = \frac{L}{D^2} = \frac{2\ \pi\ rn}{r^2} = \frac{2\ \pi\ n}{r}.$$

§ 134. **Magnetic Area.** — A rectangular coil, $a\,b\,c\,d$, of wire in the plane of this paper, would be acted upon differently in different parts by a field of force in the same plane. Suppose that the current C circulates with the hands of a watch; and that a field acts from left to right. Then (by § 131) the sides $a\,b$ and $c\,d$ (Fig. 16) will not be affected; $\overline{a\,d}$ will be depressed with a force $C \times \overline{a\,d} \times F$, and $\overline{b\,c}$ will be raised with the same force; the two forces then constitute a couple, with an arm $a\,b$ and magnitude $C\,F \times \overline{ab} \times \overline{ad}$. The couple acting on a rectangle, $a\,b\,c\,d$, is therefore equal to the product of the current and field of force multi-

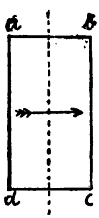
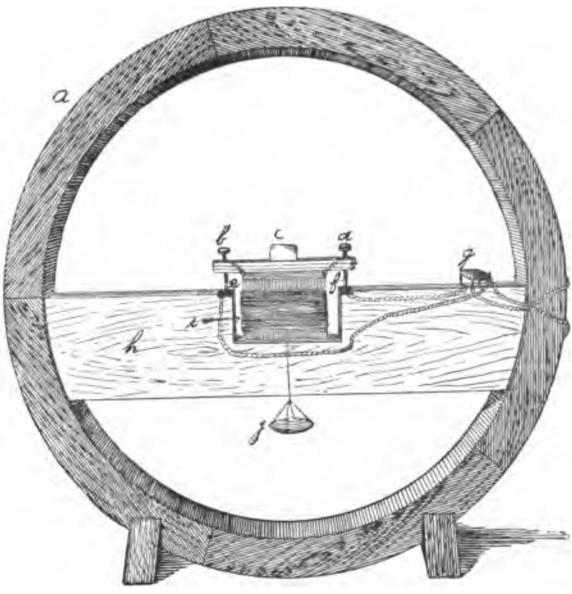


FIG. 16.

plied by the area of that rectangle. The same clearly holds for any number of rectangles or for their sum. A rectangular coil of wire consists essentially of a series of rectangles, *a b c d*, each carrying the current, C. The total area, A, enclosed by these rectangles is called the *magnetic area* of the coil, and determines the couple, $C\ F A$, acting upon the coil in a magnetic field, F, in its own plane.

§ 135. **Electro-Dynamometer.** — A common form of electro-dynamometer consists (see illustration) in a



ELECTRO-DYNAMOMETER.

coil of wire *a*, with a smaller coil *i*, at right angles to it near its centre. The larger coil is usually circular;

the smaller may be rectangular. If K is the constant of the large coil, a current C, circulating through this coil, will cause a field of force ($F = CK$) to act on the small coil; if the magnetic area of this is A, and the same current, C, passes through the small coil, the couple acting on the latter will be $CFA = C^2KA$.

When the constant K and magnetic area A are known it is only necessary to measure the couple in order to determine the current. A current is thus primarily measured by the force with which it acts on itself. We shall not need to consider currents through long conductors, except where, as in § 133 or in § 134, every portion is similarly situated with respect to the forces in question.

CHAPTER X.

ELECTROMOTIVE FORCE AND RESISTANCE.

§ 136. **Heating by Electricity.** — When a current of electricity passes through a wire, heat is developed in proportion to the square of the current and also to what we call the *electrical resistance* of the conductor. This is known as Joules's Law. When the power, or the rate at which heat is generated, reduced to watts (see § 15) is P, when the current in ampères (§ 19) is C, and when the resistance in ohms (§ 20) is R, we have $P = C^2 R$.

The resistance R of a conductor is thus easily found if we know the amount of heat developed in it by a given current in a given time. (See ¶ 172.)

§ 137. **Electrical Power.** — The work spent in one second in maintaining a current is obviously the same thing as the power, P; and the quantity of electricity flowing in one second is by definition equal to the current C; the ratio of the power to the current is therefore the same thing as the work spent per unit of electrical quantity, and is defined as electromotive force, E. Electromotive force corresponds therefore to hydrostatic pressure (see § 118), or rather, to a difference of hydrostatic pressure.

We have, therefore,

$$E = P \div C \text{ or } P = CE;$$

that is, electrical power (in watts) is equal to the product of the current (in ampères) by the electromotive force (in volts).

§ 138. **Ohm's Law.** — Since in the last section we found $E = P \div C$, and in the section before, $P = C^2 R$; we have, substituting, $E = C^2 R \div C = C R$. In other words, the electromotive force (in volts) is equal to the product of the current (in ampères) and the resistance (in ohms). It follows that the current (in ampères) is equal to the electromotive force (in volts) divided by the resistance (in ohms), or

$$C = \frac{E}{R}.$$

This is known as Ohm's Law.

A similar law discovered by Poiseuille holds for the flow of liquids through capillary tubes. If R is the resistance of such a tube as defined in § 20, E the hydrostatic pressure in *dynes per sq. cm.*, and C the current in *cu. cm. per sec.*, we have

$$C = \frac{E}{R}.$$

§ 139. **Electrical Potential.** — Electrical potential is analogous to pressure, or head of water. As water flows through a horizontal tube from places of high pressure to places of low pressure, so electricity flows from points of high potential to points of low potential. The electromotive force of a battery is the same thing as the difference in potential which it is capable of producing. Hence we may apply Ohm's Law as follows: the current (in ampères) through any con-

ductor (containing no source of electricity) is equal to the difference in potential of its two extremes (in volts) divided by the resistance (in ohms) between them, no matter how the difference of potential is kept up; and the difference of potential at the two extremes of such a conductor (in volts) is the product of the current (in ampères) and the resistance (in ohms). Denoting by c the current, by r the resistance, and by e, the difference of potential in any portion of the conductor, we have

$$e = cr.$$

Clearly, when a given current of electricity, c, travels along a wire it loses in potential by an amount, e, proportional at any point to the resistance, r, which has been overcome.

§ 140. **Resistance in Series and in Multiple Arc.** — When a current passes first through one conductor then through another, as we say in *series*, the total resistance is clearly the sum of the separate resistances; but if the current has a choice of two paths, like a congregation dispersing through two doors, it is less retarded than if confined to one alone.

Let ABC and ADC (Fig. 17) be two such channels as we say, in *multiple arc;*

Fig. 17.

if the resistance of ABC is R_1, and that of ADC, R_2, and the difference of potential between A and C is E,

then the current C_1 through $A\,B\,C$ is $C_1 = E \div R_1$; that through $A\,D\,C$ is $C_2 = E \div R_2$; the total current is $C = C_1 + C_2 = E \div R_1 + E \div R_2$. But if the combined resistance is R, we have $C = E \div R$. Equating the two values of C, and cancelling E, we have

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}\ ;\ \text{or}$$

the reciprocal of the combined resistance of two (or more) conductors in multiple arc is equal to the sum of the reciprocals of the separate resistances.

We notice also that the current through each channel is inversely as its resistance, or $C_1 : C_2 :: R_2 : R_1$, from which $C_1 : C :: R_2 : R_1 + R_2$, etc.

§ 141. **Wheatstone's Bridge.**—We have seen (§ 139) that loss of potential is proportional by a given path to the resistance overcome. Since in Fig. 17, § 140, in passing by either path from A to C, the total loss of potential must be the same, the loss in reaching B will be the same as in reaching D if the resistances of $A\,B$ and $A\,D$ bear the same proportion to the total resistances of $A\,B\,C$ and $A\,D\,C$ respectively. In this case no current will flow through a wire joining B and D (Fig. 18), since these points will have the same po-

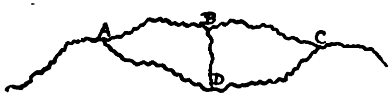


FIG. 18.

tential. A cross connection $B\,D$, between two parallel circuits $A\,C$, is called a Wheatstone's Bridge; and the absence of any current through it shows that the

four resistances AB, BC, AD, and DC are in proportion; that is,

$$AB:BC::AD:DC.$$

§ 142. **Electrolysis.** — When a current of electricity passes into and out of a fluid by means of two conductors, often called electrodes, the liquid is almost always decomposed, and its constituents liberated. The metallic elements are generally carried with the current, the acid constituents against it until they reach the electrodes. There they are either deposited, as in electroplating, or set free in the gaseous form, as in the electrolysis of water, or made to combine with the material of one of the electrodes, as the acid does with the zinc of an ordinary battery.

§ 143. **Electro-chemical Equivalents.** — As concerns the quantity of a given substance acted upon in electrolysis, neither the surface of the electrode nor the chemical nature of the reaction seems to have any effect. A given quantity of electricity always affects a given quantity of a given substance. Thus one ampère in one second causes about one 3000th of a gram of zinc to be dissolved from a zinc plate forming one of the electrodes, or deposits about three times as much mercury. The quantity of mercury is found to be the same, whether the nitrate or chloride is used; and a similar uniformity is found, in the case of other elementary substances, in regard to the quantity set free from their various salts. The weight of a substance acted upon by the unit quantity of electricity is called its electro-chemical equivalent. (See Tables 8 *b*, 11 and 12.)

§ 144. **Law of Electro-chemical Equivalents.** — Observation shows that the electro-chemical equivalents of different substances are to each other as their chemical combining proportions. Thus 2 parts of hydrogen combine with 16 parts of oxygen to form water, or with 71 parts of chlorine to form muriatic acid; again, 71 parts of chlorine or 16 parts of oxygen unite with 63 parts of copper or 65 parts of zinc; one ampère in about 192 seconds sets free 2 *mgr.* of hydrogen, 16 *mgr.* of oxygen, 71 *mgr.* of chlorine, dissolves 65 *mgr.* of zinc, and precipitates 63 *mgr.* of copper. There is evidently an intimate connection between electricity and the bonds which bind atoms chemically together; though no one as yet has offered a satisfactory explanation of the law of electro-chemical equivalents.

§ 145. **Calculation of Electromotive Force.** — Since we know the quantity of zinc dissolved by one ampère in a second ($\frac{1}{3000}$ *g*), the amount of heat which a gram of zinc gives out in combining with nitric acid (about 1500 units), and the value of one unit of heat per second in watts (4.2 nearly), we can evidently find the power spent on one ampère by multiplying these three together, and this should be (§ 137) the electromotive force developed by the action. Hence a battery in which the only reaction is the dissolving of zinc in nitric acid should have an electromotive force of about $\frac{1}{3000} \times 1500 \times 4.2$, or 2.1 volts.

The electromotive force E generated by any chemical action is accordingly 4.2 times the product of the electro-chemical equivalent and heat of combination in question. In the Daniell cell we must offset

against the electromotive force due to the solution of zinc, that due to the precipitation of copper, which is about one-half of the former, because the copper which is separated from the acid with which it is combined has very nearly half as much affinity for it as the zinc which takes its place. The electromotive force of a Daniell cell is therefore about 1 volt.

Experiment shows that electromotive forces can be calculated with more or less exactness in this way, as nearly all of the chemical energy is spent on the electric current. The actual electromotive force can never exceed its theoretical value.

§ 146. **Arrangement of Batteries.** — When we join several batteries together in multiple arc (Fig. 19), the

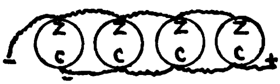


Fig. 19.

zinc poles having all the same potential, and the copper (or carbon) poles all the same potential, we gain nothing in electromotive force, any more than we should gain in pressure by connecting two reservoirs on the same level. The current is, however, often increased, owing to the diminished resistance (see § 140).

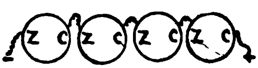


Fig. 20.

When, however, we join batteries in series (Fig. 20), so that the current passes in all cases from zinc to

copper, a given amount of work is done on the same current by each cell, as explained in the last section, and hence the electromotive force is increased in proportion to the number of cells. Unfortunately, the resistance is also increased in the same proportion, (§ 140).

In seeking to increase a current, it is as important to diminish resistance as to increase electromotive force (see Ohm's Law, § 138); and a practical rule often of service in the arrangement of a battery is to reduce the resistance of a battery by arrangement in multiple arc or to increase its electromotive force by arrangement in series until the internal resistance is equal as nearly as may be to the resistance of the outside circuit which is to be overcome.[1] In this way the greatest possible current will be obtained from a given number of cells through a given outside resistance. Thus for a very long telegraph line we prefer an arrangement of batteries in series; for a very short circuit an arrangement in multiple arc.

§ 147. **Induction of Electricity.** — When a wire of length L, carrying a current C, at right angles to the lines of force of a magnetic field F, is moved at right angles both to these lines and to itself with a velocity V, against the forces acting on it, evidently power is required of the magnitude $C L F V$ ergs per second; for the force overcome is $C L F$ (§ 132) and the distance traversed in one second is V. The power required *per unit of current* to keep up the motion is

[1] A similar rule applies to the arrangement of several electrical instruments, but from lack of space it cannot be dwelt upon here.

therefore $CLFV \div C$, or LFV. Experiment shows that this power is not spent, as one might expect, in heating the wire, but, through some agency which we do not understand, it acts upon the current in the wire. It produces, in fact, an electromotive force, E, which we have seen (§ 137) is equal to the power per unit of current.[1] That is $E = LFV$. The current is given accordingly by Ohm's Law (§ 138), if the resistance of the circuit is known. We are thus able, given the phenomenon, to anticipate the law governing what is called the *induction of electricity*.

We make use of induced currents, in Experiment 76, to compare the intensity of two fields of force; and in Experiment 77, to compare the intensity of the same field in two directions. In each case the motion of the wires is limited to a certain distance. If the distance is traversed rapidly we get a strong current for a short time; if slowly, a small current for a long time; the sudden throw of a galvanometer-needle (see § 109) is therefore dependent simply upon the strength of the magnetic field.

§ 148. **Thermo-Electricity.** — In regard to the electric current generated by heating or cooling a junction of two dissimilar metals, we observe that the electromotive force is approximately proportional to the temperature of the junction, within narrow limits. As

[1] The electromotive force in this formula is expressed in ergs per second per unit of current. Reducing the power to watts and the current to ampères, we find that the electromotive force in volts is equal to the product of the length of wire in centimetres, its velocity in centimetres per second, and the strength of the field in dynes per unit of magnetism divided by 100,000,000.

one junction in an electrical circuit implies another, it is the difference of temperature of these two junctions which we take into account.

When the range of temperature is considerable, the thermo-electric force is rarely proportional to the difference of temperature of the two junctions. Thus the current which flows ordinarily from copper to iron through a hot junction, increases up to 275°, then diminishes, and is reversed at a still higher temperature.

§ 149. **Conservation of Energy.** — The principle of the conservation of energy explained at the end of chapter VIII., applies to all transformations of energy, and forms the basis, as we have seen, of most important calculations. Whatever light, electricity, and magnetism may be, they return to us eventually in some form the energy spent in creating them. Energy, like matter, may be transformed or scattered, but cannot be destroyed.

ADDENDA.

AMBIGUOUS TERMS.

§ 150. **Gravity.** — Ordinary matter has two characteristic properties: inertia (§ 151), and gravity. The continual changes which take place in the velocities of heavenly bodies, or in the directions of their motions, are attributed to gravity. To account for these changes, it is necessary to suppose an attraction between different bodies which, other things being equal, varies inversely as the square of the distance between them. This is known as Newton's Law of Universal Gravitation. It is not confined to heavenly bodies alone, but holds for any two bodies of matter, however small; though the operation of the law may be concealed by other phenomena. That property in matter which makes it attract other matter is properly called its *gravity*. We say, for instance, that "gravity" draws all bodies toward the centre of the earth. In such expressions as the "acceleration of gravity," the earth's gravity is usually referred to. A body cannot strictly be said to fall under the influence of *its own* gravity. Gravitation is a *mutual* attraction, existing only between two different bodies of matter. We must distinguish

between forces of gravitation, which depend upon the distances between bodies, and their gravity proper, which is invariable so long as no change is made in the *quantity of matter* which they contain. An estimate of the quantity of matter, founded upon this invariable property is usually designated by the word *mass*, notwithstanding the fact that "mass" is strictly defined without any reference to gravity whatsoever (see § 152). It is also designated by the word "weight," though this has properly an entirely different signification (see § 153).

Either the word "mass" or the word "weight" *may* mean, accordingly, an *estimate of the quantity of matter which a body contains, founded upon gravitation.* Thus the number of grams (§ 6) by which a body can be balanced determines its "weight in grams." The word "weight" should always be qualified in this way when it refers to a quantity of matter; and when thus qualified it is preferable to the word "mass" as applied to measurements depending upon gravity.

§ 151. **Inertia.** — Bodies do not move instantly from one place to another under the action of forces. More or less time is always required to set a body in motion, to turn it one side, or to bring it to rest. These facts are explained as the result of a universal property of matter called *inertia.* There is, however, no agreement amongst scientific men as to the exact meaning of this term. Inertia is described by some writers (in accordance with the original meaning of the Latin word) as the "inability" of matter

to move itself. According to Ganot,[1] "Inertia is a purely negative, though universal, property of matter." Other writers associate with inertia a certain power or necessity. An old term, *vis inertiæ* (force of inertia), illustrates this view. Inertia has been defined as "that property of matter which makes the application of a force necessary for any change in the magnitude or direction of a body's motion."[2] "The fundamental principle of physics," says Deschanel,[3] "is the inertia of matter."

We must distinguish between the so-called *forces of inertia* — that is, forces of greater or less magnitude required under different conditions to produce changes in the motion of a body — and the inertia proper of a given body, which, like its gravity (§ 150), depends only upon the quantity of matter which it contains. An estimate of a quantity of matter, founded upon this invariable property, is designated by the word *mass* in its strict scientific signification (see § 152).

§ 152. **Mass.** — The word *mass* is thought to have the same origin as the German maaß, and to denote, literally, a *measure* of the quantity of matter which a body contains. The mass of a body is strictly defined as the number of standard units of quantity (§ 6) to which a body is equivalent in respect to inertia (§ 151). This is what is always meant by the "dynamical mass" of a body. There are various dynamical devices by which masses may be compared

[1] Ganot's Physics, § 19. [2] Hall's Elementary Ideas, page 5.
[3] Deschanel's Natural Philosophy, 1878, § 6.

(Exps. 59–60); but none leading to very accurate results. It is, however, inferred from results obtained with pendula constructed of different materials (Exp. 58), that there is no perceptible difference between the mass and the weight of a body when both are estimated in grams. The best comparisons of mass are made, accordingly, by means of an ordinary balance. In practice the word "mass" means the number of grams to which a body is equivalent in respect to weight. It is in other words (practically) the same thing as "weight in grams" (§ 150).

§ 153. **Weight.** — Weight is, as we have seen (§ 150), sometimes used to denote the quantity of matter which a body contains. The proper use of the term is, however, in the sense of a force. The weight of a body is strictly defined as the force with which it is attracted by the earth's gravity. In this sense weights should be accordingly expressed in dynes (§ 12). To avoid confusion between the different meanings of the word "weight," it is well to qualify it even when used in its strictest sense. To speak, for instance, of the "weight in dynes" of a body leaves no doubt that it is the idea of force which we wish to convey.

It may be observed that the "weight in dynes" of a body varies with the intensity of the force of gravity exerted upon it; but that the "weight in grams," being practically the same thing as the mass of the body, remains always the same.

§ 154. **Density.** — The density of a body is strictly defined as the ratio of its mass to its volume (§ 9).

Since, however, we usually estimate masses by balancing them with gram weights, and since volumes are measured in cubic centimetres (§ 9), density means in practice the quotient obtained when the weight in grams of a body is divided by its volume in cubic centimetres. The weight is supposed in all cases to be corrected for the buoyancy of air, or in other words, *reduced to vacuo* (§ 67); the volume is supposed to be measured at 0° or reduced to 0°, unless the temperature of the experiment is stated.

If V is the volume of a body in *cu. cm.*, M its mass (or practically its weight) in grams, and D its density, we have accordingly —

$$D = \frac{M}{V}, \qquad (1)$$

whence

$$M = DV, \qquad (2)$$

and

$$V = \frac{M}{D}. \qquad (3)$$

It follows that the density of a substance is numerically equal to the number of grams contained in 1 *cu. cm.* Thus 1 *cu. cm.* of lead weighs (see Table 8) from 11.3 to 11.4 grams; and 1 *cu. cm.* of dry air usually weighs (see Table 19) from .0011 to .0013 grams.

§ 155. **Specific Volume.** — The specific volume of a body is defined as the ratio of its volume to its mass. It is found in practice by dividing its volume in cubic centimetres by its weight in grams. The

specific volume (S) of a substance is accordingly the reciprocal of its density; that is (see § 154),

$$S = \frac{1}{D}, \tag{1}$$

whence

$$S = \frac{V}{M}, \tag{2}$$

$$V = MS, \tag{3}$$

and

$$M = \frac{S}{V}. \tag{4}$$

We must distinguish apparent specific volumes from true specific volumes. The true specific volume of a substance is the space occupied by a quantity of that substance weighing 1 gram *in vacuo*. The *apparent* specific volume is the space occupied by a quantity weighing apparently 1 gram in air. Apparent specific volumes are accordingly affected by the density of air. The apparent specific volumes of water under different conditions are contained in Table 22, and are useful in calculations of volumes in hydrostatics. If w is the apparent weight of water, and s its apparent specific volume, the true volume v is given by the equation (see 3),

$$v = ws. \tag{5}$$

§ 156. **Correction and Error.** — Mistakes sometimes arise from confusion between the terms "correction" and "error." If o is the observed magnitude of a quantity, q, the error of observation is $o - q$. A correction is defined as a quantity which added algebraically

to an observed magnitude (o) will give the true magnitude (q). It is equal, accordingly, to $q - o$. If the observed value is greater than the true value, it follows that the error is positive, the correction negative; but if the observed value is less than the true value, the error is negative and the correction positive. In every case the correction and the error are equal and opposite.

If e is the "*probable error*" of observation (see § 50), we have by definition,

$$o + e > q > o - e, \textit{ probably},$$

or in the conventional system of representation (§ 53),

$$q = o \pm e.$$

The student must not be led by this expression to imagine that the "probable error" of a result is to be added to it or subtracted from it. He should bear in mind that the so-called "probable error" is not literally a probable error (see § 50), but simply a limit within which the error is probably confined. Even if we knew the magnitude of the error, it would still be impossible to correct for it, since the sign is unknown. No matter how great the probable error of our observations may be, results strictly calculated from these observations are generally *less improbable* than those obtained by making allowances for errors which we do not know to exist.

NOTES

ON THE

ARRANGEMENT OF MATHEMATICAL AND PHYSICAL TABLES.

METHODS OF CONDENSATION.

THE object of constructing mathematical or physical tables is to condense into a small space a large number of results obtained either by calculation or by observation. There are various well-known methods by which condensation may be effected. Thus, instead of writing

The square of the number 1 is 1.
The square of the number 2 is 4. I.
The square of the number 3 is 9.
etc. etc. etc.

we may express these results more concisely as follows:—

Numbers.	Squares.	Numbers.	Squares.	Numbers.	Squares.	
1	1	3	9	5	25	II.
2	4	4	16	6	36	

or in a still more condensed form:—

Numbers.	1	2	3	4	5	6	7	8	9	III.
Squares.	1	4	9	16	25	36	49	64	81	

The fact that a certain column or line of figures contains numbers, another the squares of these numbers, is indicated by the words "numbers" or "squares" at the beginning of the column or line. It is not, however, explicitly stated which number each square corresponds to; this is left to be inferred from the proximity of the printed figures by which the squares and the numbers are represented. Thus in either of the tables II. or III. above, the fact that 25 is the square of 5 is indicated by printing the 5 much nearer to the 25 than to any of the other squares contained in the table.

Sometimes a heavy or a double line is used, as between the 9 and the 5 of the second table (II.), to indicate a wide separation. In this case an arrangement of figures similar to that in Table II., is to be interpreted in accordance with the fact that 25 (not 9) is the square of 5, even if the 9 is closer than the 25 to the figure 5.

It is occasionally desirable to print side by side on the same page the results of performing different operations upon a given number. "Reciprocals," "square roots," "squares," and "cubes" might thus be represented: —

Numbers.	Reciprocals.	Numbers.	Square Roots.	
1	1	1	1	
2	0.5	2	1 41	
&c.	&c.	&c.	&c.	
				IV.
Numbers.	Squares.	Numbers.	Cubes.	
1	1	1	1	
2	4	2	8	
&c.	&c.	&c.	&c.	

It is obviously unnecessary in such cases to repeat the same numbers in each alternate column; and by omitting to do so, as in V., considerable space is gained.

Numbers.	Reciprocals.	Square Roots.	Squares.	Cubes.	
1	1	1	1	1	
2	0 5	1 41	4	8	V.
&c.	&c.	&c.	&c.	&c.	

ARGUMENT, VARIABLE, AND FUNCTION DEFINED.

Starting in such a table (see Table 2, page 798), in the left-hand column, with any number between 1 and 100, we find in a line with it its reciprocal, square root, square, or cube. The number which one starts with is called the *argument.* Different values of the "argument" are almost always placed in the left-hand column of a table, and are printed in heavy type, so as to be distinguished from the rest of the table. The "arguments" represent certain values of a quantity which may or may not vary between wide limits. This quantity is called in any case the "variable." It will be seen by reference to Table 2 (page 798) that when a number increases, its reciprocal diminishes; but that its square and its cube increase faster than the number itself. The reciprocal, square, cube, &c., of a variable are called *functions* of that variable (*fungo*, to perform). Logarithms, sines, cosines, &c., are also called "functions," and in general, whenever two variable quantities are connected together, either by mathematical or by physical laws, so that if the first

is given the second may be found, the second is called a "function" of the first. The name of a table relates to the function which it represents. If several functions are given in the same table (see V.) the name of each is usually printed at the head of each column or at the beginning of each line containing the function in question.

ORDINARY MATHEMATICAL TABLES.

When the argument and the function require each 3 or 4 figures to represent it, the same page cannot conveniently contain more than 200 or 300 values of each. If, however, the argument increases regularly (as is generally the case), it is not necessary that it should be printed opposite each value of the function. It is, in fact, sufficient that the argument should be given for every 10th value of the function, since the intermediate values of the argument can be easily supplied. This principle is utilized in the ordinary arrangement of mathematical tables, and affords a considerable saving of space.

Different values of the argument, corresponding in such tables to every 10th value of the function, are placed in a column at the left of the page. Opposite them, in a second column, the corresponding values of the function are given.

Thus in the first two columns of Table 3, *G* (page 810), relating to the areas of circles, we find

10	78.5
11	95.0
12	113.1
etc.	etc.

VI.

The letters *Diam.* are printed over the first column to show that it relates to the diameters of circles. The words "Areas of Circles" apply to the second as well as to the succeeding columns. We see, therefore, that a circle having a diameter equal to 10 units of length, must have an area equal to 78.5 units of area (as nearly as the result can be expressed by three figures). The use of the first two columns by themselves does not differ in any respect from cases which we have already examined.

It has, however, been stated that the first two columns give only every 10th value of the argument and function. The functions of "round numbers" are in fact confined to the second column, which is accordingly headed **0**. Intermediate values of the function are contained in the succeeding columns, headed by the numbers **1, 2, 3, 4, 5, 6, 7, 8, 9**. The values are arranged so as to follow in regular succession when read from left to right like a page of ordinary print. This succession should be continued in passing from a number in the column headed **9**, to the number in the next line in the column headed **0**. The table for the areas of circles becomes accordingly: —

Diam.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	
10	78.5	80.1	81.7	83.3	84.9	86.6	88.2	89.9	91.6	93.3	
11	95.0	96.8	98.5	100.3	102.1	103.9	105.7	107.5	109.4	111.2	VII.
12	113.1	115.0	116.9	118.8	120.8	122.7	124.7	126.7	128.7	130.7	
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	

The chief peculiarity of a table constructed in this way is that, instead of printing the argument at the left of each value of the function, as in IV., part of

the argument is to be found at the left of the line containing the function, while the remainder of the argument — usually a single figure — is placed at the head of the column containing the function. The areas in the first line of the main body of the table (VII.) correspond, accordingly, to the diameters 10.0, 10.1, 10.2, &c., those in the next line to 11.0, 11.1, 11.2, &c., &c.

The argument corresponding to any number in a given column and line may always be found by the following rule: *Add the figure at the head of the column to the figures at the left of the line to find the argument in question.* In making this addition, attention must of course be paid to *decimal points*, which in cases of doubt are given both at the head of each column and at the left of each line. If the decimal point is omitted in either of these two places, it may be taken for granted that the figure at the head of the column is to be written *after* the figures at the left of the line.[1] Thus in Tables 47 and 48, page 897, since the first column contains latitudes 0°, 10°, &c., while at the head of the columns we find 0°, 1°, 2°, &c., we infer that the first line refers to latitudes 0°. 1°, 2°, &c., while the second refers to 10°, 11°, 12°, &c. In table 3, *F* (page 809), however, in the absence of any decimal point in the left-hand column, we infer that the figures in that column, 10,

[1] For an arrangement of tables (having certain advantages) in which the *reverse* is taken for granted, see Pickering's Physical Manipulation, Vol. II.

11, &c., are simply to be prefixed to the figures 10, 20, &c., in the top line.

The first line of "circumferences" relates, accordingly, to circles with the following diameters: 1000, 1010, 1020, &c.; while the diameters corresponding to the second line of circumferences are 1100, 1110, 1120, &c.

EXTENSION OF TABLES.

Most tables contain arguments reaching from 1, 10, or 100 to a value 10 times as great,[1] so that it is possible to find the value of a function corresponding, if not to a given argument, at least to some decimal multiple or submultiple of that argument. From this the desired result may often be obtained by pointing off the proper number of decimal places. Thus to find the circumference of a circle 300 *cm.* in diameter, we observe that 300 *cm.* = 3000 *mm.*, and that the corresponding circumference is (see Table 3, *F*, page 808) 9425 *mm.*, or 942.5 *cm.* Again, in finding the area of this circle, we reduce the diameter (300 *cm.*) to decimetres; and starting with the result (30.0 decim.) as an argument, in Table 3, *G* (page 810), we find the area to be 706.9 *sq. decim.* or 70690 *sq. cm.* (since 1 *sq. decim.* = 100 *sq.*

[1] Tables of reciprocals, squares, cubes, logarithms, &c., often reach from 1 to 11 instead of from 1 to 10. The extension of such tables from 10 to 11, though strictly involving a repetition, is of great convenience in physical problems in which factors just above unity are of comparatively frequent occurrence.

cm.). In finding the volume of a sphere with the same diameter (300 *cm.*), we should reduce this diameter to metres; then with the result (3.00 metres) as an argument, we should find the volume of the sphere to be 14.14 *cubic metres*, according to Table 3, *H* (page 812), or 14,140,000 *cu. cm.* (since 1 cubic metre = 1,000,000 *cu. cm.*). For the extension of trigonometric or logarithmic tables beyond their natural limits, special rules must be observed (see explanation of the tables, page 761 *et seq.*).

OMISSION OF CIPHERS, ETC.

It may be remarked that it is not customary to repeat *initial* ciphers or decimal points throughout the whole of a table. These are given either at the head of each column, or at the beginning of each 5th line. In some books other omissions take place. It is well always to look through a new table carefully before deciding how it is to be read, and where the decimal point is to be placed. A negative sign placed *before* a number applies not only to the integral part of that number, but also to the decimal part which follows. A negative sign placed *over* a figure applies only to that figure. If the figure is an integer followed by a decimal, the integer is negative, the decimal positive. In logarithmic tables, decimal points are frequently omitted both in the argument and in the logarithm. In such cases they are always understood to exist *after* the first figure of the argument and *before* the first figure of the logarithm.

COMPLEMENTARY ARGUMENTS.

Some tables (for instance, Table 4, page 814) contain two arguments. One of these is printed in the ordinary manner, partly at the left and partly at the top of the page, and is to be used in connection with the function mentioned at the top of the page. The other argument is printed partly at the right and partly at the bottom of the page, and is to be used in connection with the function named at the bottom of the page. The object of this arrangement is to make a *double use* of the figures in the body of the table. An extra column of figures is usually added to avoid certain difficulties. No number is placed at the head of this column, and no attention is to be paid to it in dealing with the functions named at the top of the page. The argument corresponding to the function at the bottom of the page is found, in the case of a number in a given column and line, by adding the figure at the bottom of the column to the figures at the right of the line. The values of the argument at the right of a page increase upwards; those at the bottom of the page increase from right to left.

INDEPENDENT ARGUMENTS.

The two arguments employed in the class of tables mentioned above are not independent, but represent quantities each of which is usually the "complement" of the other. The use of *two independent arguments* introduces an entirely different kind of

tables. The two arguments correspond in these tables to *two independent variables* upon which the value of the function depends. The first argument is arranged in a column, usually at the left of the table; the second is arranged in a line, usually across the top of the table. To find the value of a function corresponding to given values of both arguments, we follow the line containing the given value of the first argument until we reach the column containing the given value of the second argument. Table 1 (which is a form of multiplication table, see page 797) is an example of the use of two independent arguments. The first argument is a series of factors from 1 to 100, arranged in column at the left of either half of the table. The second argument is an independent series of factors, .1, .2, .3 .4, .5, .6, .7, .8, .9, in the head-line of either half of the table. The body of the table consists in results obtained by multiplying these two sets of factors together. The number occupying a place in a given column and line is the product of the number at the left of the line and the number at the head of the column.

In a table with two independent arguments, the nature of the function is usually given either in the title or at one side of the figures representing the function; the nature of the first argument is given at the head or at one side of the column containing it; while the nature of the second argument is given either at the beginning of the head-line of the table, or just above this head-line.

There is a second method of arranging tables with two arguments, namely: to calculate a separate table of the ordinary sort for each value of one of the arguments. Thus Table 16, *A*, consists of two parts, one calculated for a value of the acceleration of gravity equal to 980, the other for the value 981 *cm. per sec. per sec.* A still greater number of such tables would be necessary to cover all variations in gravity (from 978 to 983) on the earth's surface.

The only way in which it is practicable to represent the value of a function depending upon three independent variables is by means of a series of tables containing two independent arguments, each table being calculated for a special value of the third variable. A complete 2-place table containing three independent arguments, each varying from 1 to 10, would ordinarily occupy about the same space as a 4-place table with a single argument, varying from 1 to 1000, let us say 2 pages. A table with two independent arguments must occupy about 20 pages in order that 3 figures should be significant, and about 2000 pages to give significance to 4 figures. The addition of a third independent argument in the latter case would increase the table to about 2,000,000 pages. It is obvious that the use of tables containing more than 1 independent argument is practically reduced to cases where a rough knowledge of a function is sufficient (as in the calculation of corrections) or where one at least of the variables, like the acceleration of gravity on the

earth's surface, or the ordinary condition of atmospheric temperature and pressure, is confined within narrow limits.

PHYSICAL TABLES.

We have seen that, in representing functions of two variables, one argument is usually printed at the left of the table, the other at the head of the table. A similar arrangement is adopted when it is desired to represent simultaneous variations in different physical quantities due to temperature, pressure, or any other single cause. The values of a given physical quantity are arranged either, as in Table 28, in a column opposite the values of the argument to which they correspond, or else, as in Table 31, in a line *underneath* the corresponding values of the argument. The second argument in such tables is *replaced by names*, referring to a series of physical quantities. These are usually different properties of a given substance, or a given property of different substances; but the arrangement may be applied to any set of quantities which are affected by changes in a given variable.

We have, furthermore, an arrangement peculiar to purely physical tables, in which one argument consists of a series of physical properties, while the other argument consists of a series of substances to which these properties belong. This arrangement is adopted in Tables 8, 9, 10, 11, 12, &c. The names of different substances are arranged in a column at

the left of the table; the names of different physical properties are printed at the heads of a series of columns so as to form a line across the top of the table. The body of the table contains numerical values. The name of the property to which a given number relates is to be found at the head of the column containing that number; the name of the substance to which it applies is to be found at the left of the table in line with the number in question. The names of the properties and of the substances should be such that, when combined together, they form complete definitions of the physical quantities to which the table relates. The numerical values are in each column reduced, when practicable, to the C. G. S. system; when this is not practicable, a factor by which this reduction may be effected, is placed in the first line of the column. In any case the reduction consists simply in moving the decimal point.

DIFFERENCES.

The differences between adjacent numbers in a purely physical table (especially when, as in the cases which follow, an alphabetical order is observed) have in general no special significance. In mathematical tables, on the other hand, the use of such differences is exceedingly important.

The difference between two adjacent numbers in a table should theoretically, if represented at all, be printed half-way between them as in VIII.

1	1	2	1	3	1	4	1	5	
5		5		5		5		5	
6	1	7	1	8	1	9	1	10	VIII.
5		5		5		5		5	
11	1	12	1	13	1	14	1	15	

It is, however, customary if a given line or column of differences is constant, or nearly constant, to omit this line or column, and instead to print the average value of the differences thus omitted near where the end of the line or column of differences would naturally have come. Table VIII. would thus assume one of the following forms:—

						Dif.	
	1	2	3	4	5	1	
	6	7	8	9	10	1	IX.
	11	12	13	14	15	1	
Dif.	5	5	5	5	5		

										Dif.	
	1		2		3		4		5		
	6		7		8		9		10	5	X.
	11		12		13		14		15	5	
Dif.		1		1		1		1			

	1		2		3		4		5	
	6		7		8		9		10	
	11		12		13		14		15	XI.
Dif.	5		5		5		5		5	
Dif.		1		1		1		1		

					Dif.	Dif.	
1	2	3	4	5	1		
6	7	8	9	10	1	5	XII.
11	12	13	14	15	1	5	

Differences printed, as in IX., *on* a given line or *in* a given column relate accordingly to pairs of adjacent

numbers in that line or column. On the other hand, differences printed, as in X., *between* two lines or *between* two columns relate to pairs of adjacent numbers *one in each line* or *one in each column*. Either set of differences, if not needed, may of course be omitted. Table 3, *D* (page 806), corresponds, for instance, to form IX. without the lower line, or to form XII. without the right-hand column of differences.

Instead of printing a series of numbers in the column of differences when they are exactly alike, it is customary to print only one of them, situated as nearly as possible in the middle of the space which the whole series would occupy. This method of representing differences is adopted in Tables 3 *A*, 3 *C*, 3 *G*, 4, 4 *A*, 5, 5 *A*, &c. The difference between any two consecutive values of the function is, in these tables, approximately equal to the *nearest* number in the column of differences. The use of this column of differences will be found to effect a considerable saving of time[1] in processes of interpolation. To effect a still greater saving of time in these processes, a small table of "proportional parts" has been printed in the table of logarithms (Table 6), beneath each difference. The use of proportional parts for interpolation will be explained below (see explanation of Table 1).

[1] It may be remarked that owing to necessary irregularities in the differences which most tables of functions contain, the most accurate results require that these differences should be calculated by actual subtraction in each case.

USE AND EXPLANATION OF MATHEMATICAL AND PHYSICAL TABLES.

TABLE 1 consists of products obtained by multiplying any of the whole numbers (from 1 to 100) in the left-hand column of either half of the table by the decimals .1, .2, .3, .4, .5, .6, .7, .8, .9 at the head of the table. The decimal part of the product is rejected in every case, the units being increased by 1 if the fraction is .5 or more. The table is useful in dividing differences into *parts proportional* to the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, whence the name of the table. It may be used in connection with any of the tables which follow. Let us suppose that it is required to find the sine of 12°.34 in Table 4, page 814. We find the sine of 12°.3 (in the line with 12° and in the column with .3) to be .2130, while the sine of 12°.4 is .2147. The first number (.2130) is too small; the second (.2147) is too great. The difference between them is .0017, or 17 units in the last place, as indicated by the nearest number in the column of differences. If 0°.1 makes a difference of 17 units, 0°.04 should make a difference of $.04 \div 0.1 \times 17$, that is, 6.8 or (nearly) 7 units in the last place. The *same result* may be found by seeking in Table 1 a number

opposite the difference (17) and under the figure (4) for which the interpolation takes place. The result (7 units in the last place) is to be *added to* the sine of 12°.3, because the sines increase when the angles increase — in other words, because the differences are positive. The sine of 12°.34 is accordingly 0.2130 + .0007 = 0.2137.

Again, to find the reciprocal of 6.789, by Table 3 A, page 802, we observe that the reciprocal of 6.78 is .14749, while that of 6.79 is .14728. The difference between these reciprocals is —.00021, because the reciprocals decrease as the numbers increase. Opposite 21 and under .9 in Table 1 we find 19; hence the answer is .14749 — 00019 = .14730. If we had used the nearest number (22) in the column of differences of Table 3 A., instead of the actual difference (21), we should have found similarly .14729 instead of .14730. The true reciprocal happens to lie between these two values.

Table 1 can be used also in *inverse* processes. Let us suppose that it is required to find the cube root of 800, by Table 3 D, page 806. We notice that the cube of 9.28 is 799.2, just below 800, while the cube of 9.29 is 801.8, just above 800; the difference being 26 units in the last place. The difference between 799.2 and 800.0 is 8 units in the last place. In line with the number 26 in the left-hand column of Table 1, and *over* the number 8,[1] we find .3. We see

[1] When the exact number cannot be found amongst the proportional parts we choose the one nearest to it.

therefore that the cube of 9.283 would be 800.0; hence, conversely, 9.283 is the cube root of 800.

The use of proportional parts is especially recommended when accuracy in the last figure is important. The tables which follow have, however, been constructed with such fulness that interpolation will generally be unnecessary, or readily carried on in the head.

TABLE 2 contains several functions often needed, and is intended for rough and rapid work. More exact values of the functions will be found in Tables 3 A — 3 H, which follow.

Column a contains the "reciprocals" of the numbers in the first column from 1 to 100. The reciprocal of a number is defined as the quotient obtained when unity is divided by the number in question. Example: the reciprocal of 30 is .0333.

Column b contains the square roots of numbers from 1 to 100. The square root of a number is defined as a number which multiplied by itself would give a product equal to the original number. Example: the square root of 49 is 7.00.

Column c contains the squares of numbers from 1 to 100; that is, the products obtained when each number is multiplied by itself. Example: the square of 40 is 1600.

Column d contains the cubes of numbers from 1 to 100. The cube of a number is defined as the result of multiplying that number by the square of that number; or as the result of multiplying that number three times into unity. Example: the cube of 5 is 125.

Column e contains three-place logarithms (see under Table 6) from 0.1 to 10.0. Example: the logarithm of 2 is 0.301, correct to 3 places of decimals.

Column f contains the circumferences of circles having diameters from .1 to 10.0. The circumference is given in the same units as the diameter. Example: given the diameter 2.0 *cm.*, the circumference is 6.28 *cm.*

Column g contains the areas of circles having diameters from .1 to 10.0. The area is given in units corresponding to the unit of length employed in measuring the diameter. Example: given the diameter 2.0 *cm.*, the area of the circle is 3.14 *sq. cm.*

Column h contains the volumes of spheres having diameters from .1 to 10.0. The volume is given in units corresponding to the unit of length employed in measuring the diameter. Example: given the diameter 2.0 *cm.*, the volume of the sphere is 4.19 *cu. cm.*

TABLE 3 contains principally 3-place trigonometric functions, and is, like Table 2, intended for rough and rapid work.

Column a contains angles from 0° to 90°; covering in all a right-angle.

Column b contains the tangents of angles. The tangent of an (acute) angle is defined, with reference to a right-angled triangle, as the ratio of the side opposite it to the (shorter) adjacent side. Example: the tangent of 15° is 0.268.

Column c contains "arcs;" that is, in a circle of radius unity, the length of the arcs intercepted by

angles with their vertices at the centre of the circle. "Arcs" are also called the "circular measures" of angles. Example: 15° is equal to 0.262 in circular measure; or the arc of 15° is 0.262.

Column d contains the "chords" of angles. The chord of an angle is defined, with reference to an isosceles triangle, as the ratio of the side opposite the vertical angle to either of the two equal sides. Example: the chord of 15° is 0.261.

Column e contains natural sines. The sine of an angle is defined, with respect to a right-angled triangle, as the ratio of the side opposite that angle to the longest side, or hypothenuse. Example: the sine of 15° is 0.259.

Column f contains natural cosines. The cosine of an angle is defined as the sine of the complement of that angle (see *i*). Example: the cosine of 15° is 0.966.

Column g contains rates of vibration corresponding to different arcs from 0° to 45°, through which for instance a pendulum is vibrating. The arcs are measured from one side of the vertical to the other. The rate of vibration in a very small arc is taken as 1. Example I.: if a pendulum vibrates once a second in a very small arc, it will vibrate .99893 times a second in an arc of 15° (i. e. 7½° on each side of the vertical). Example II.: given the time of oscillation of a magnet equal to 10 seconds in an arc of 45°; required its time of oscillation in a very small arc. Answer, $10 \times .99037 = 9.9037$ sec. Column *g* contains also *coversines* from 45° to 90°.

The coversine of an angle is defined as unity less the sine of the angle. It is the same thing as the versine of the complement of the angle. Versines and coversines measure various errors introduced into physical measurement when two lines which ought to be parallel or perpendicular are inclined at a given angle. The inclination of the two lines is to be found in column *a* or in column *i* as the case may be. Example I.: the shaft of a cathetometer (¶ 262) makes an angle of 89° with the horizon; required the error introduced in the measurement of vertical distances. Answer, .00015 parts in 1, or $\frac{15}{1000}$ of 1 %. Example II.: a magnet which should be horizontal dips 10°; required the error in estimating its magnetism. Answer, .0152, or 1 $\frac{52}{100}$ %.

Column h contains secants, or the reciprocals of cosines. Example: the secant of 15° is 1.035.

Column i contains the complements of the angles contained in column *a*; that is, the results of subtracting these angles from 90°. Example: the complement of 15° is 75°.

It may be remarked that the cotangent of an angle is the tangent of its complement; the cochord of an angle is the chord of its complement; the cosecant of an angle is the secant of its complement. These may all be found, accordingly, by Table 8. Examples:—

The cotangent of 15° = tangent of 75° = 3.732
The cochord of 15° = chord of 75° = 1.218
The cosecant of 15° = secant of 75° = 3.864

To find any function of the *complement* of an angle,

we have only to look up that angle in *column i*, instead of in *column a*.

TABLE 3 A is essentially a 4-place table of reciprocals from 1.00 to 11.09, carried out, however, to 5 places between 6.00 and 9.99. Examples: the reciprocal of 2.73 is .3663; the reciprocal of 273 is .003663.

TABLE 3 C is a 4-place table of squares from 1.00 to 9.99, carried out to 5 places between 10.0 and 11.09. Examples: the square of 3.14 is 9.860; the square of 31.4 is 986.0. The square root of 1.25 is is 1.12 nearly, or more exactly, 1.118 (see under Table 1).

TABLE 3 D is a 4-place table of cubes from 1.00 to 9.99, carried out to 5 places from 10.0 to 11.09. Examples: the cube of 5.55 is 171.0; the cube of .555 is .1710. The cube root of 800 is 9.283 (see under Table 1).

TABLE 3 F contains the circumferences of circles with diameters (*diam.*) varying from 1000 to 10090 by 10 units at a time. The results are carried out to units. The differences in this table are either 31 or 32, from beginning to end. The mean difference is 31.416. Proportional parts corresponding to this mean difference are printed at the bottom of the table. The circumference is given in units of the same magnitude as the diameter. Example I.: the circumference of a circle 3600 *cm.* in diameter is 11310 *cm.* Example II.: given a circumference 10,000 metres, the diameter is 3180 metres, nearly; or more exactly, 3183 metres (see under Table 1).

TABLE 3 G is a 4-place table containing the areas of circles corresponding to diameters (*diam.*) from 10.0 to 100.9. The area is given in units corresponding to the unit of length in which the diameter is measured. Example I.: diameter = 15.0 *cm.*, area = 176.7 *sq. cm.* Example II.: diameter = 55.5 *mm.*, area = 2419 *sq. mm.* = 24.19 *sq. cm.* Example III.: area = 4000 *sq. cm.*, diameter = 71.4 *cm.*, nearly; more exactly, 71.36 *cm.* (see under Table 1).

TABLE 3 H contains the volumes of spheres corresponding to diameters from 1.00 to 10.09. The volume is given in units corresponding to the unit of length in which the diameter is measured. Example I.: diameter = 11.1 *mm.* = 1.11 *cm.*: volume = .539 *cu. cm.* = 539 *cu. mm.* Example II.: volume = 35.00 *cu. cm.*, diameter = 4.06 *cm.*, nearly; or more exactly, 4.058 *cm.* (see under Table 1).

TABLE 4 is a 4-place table giving the natural sines of angles from 0°.0 to 89°.9, when interpreted in the ordinary manner by means of the argument at the left and at the top of the page. Natural cosines may also be found by means of this table, by using the argument at the right and at the bottom of the page. Example I.: the sine of 30°.0 is 0.5000. Example II.: the cosine of 30°.0 is .8660.

TABLE 4 A is a 4-place table giving the logarithmic sines (that is the logarithms of the sines) of angles from 0°.0 to 89°.9, when read in the ordinary way. Logarithmic cosines may be found through the argument at the right and bottom of the page. Example I.: the logarithm of the sine of 30° is $\bar{1}.6990$.

Example II.: the logarithm of the cosine of 30° is $\bar{1}.9375$.

TABLE 5 contains the natural tangents of angles from 0°.0 to 89°.9. Natural cotangents from 45°.0 to 89°.9 may also be found by using the argument at the right and bottom of the first half of the table. Below this limit, they are not given; but they may be found by calculating the complement of the angle and looking up its tangent. Example I.: the tangent of 30° is 0.5774. Example II.: the cotangent of 22°.5 = tangent of 77°.5 = 4.511.

TABLE 5 A is a 4-place table giving the logarithmic tangents (that is, the logarithms of the tangents) of angles when read in the ordinary way. Logarithmic cotangents may also be found by using the argument at the right and at the bottom of the page. Example I.: the logarithm of the tangent of 30°.0 is $\bar{1}.7614$. Example II.: the logarithm of the cotangent of 30° is 0.2386.

TABLE 6 is a 5-place table of the logarithms of numbers from 1,000 to 11,009. A decimal point is understood after the first figure of each number and before the first figure of each logarithm. Example: the logarithm of 2.000 is .30103.

When the decimal point of a number does not follow the first figure, the corresponding logarithm consists of two parts. The first part is a whole number called the "characteristic" of the logarithm; the second or decimal part is called the "mantissa."

The "characteristic" of a logarithm is not to be found in Table 6, but is to be supplied by inspection.

Its numerical value is equal to the number of spaces between the decimal point of the argument and the space following the first figure of the argument.

Thus the logarithm of the number 1.11 has the characteristic 0; while the characteristics of 11.1 and 111 are 1 and 2 respectively. The sign of the characteristic is positive if the decimal point is at the right of the first figure of the argument; if it is at the left, the sign is negative. Thus the characteristic of the logarithm of .1111 is — 1., the characteristic of the logarithm of .01111 is — 2., &c. The negative sign is in practice written *over* the characteristic, as it affects this characteristic alone.

It is a peculiarity of logarithms that the "mantissa" is not affected by the location of the decimal point in the original number. The logarithm of 1.111 (namely, 0.04571) is, for instance, the same as the logarithm of 1,111. (namely, 3.04571), as far as the mantissa is concerned. The mantissa or decimal part of the logarithm of any number may be found, accordingly, by Table 6, by considering only the figures of which the number is composed.

Initial and final ciphers may be thrown off *ad libitum* in this process; but ciphers in the middle of a number form an essential part of it. Thus in finding the decimal part of the logarithm of .000,100,100, we need to consider only the figures 1001, since these are preceded and followed only by ciphers; but the ciphers between the first and last figures cannot be neglected. The following logarithms from Table 6 may also serve as examples: —

The logarithm of	3.1416	is 0.49715
" " "	980	" 2.99123
" " "	41,700,000	" 7.62014
" " "	.00367	" $\bar{3}.56467$

Conversely, in finding the number corresponding to a a given logarithm, we first obtain the figures of which the number is composed by considering simply the mantissa, or decimal part of the logarithm, and to these figures we add as many initial or final ciphers as may be needed; then starting with the space at the right of the first figure (disregarding initial ciphers) we count off to the right if the characteristic of the logarithm is positive (or to the left if negative) a number of spaces equal to the characteristic in question, in order to locate the decimal point. In any case the number of figures between the decimal point and the space following the first figure of the answer must be equal to the characteristic of the logarithm.

Example I.: given the logarithm 0.14860, the figures of the corresponding number are 1408; the characteristic of the logarithm being 0, the answer is 1.408. Example II.: given the logarithm 3,14860, the mantissa being .14860 as before, we find the same figures, 1408. Since the characteristic (3) is positive, the decimal point is at the right of the first figure, and since 3 figures must come between the decimal point and the space following the first figure, the answer is 1,408.

The following rules embody the most important applications of logarithms,—namely, to problems of multiplication and division.

Rule 1. To multiply two or more numbers together, find the logarithm of each and add the logarithms together. The number corresponding to their sum is the required product. Example: to multiply 2×4.

The logarithm of 2 is	0.30103
" " " 4 is	0.60206
The sum of these logarithms is	0.90309,

which is the logarithm of 8, the answer. Numbers involving more than 3 significant figures may be multiplied together by the aid of logarithms with greater ease than by arithmetical processes.

Rule 2. To divide one number by another, find the logarithm of the first, subtract the logarithm of the second; the remainder is the logarithm of the answer. Example: to divide 4 by 8,

The logarithm of 4 is	0.60206
" " " 8 "	0.90309
The difference is	$\bar{1}.69997$,

which is the logarithm of 0.5, the answer.

Rule 3. To find the value of a fraction with several factors, find the logarithm of each factor in the numerator, and add the logarithms together. Then find the logarithm of each term in the denominator, and add these logarithms together. Subtract the

latter sum from the former sum. The remainder is the logarithm of the answer. Example: to find the value of the fraction

$$\frac{.2345 \times 45.67 \times 6{,}789}{1.234 \times 34.56 \times 567.8}, \text{ we find—}$$

(1) log. .2345	$= \bar{1}.37014$	(5) log. 1.234	$= 0.09132$
(2) " 45.67	$= 1.65963$	(6) " 34.56	$= 1.53857$
(3) " 6789	$= 3.83181$	(7) " 567.8	$= 2.75420$
(4) sum	$= 4.86158$	(8) sum	$= 4.38409$
(9) subtract	4.38409		
(10) remainder	$= 0.47749 =$ log. 3.002 +, ans.		

Rule 4. To raise a number to any power, find its logarithm, multiply by the power, and the product is the logarithm of the answer. Example: to find the 4th power of 2. The logarithm of 2 is 0.30103; which multiplied by 4 gives 1.20412. This is the logarithm of 16, the answer.

Rule 5. To extract any root of a number, find the logarithm of the number and divide by the root in question; the quotient is the logarithm of the answer. Example: to find the 12th root of 2. The logarithm of 2 is 0.30103; this divided by 12 gives .02509, which is the logarithm of 1.0595, the answer. (This is the value of the interval called 1 semitone on the tempered scale.)

Table 7 contains the probability of an error's exceeding limits bearing to the "probable error" (§ 50) the ratios represented in the left-hand column. The probability is expressed as so many chances in 1. Example I.: the probable error of a weighing is 1

centigram; what are the chances of an error greater than 1 centigram? Answer, by definition, an *even chance* or 0.50000. Example II.: under the same circumstances, what are the chances of an error's exceeding 2 centigrams? Answer, 0.17734, i. e. 17,734 chances in 100,000, or about 1 chance in 6. Example III.: under the same circumstances, what are the chances of an error's exceeding 5 centigrams? Answer, 0.00075, or less than 1 in 1000.[1]

TABLES 8, 9, 10, 11, and 12 contain (1) the names, (2) the chemical symbols, and (3) the atomic weights of various substances, and deal with the following physical properties: (4) the specific gravity (§ 69) of gases and vapors referred to hydrogen at the same temperature and pressure; (5) the density (§ 15) of substances at 0° under the ordinary atmospheric pressure (6) the "viscosity" of liquids at about 20°, or the force in dynes required to maintain a relative velocity of 1 *cm. per sec.* between two surfaces 1 *cm.* square and 1 *cm.* apart; (7) the "surface tension" of liquids (¶ 169) at about 20°, or the force in dynes with which *each surface* of a liquid film 1 *cm.* broad tends to contract; (8) the "breaking strength" of solids, or the force in kilo-megadynes[2] required to break a wire 1 *sq. cm.* in cross section; (9) the "crushing strength" of solids, or the force in kilo-megadynes required to crush a block 1 *sq. cm.*

[1] The chances relate only to "accidental errors" (§ 24). The chances of "mistakes" are not included.

[2] 1 kilo-megadyne = 1.02 "tonne weight," or 1 English ton weight, nearly.

in cross section; (10) the "shearing strength" of solids, or the force in kilo-megadynes required to cut a wire 1 *sq. cm.* in cross section; (11) the "hardness" of solids according to Mohs' arbitrary scale (page 587); (12) the "simple rigidity" of solids, or the force in kilo-megadynes required to make two surfaces 1 *cm.* square and 1 *cm.* apart move parallel to one another through a thousandth of a centimetre (.001 *cm.*), (13) "Young's modulus," or the force in kilo-megadynes required to pull two such surfaces apart through one thousandth of a centimetre (.001 *cm.*); (14) the "resilience of volume" or the pressure in kilo-megadynes required to compress a centimetre cube by one cubic millimetre; (15) the average cubical "coefficient of expansion" of substances[1] (§ 83) between 0° and 100° under a constant pressure of 76 *cm.* of mercury; (16) the "melting-point" of solids, or the "freezing-point" of liquids on the Centigrade scale; (17) the "boiling-point" of liquids, or the "temperature of condensation" of vapors at the atmospheric pressure; (18) the "critical temperature" of liquids and vapors,—that is, the temperature at which the properties of the liquid and its vapor become indistinguishable; (19) the "critical pressure" of liquids and vapors, that is the pressure of the vapor of a liquid at the critical temperature in megadynes *per sq. cm.*; (20) the "pressure

[1] When a change of state takes place between 0° and 100°, the averages in question refer only to that part of the interval (0° to 100°) in which the substance exists in the state named at the head of the table.

of vapors" at 20°, in megadynes *per sq. cm.*; (21) the average specific heat of substances[1] (§ 86) between 0° and 100°, under the "constant pressure" of 76 *cm.* of mercury; (22) the average specific heat of substances between 0° and 100°, when prevented from expanding; that is, confined to a "constant volume;" (23) the "latent heat of melting" of solids, or the "latent heat of liquefaction" of liquids; that is, the number of units of heat required to convert 1 gram of a solid, at its melting-point, into a liquid at the same temperature under a pressure of 1 atmosphere; (24) the "latent heat of vaporization" of liquids, or the "latent heat of condensation" of vapors; that is, the number of units of heat required to convert 1 gram of a liquid at the boiling-point into vapor at the same temperature under the atmospheric pressure; (25) the "heat conductivity" of substances, or the number of units of heat conducted in one second between two opposing faces of a centimetre cube differing 1° in temperature; (26) the "electrical conductivity" of substances, or the current in ampères flowing between two opposing faces of a centimetre cube differing 1 microvolt (.000,001 volt) in electrical potential (§ 139); (27) the "thermo-electric heights" of conductors, or the electromotive force in microvolts developed by a thermo-electric junction of which one element is lead, corresponding to a difference of temperature of 1°; (28) "electro-chemical equivalents," or the weight in milligrams of various

[1] See footnote, page 775.

elementary substances affected by a current of 10 ampères in 1 second; (29) the specific inductive capacity of substances determined by currents alternating several hundred times per second (¶ 256); (30) the minimum "extraordinary index of refraction" of optical materials; (31) the "ordinary index of refraction" of uniaxial crystals, or the "medium" index of refraction of biaxial crystals; (32) the maximum "extraordinary index of refraction" of different substances — these three indices referring to the sodium (D) line; (33) the ordinary (or medium) "index of dispersion," or the difference between the ordinary (or medium) indices of refraction for the lines *A* and *H* of the solar spectrum; and finally (34) the solubility of solids in water, expressed in per-cents by weight, and the solubility of gases, also in per-cents by weight, under a pressure of 1 atmosphere.

The first line of each table contains factors by which the values given in the column below them may be reduced to the C. G. S. system. Thus the coefficient of resilience of aluminum (Table 8) is 0.5 (?) $\times\ 10^{12} = 500{,}000{,}000{,}000$(?), and the thermo-electric height of copper is about $4 \times 100 = 400$ absolute units.

TABLE 8 contains the properties of elementary substances.

TABLE 9 contains the properties of solids remarkable especially for their strength or for other properties rendering them suitable for building materials or for the construction of machines.

TABLE 9 A contains the properties of certain chemical salts and other substances in ordinary use.

TABLE 10 contains the properties of solids remarkable for their optical or other allied properties.

TABLE 11 contains properties of liquids.

TABLE 12 contains properties of gases and vapors.

TABLES 13 A, B, and C, give the (maximum) pressure in megadynes *per sq. cm.* of the vapor arising from various liquids at different temperatures.

TABLE 13 A contains substances which are for the most part gaseous at ordinary temperatures.

TABLE 13 B contains more or less volatile liquids.

TABLE 13 C gives the pressure of the vapor of mercury, sulphur, and water, including the vapor of water arising from sulphuric acid of different strengths.

TABLE 13 D contains the "density of steam," or the maximum density of aqueous vapor at different temperatures.

TABLE 14 gives the boiling-points of water corresponding to different barometric pressures from 68.0 to 77.9 centimetres of mercury reduced to latitude 45° (see Landolt and Börnstein, Table 20). Example: when the barometer stands at 75.0 *cm.*, water boils at 99°.63.

TABLE 14 A gives dew-points (calculated from Regnault's data) corresponding to different degrees of temperature and "relative humidity." The "dew-point" means that temperature at which moisture would (barely) be precipitated out of the air (as when dew is formed); the "relative humidity" is the

proportion which the moisture contained in the air at a given temperature bears to the maximum possible amount which it can hold at that temperature. Example I.: the air of a room at 20° is half saturated with moisture (i. e. the relative humidity = 50 %); required the dew-point. Answer, 9° Centigrade by Table 14 A. Example II.: sea air saturated at 9° with moisture is warmed to 20°; required the relative humidity. Answer, 50 %.

TABLE 15 shows at a given temperature (T) the maximum pressure (P) of aqueous vapor in centimetres of mercury, the maximum density (D) of aqueous vapor, and the factor (F) by which the difference between the readings of a wet and a dry bulb thermometer must be multiplied in order to find the difference between the dew-point and the temperature (T) of the air. The data have been taken from Kohlrausch, Table 13, Landolt and Börnstein, Tables 18 *a* and 23, and from Everett's "Units and Physical Constants," Art. 124. The first three columns are an amplification of results contained in Table 13. The last column is useful in hygrometry. Example: if the dry-bulb thermometer reads 20°, and the wet-bulb thermometer reads 15°, so that the difference between them is 5°, we have (since F = 1.8), 5° × 1.8 = 9°, which subtracted from 20° gives 11° for the dew-point.

TABLE 16 A gives the specific heat of moist air at about 50°, corresponding to different dew-points under a constant pressure of 76 *cm.* of mercury. The specific heat of dry air at 0° (.2383) is the mean be-

tween the results obtained by Regnault and E. Wiedemann. The other specific heats have been calculated by interpolation between the specific heats of air and of steam (.4805).

TABLE 15 B gives the velocity of sound in atmospheric air calculated for different degrees of temperature and relative humidity, allowing for the effect of moisture on the density of air and on the ratio of the two specific heats of air under constant pressure and under constant volume. The barometric pressure (which has hardly a perceptible influence on the result) was assumed to be 76 *cm.* of mercury.

TABLE 15 C contains coefficients of interdiffusion of gases. The values (due to Maxwell) are taken from Everett's "Units and Physical Constants" (Art. 131). If two reservoirs filled with different gases are connected by a tube 1 *cm.* long, the numbers in Table 15 C show the mean velocity in *cm. per sec.* with which a stream of gas flows through the tube from each reservoir into the other.

TABLE 16 is intended for the reduction of barometric readings, when given in inches, to centimetres. The last line of the table contains "proportional parts" (see under Table 1).

TABLES 16 A and B are intended for the reduction of barometric readings in *cm.* of mercury at 0° to megadynes *per sq. cm.* A is calculated for a value of the acceleration of gravity (g) equal to 980 *cm. per sec. per sec.*; B for the value $g = 981$. The two tables differ by about 10 units in the last place. For values of g between 980 and 981, or just outside of these limits,

results may be easily obtained by interpolation. Example: $g = 980.4$; required the value of 1 atmosphere (76 *cm.*) in megadynes *per sq. cm.* Answer, $1.0127 + 10 \times .4 = 1.0131$ megadynes *per sq. cm.*

Table 17 gives the elevation in metres above the sea-level corresponding to different barometric pressures at 10° Centigrade. It has been calculated for dry air in latitude 45° by the formula

$$h = 190790 \,(\log. 76 - \log. p)\,(1 + .000,000\,1\,h).$$

It is used in estimating heights by the barometer. Example I.: the mean barometric pressure is 70.0 *cm.* at the top of a hill rising out of the sea, the sides of the hill having a mean temperature of about 10°; required the height of the hill. Answer, about 681 metres. Example II.: the barometric pressures at a given instant are 75.1 *cm.* at the foot of a hill, and 74.2 *cm.* at the top of the hill,—the mean temperature being about 10°; required the height of the hill. Answer, $199 - 99 = 100$ metres.

TABLES 17 A and 17 B give corrections in per cent to be added to or subtracted from the results of Table 17, according to the mean temperature and dew-point between the observing stations. Thus for a mean temperature 23° and the dew-point + 8° add $4.6 + 0.4 = 5.0$ % to all results. This would make the height of the hill in Example II., 105 (instead of 100) metres.

TABLE 18 *a* gives the correction in centimetres to be subtracted (on account of expansion) from the reading of a mercurial barometer provided with a

brass scale reaching from its zero in the surface of mercury in the reservoir to the free surface of mercury in the tube. In calculating this table, the coefficient of expansion of mercury was assumed to be $.000180 + .000,000,036\,t$; the value .000019 was taken for the coefficient of expansion of brass. Example I.: the mercurial column is 76 *cm.* long, measured by a brass scale, its temperature is 20°, we subtract 0.245 *cm.*, and find 75.755 *cm.* for the value at 0°. Example II.: same as I. except that a *glass* scale is used; corrected value the same less .016 *cm.*, that is, 75.739 *cm.*

TABLE 18 *b* gives the mean correction to be *added to* the apparent height of the mercurial column on account of "capillarity," that is, the tendency of capillary or in general *small* tubes to depress a mercurial column (see Everett, 46 A, and Pickering, Table 12). The correction depends, however, not only upon the internal diameter of the barometer tube at the point where the mercury stands, but also upon the height of the "meniscus," which is different according to the direction in which the mercurial column has been moving. Corrections corresponding to different heights of the meniscus are taken from Kohlrausch, Table 15, 6th ed. The results in this table differ widely from those quoted in the 2d edition.

TABLE 18 *c* contains corrections for the pressure of mercurial vapor. They have been obtained by averaging the results of Regnault, Hagen, and Hertz, quoted in Landolt and Börnstein, Table 27. The

results in question differ in some cases even in regard to the position of the decimal point.

On account of the great discrepancy between the results obtained by different observers, barometric readings, even when corrected by Tables 18 *a*, 18 *b*, and 18 *c*, are significant only as far as hundredths of a centimetre.

TABLES 18 *d*, 18 *e*, 18 *f*, and 18 *g*, contain factors for the reduction of either the density or the volume of a gas to 0° or to 76 *cm*. Example I.: the density of coal-gas being .0005 at 20° and 75.0 *cm*., required its density at 0° and 76 *cm*. Answer, $.0005 \times 1.0734 \times 1.0133 = .00054 +$. Example II.: the volume of a gas at 20° and 75 *cm*. is 100 *cu. cm.*; required its volume at 0° and 76 *cm*. Answer, $100 \times 0.9316 \times 0.9868 = 91.9$ *cu. cm.* If the gas were collected over water at 20° we should subtract 1.74 *cm*. (see Table 15) from the apparent pressure (75 *cm*.) and find 73.26 *cm*. for the pressure of the gas. This would give a factor .9640 instead of .9868, and a result 89.8 *cu. cm.* in the example above.

TABLE 19 contains the density (or weight of 1 *cu. cm.*) of air corresponding to different temperatures and pressures, and has been taken from Kohlrausch, 2d ed., Table 6. It was calculated from Regnault's observations for latitude 45°.

TABLE 20 contains corrections for the results in Table 19 to be applied on account of moisture. Example: required the density of air at 20° and 76 *cm*. pressure when the dew-point is + 4° Centigrade. Answer, $.001204 - .000004 = .001200$.

TABLE 20 A contains the weight of air displaced by 1 gram of brass of the density 8.4, and is useful in calculating effective weights (§ 64). Example: a body is balanced by 100 grams of brass in air of the density .001200; required the effective weight of the body. Answer, 100 grams *minus* 100 × 0.000143 grams, or 99.9857 grams.

TABLE 21 contains factors for reducing apparent weighings with brass weights to vacuo. The factors correspond to different densities of the substance weighed, as well as of the air in which the weighing takes place. Example: a piece of glass of the density 2.5 is balanced by 100 grams of brass, in air of the density .00120; required its true weight *in vacuo.* Answer, 100 × 1.00034 = 100.034 grams.

TABLE 22 contains "apparent specific volumes" of water; that is, the space in cubic centimetres occupied by a quantity of water weighing apparently 1 gram when counterpoised in air with brass weights of the density 8.4. The apparent specific volumes correspond to different temperatures and different conditions of atmospheric density, and are useful especially in calculations of volume or capacity in hydrostatics. Example: a flask holds apparently 1000 grams (1 litre, nearly) of water at 20°, when weighed in air of the density .00120; required the capacity of the flask. Answer, 1000 × 1.00279 = 1002.79 *cu. cm.*

TABLE 23 contains true "specific volumes" of water; that is, the space in cubic centimetres occupied at various temperatures by a quantity of water weighing actually 1 gram *in vacuo.* These values

are reciprocals of those in Table 24, and are to be used for the calculation of volumes corresponding to *true* weights *in vacuo*. Example: a piece of steel displaces 100 grams of boiling water; required its volume. Answer, $100 \times 1.04311 = 104.311$ *cu. cm.*

TABLE 23 A gives the true specific volume of mercury at different temperatures, and is used like Table 23. In calculating this table Regnault's value (13.596) for the density of mercury at 0° was used, and a coefficient of expansion $.000180 + .000,000,036\ t$.

TABLE 23 B gives apparent specific volumes of mercury when balanced by brass weights of the density 8.4 in air of the density .0012. It is used, like Table 22, to calculate volumes and capacities. Example: the apparent weight of mercury required to fill a tube at 20° is 100 grams; required the capacity of the tube. Answer, $100 \times 0.073812 = 7.3812$ *cu. cm.*

TABLE 24 contains the density of mercury at different temperatures. The values are reciprocals of those contained in Table 23 A.

TABLE 25 contains the density of water at different temperatures. A mean value, 1.00001, was taken for the maximum density of water (Kupfer's value is 1.000013). The relative densities lie between the estimates of Rossetti and Volkmann, founded upon observations by Despretz, Hagen, Jolly, Kopp, Matthiessen, Pierre, and Rossetti.

TABLE 26 contains the density of commercial glycerine, calculated from observations made in the Jefferson Physical Laboratory.

TABLE 27 contains the density of dilute alcohol corresponding to different temperatures and different strengths. The values are a mean between results obtained by numerous observers.

TABLE 28 gives the density, at 15°, of acids and saline solutions corresponding to various strengths, and is useful in making tests with a densimeter. See Storer's "Dictionary of Solubilities." Example: the density of some sulphuric acid is 1.807 at (about) 15°; required its strength. Answer (about) 88 %.

TABLE 29 gives the boiling-points of solutions of various strengths estimated by interpolation from data contained in Storer's "Dictionary of Solubilities." It furnishes an independent (and in processes of concentration by boiling a very convenient) method of estimating the strength of such solutions. Thus a solution of hydrate of sodium boiling at 120° is known to have a strength of about 40 %.

TABLE 30 gives the specific heats of solutions of different strengths at about 20°. It is useful in certain processes in calorimetry (see ¶¶ 99–100). The numbers were obtained by interpolation from results contained in Landolt and Börnstein, Tables 71 and 72. Those nearest the observed values are printed in heavier type.

TABLE 31 A gives the electrical conductivity of solutions at about 18°. It shows the current in ampères which an electromotive force of one volt would cause to flow through a metre-cube of the solutions in question, or through a voltameter with plates 1 decimetre square and 1 *cm.* apart, filled with these solu-

tions, neglecting the effects of polarization. The results must be multiplied by 10^{-11} (.000,000,000,01) to reduce them to the C. G. S. system. The relative values of different results are probably accurate within 5 or 10 per cent, but their absolute values are much less reliable.

TABLE 31 B gives Refractive and Dispersive indices corresponding to the sodium (D) line for solutions of different strengths, and was obtained by interpolation from results quoted by Landolt and Börnstein.

TABLE 31 C is intended to facilitate the preparation of solutions of any desired strength, and for the calculation of per cent contents from the ratio of two constituents. Example: how many parts of salt must be added to 100 of water to make a 20 % solution? Let A = salt; B = water, — the answer is 25 parts. Example II.: a solution contains 100 parts of sulphuric acid to 150 of water; required its strength. Let A = water, B = sulphuric acid; the answer is: 60 % water, 40 % sulphuric acid.

TABLE 31 D gives coefficients of diffusion of saline solutions in water at about 20°. The values were calculated from Graham's data quoted in Cooke's "Chemical Physics." Example: how much common salt would escape by diffusion into pure water from a 20 % solution in 600,000 seconds through a layer 1.2 *cm.* thick and 8 *sq. cm.* in cross section? Answer, 20 % of $600{,}000 \times 8 \times .000{,}0046 \div 1.2 = 3.68$ grams.

TABLES 31 E and F give the rotation in degrees of the plane of polarization of different kinds of light

corresponding to the Fraunhofer lines A to H. *E* refers to dilute solutions having such a depth that a beam of light passing through an orifice 1 *cm.* square meets just one gram of the dissolved substance.[1] *F* refers to the effect of plates 1 *cm.* thick.

TABLE 31 G relates to the effect of a magnetic field in rotating the plane of polarization of light parallel to the lines of force.

TABLE 31 H relates to (1) Magnetic Susceptibility, (2) Saturation, and (3) Permanent Magnetism,—that is, the magnetic moment of a unit cube of different materials (1) in a unit magnetic field, (2) in an infinite magnetic field, and (3) in space after the magnetizing influence has been removed. The results are taken from Everett and Ganot.

TABLE 31 I contains some of Weisbach's results for the coefficient of friction of water moving with different velocities through tubes not far from 1 *cm.* in diameter. The results have been reduced to the the C. G. S. system.

TABLE 31 J gives coefficients of friction of solids on solids, taken from De Laharpe's "Notes et Formules de l'Ingénieur."

TABLE 31 K contains coefficients of reflection, absorption, and transmission of radiant heat, from Ganot's Physics.

TABLE 31 L contains estimates (by the author) of the heat radiated at different temperatures by 1 *sq.*

[1] The rotation is proportional, within more or less narrow limits, to the strength of the solution; but may vary widely outside of these limits. Cases of reversal even occur. See Landolt and Börnstein.

cm. of blackened or perfectly radiating surface surrounded by perfectly absorbing walls, or space at 0°. The table was calculated by the formula —

$$q = \log.^{-1}(.0013 \times (t^\circ + 273^\circ) - 1.8249) - .034,$$

which was found to reconcile various well-known facts. Example: how much heat is required to maintain 1 *sq. cm.* of platinum at its melting-point (1900°) for 1 sec.? Answer, 10 (?) units.[1]

TABLES 32 A and 32 B give heats of combustion[2] in oxygen and in chlorine respectively, from data quoted by Everett, by Landolt and Börnstein, and by other authorities. The chemical reactions are not in all cases such as actually take place; but the table gives the heat which it is supposed *would be* developed if the reactions did take place. The last column gives the electromotive forces developed by or necessary to undo some of the reactions. Example: 2 grams of hydrogen uniting with 16.0 grams of oxygen give out 69,000 units of heat, or 34,500 units per gram of hydrogen. This is equivalent to 1440 megergs per *mgr.* of hydrogen consumed. To decompose water, an electromotive force of 1.49 volts is required.

TABLE 33 gives "heats of combination" involving more complicated chemical reactions than those which take place in simple combustion.

[1] This corresponds to 8 + volt-ampères per candle-power.

[2] The heat of combustion of many substances can be inferred only from indirect processes. See experiment 38.

TABLE 34 gives contact differences of electrical potential in volts. The data are taken from Everett's "Units and Physical Constants," Art. 206. Example: a piece of zinc is brought into contact with a piece of copper; required the difference of electrical potential. Answer, the zinc is positively electrified with respect to the copper; the difference of potential is 0.750 volts.

TABLE 35 gives the electromotive force in volts of voltaic cells of various sorts.

TABLE 36 gives the relation between electromotive forces and the length in *mm.* of the spark which they produce in ordinary atmospheric air, calculated from Everett, Art. 192. Example: an induction machine produces sparks 2.5 *mm.* long; required the difference of potential between its poles at the instant. Answer, 9000 volts. Only the first two figures are significant in this answer.

TABLES 37 *a* and *b* give specific resistances of conductors and insulators at 0°. The last column gives the per cent of increase of all these resistances due to a rise of temperature of 1° Centigrade.

TABLE 38 gives the specific resistances of electrolytes corresponding to various strengths. The resistances are in ohms, and apply to a centimetre cube of the liquid. The probable error of the results is about 10 %. *Relative* values are probably not so inaccurate. Example: required the resistance of a cubical Daniell cell, with a plate of copper 10×10 *cm.*, separated by a layer of 20 % (crystallized) sulphate of copper 5 *cm.* deep, and by a layer of 20 % (crystallized) sul-

phate of zinc, also 5 *cm.* deep, from a plate of zinc 10 $\times$ 10 *cm.* Answer: the resistance of the copper solution is $20 \times 5 \div (10 \times 10) = 1$ ohm; that of the zinc solution is the same; hence the resistance of the battery is 2 ohms.

TABLE 39 gives a comparison between the Fahrenheit and Centigrade thermometers. Example: 98°.6, F = 37°.0, C.

TABLE 40 (Pickering, Table 14) gives a comparison of hydrometer scales. Example: 40 Beaumé for liquids lighter than water corresponds to the density 0.830.

TABLE 41 gives lengths of waves of light in air, intermediate between the numerous results quoted by Landolt and Börnstein. The probable error is about 1 unit in the last figure. Example: the Fraunhofer lines D_1 and D_2, together designated Na (or D), are due to sodium (symbol, Na) and occur in the yellow of the spectrum. They correspond to number 50 on Bunsen's scale, to numbers 1003 and 1007 on (Bunsen and) Kirchoff's scale, and have the wave-lengths 0.00005896 *cm.* and 0.00005890 *cm.* respectively.

TABLE 42 A refers to the imperial wire gauge adopted by the Board of Trade (Stewart & Gee, I. B.).

TABLE 42 B gives the Birmingham wire gauge (B. W. G.). The results are intermediate between those quoted in English, French, and German books. The probable error is about 1 unit in the last figure.

TABLE 43 gives the number of vibrations corresponding to a series of musical notes on the tempered or isotonic scale, one half of a semitone apart. The

designation of some of these notes is given in the left-hand or in the right-hand column. The former is to be used for "physical pitch," in which all powers of the number 2 represent the note C; the right-hand column may be used for notes given by American instruments tuned to "concert pitch." The numbers *between* those corresponding to a given note in the first and last columns may be taken to represent the same note according to the old Stuttgart standard of pitch (A = 440, C = 264). Example: the "middle C" of an American piano (in the little octave), makes about 135.6 vibrations per second, and corresponds to C♯ physical pitch.

TABLE 44 A gives reductions of minutes and seconds to thousandths of a degree. The number of minutes is first sought; the tenths of a degree will be found next to it. Then *in the same section of the table* (there are 6 sections) the nearest number of seconds is found, and next to it the hundredths and thousandths of a degree. Example: $23°27'13'' = 23°. + 0°.4 + 0°.054$, nearly, or $23°.454$.

TABLE 44 B gives the correction to be added to dates in different years to compare them with the year 1891. Thus, Jan. 1, 10h. 0m. 0s., A. M., 1899; corresponds to Jan. 1, 10h. 0m. 0s. + 1h. 29m. 28s. A.M. = Jan. 1, 11h. 29m. 28s. A.M. 1891. The declination of the sun and the equation of time will, for instance, be the same on these two dates.

TABLE 44 C gives the gain of sidereal over mean solar time.

TABLE 44 D gives the sidereal time at Greenwich mean noon for the 10th, 20th, and last day of every month of the year 1891.

TABLE 44 E gives the semidiameter of the sun at different times in the year.

TABLE 44 F gives the declination of the sun at Greenwich mean noon for the year 1891. The sign of the declination is to be found at the head of the several columns (+ north, — south).

TABLE 44 G gives the "equation of time" at Greenwich mean noon for the year 1891. The signs + and — at the head of the columns and elsewhere show whether the sun is "fast" or "slow" respectively; + indicates that the sun passes the meridian before noon; — after noon.

TABLE 44 H gives certain astronomical data relating to the solar system.

TABLE 45 gives the Right Ascension and Declination of some of the most important stars.

TABLE 46 gives latitudes, longitudes, and elevations of certain important places.

TABLES 47 and 48 give respectively the acceleration of gravity and the length of the seconds pendulum corresponding to different latitudes. Example: since the latitude of the Jefferson Physical Laboratory of Harvard College is 42°22½′ or 42°.38, the acceleration of gravity is 980.37 *cm. per sec. per sec.*

TABLE 49 A and 49 B relate to the reduction of measures to and from the C. G. S. system.

TABLE 50 contains mathematical and physical constants in frequent use.

SOURCES OF AUTHORITY.

TABLES 1–3 H were prepared, in so far as possible, from existing tables, by rejecting decimal places when necessary. More than 3,000 values (including all doubtful cases) were confirmed or determined by an independent calculation. The results were printed with the ordinary precautions to avoid typographical errors. Tables 4–5 A were obtained by transposing Pickering's tables 6–9.

The logarithms of numbers from 1,000 to 10,000, in Table 6, were printed directly from a copy of the tables arranged by Mr. Oliver Whipple Huntington, of Harvard College. The proofs were compared with Bowditch's 5-place tables (Government Printing Office, Washington, 1882). The logarithms of numbers from 10,000 to 11,000 were obtained by rejecting figures in Chamber's 7–8-place tables. A special investigation was made in cases where the rejected figures were 50 or 500. Stereotype-proofs of all the logarithms were compared with the tables of Gauss. The table of probabilities as far as 5.0 is due to Chauvenet. The remainder of the table was the result of special calculation.

The physical tables (Nos. 8 to 50) were compiled for the most part by the aid of results contained in Landolt and Börnstein's "Physico-Chemical Tables,"[1] to which the reader is referred for a full exposition of the evidence upon which the selection of values has been made. The author wishes to thank Professors Landolt and Börnstein for looking over his manuscript, for several useful suggestions, and for their kind permission to utilize their results.

The author has quoted numerous data from Everett's "Units and Physical Constants" (Macmillan, 1886). He has also made use of information given by Professor Everett in choosing the unusually low value (4.17×10^7) for the mechanical equivalent of heat.

Among other books from which results have been taken are the following: Cooke's Chemical Philosophy, Deschanel's Natural Philosophy, Ganot's Physics, Hoffmann's Tabellen für Chemiker, Kohlrausch's Leitfaden der Praktischen Physik, das Nautisches Jahrbuch, 1891, Pickering's Physical Manipulation, Stewart and Gee's Practical Physics, Storer's Dictionary of Solubilities, Trowbridge's New Physics, and Weisbach's Mechanics.

These and other sources of authority have been acknowledged in connection with the explanation of the tables above; but it was found impossible, in the limited space which could be devoted to the tables, to give authority for the separate data. It was,

[1] Physikalisch-Chemische Tabellen von Dr. H. Landolt und Dr. Richard Börnstein, Professoren. Verlag von Julius Springer, Montbijou Platz 3, Berlin.

moreover, considered inexpedient to present to students, who would naturally be unaccustomed to weighing evidence, the conflicting statements from which the probable values of many of the physical constants have to be estimated by scientific men.

Care has been taken, in all such cases, to give results *intermediate between* those obtained by different observers. To do this, a considerable number of figures was sometimes required; but the use of figures, *not really significant*, has been in so far as possible avoided. The last figure quoted in the results is in general the only one in regard to which a difference of opinion was found to exist.

It is regretted that, owing to the necessity of entrusting the composition to foreign printers, obvious imperfections of type will be found, especially in the mathematical tables. In the expectation of reprinting these tables at no distant date, corrections and suggestions will be most gladly received.

№	.1	.2	.3	.4	.5	.6	.7	.8	.9	№	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0	0	0	0	0	0	0	0	0	**50**	5	10	15	20	25	30	35	40	45
1	0	0	0	0	1	1	1	1	1	**51**	5	10	15	20	26	31	36	41	46
2	0	0	1	1	1	1	1	2	2	**52**	5	10	16	21	26	31	36	42	47
3	0	1	1	1	2	2	2	2	3	**53**	5	11	16	21	27	32	37	42	48
4	0	1	1	2	2	2	3	3	4	**54**	5	11	16	22	27	32	38	43	49
5	1	1	2	2	3	3	4	4	5	**55**	6	11	17	22	28	33	39	44	50
6	1	1	2	2	3	4	4	5	5	**56**	6	11	17	22	28	34	39	45	50
7	1	1	2	3	4	4	5	6	6	**57**	6	11	17	23	29	34	40	46	51
8	1	2	2	3	4	5	6	6	7	**58**	6	12	17	23	29	35	41	46	52
9	1	2	3	4	5	5	6	7	8	**59**	6	12	18	24	30	35	41	47	53
10	1	2	3	4	5	6	7	8	9	**60**	6	12	18	24	30	36	42	48	54
11	1	2	3	4	6	7	8	9	10	**61**	6	12	18	24	31	37	43	49	55
12	1	2	4	5	6	7	8	10	11	**62**	6	12	19	25	31	37	43	50	56
13	1	3	4	5	7	8	9	10	12	**63**	6	13	19	25	32	38	44	50	57
14	1	3	4	6	7	8	10	11	13	**64**	6	13	19	26	32	38	45	51	58
15	2	3	5	6	8	9	11	12	14	**65**	7	13	20	26	33	39	46	52	59
16	2	3	5	6	8	10	11	13	14	**66**	7	13	20	26	33	40	46	53	59
17	2	3	5	7	9	10	12	14	15	**67**	7	13	20	27	34	40	47	54	60
18	2	4	5	7	9	11	13	14	16	**68**	7	14	20	27	34	41	48	54	61
19	2	4	6	8	10	11	13	15	17	**69**	7	14	21	28	35	41	48	55	62
20	2	4	6	8	10	12	14	16	18	**70**	7	14	21	28	35	42	49	56	63
21	2	4	6	8	11	13	15	17	19	**71**	7	14	21	28	36	43	50	57	64
22	2	4	7	9	11	13	15	18	20	**72**	7	14	22	29	36	43	50	58	65
23	2	5	7	9	12	14	16	18	21	**73**	7	15	22	29	37	44	51	58	66
24	2	5	7	10	12	14	17	19	22	**74**	7	15	22	30	37	44	52	59	67
25	3	5	8	10	13	15	18	20	23	**75**	8	15	23	30	38	45	53	60	68
26	3	5	8	10	13	16	18	21	23	**76**	8	15	23	30	38	46	53	61	68
27	3	5	8	11	14	16	19	22	24	**77**	8	15	23	31	39	46	54	62	69
28	3	6	8	11	14	17	20	22	25	**78**	8	16	23	31	39	47	55	62	70
29	3	6	9	12	15	17	20	23	26	**79**	8	16	24	32	40	47	55	63	71
30	3	6	9	12	15	18	21	24	27	**80**	8	16	24	32	40	48	56	64	72
31	3	6	9	12	16	19	22	25	28	**81**	8	16	24	32	41	49	57	65	73
32	3	6	10	13	16	19	22	26	29	**82**	8	16	25	33	41	49	57	66	74
33	3	7	10	13	17	20	23	26	30	**83**	8	17	25	33	42	50	58	66	75
34	3	7	10	14	17	20	24	27	31	**84**	8	17	25	34	42	50	59	67	76
35	4	7	11	14	18	21	25	28	32	**85**	9	17	26	34	43	51	60	68	77
36	4	7	11	14	18	22	25	29	32	**86**	9	17	26	34	43	52	60	69	77
37	4	7	11	15	19	22	26	30	33	**87**	9	17	26	35	44	52	61	70	78
38	4	8	11	15	19	23	27	30	34	**88**	9	18	26	35	44	53	62	70	79
39	4	8	12	16	20	23	27	31	35	**89**	9	18	27	36	45	53	62	71	80
40	4	8	12	16	20	24	28	32	36	**90**	9	18	27	36	45	54	63	72	81
41	4	8	12	16	21	25	29	33	37	**91**	9	18	27	36	46	55	64	73	82
42	4	8	13	17	21	25	29	34	38	**92**	9	18	28	37	46	55	64	74	83
43	4	9	13	17	22	26	30	34	39	**93**	9	19	28	37	47	56	65	74	84
44	4	9	13	18	22	26	31	35	40	**94**	9	19	28	38	47	56	66	75	85
45	5	9	14	18	23	27	32	36	41	**95**	10	19	29	38	48	57	67	76	86
46	5	9	14	18	23	28	32	37	41	**96**	10	19	29	38	48	58	67	77	86
47	5	9	14	19	24	28	33	38	42	**97**	10	19	29	39	49	58	68	78	87
48	5	10	14	19	24	29	34	38	43	**98**	10	20	29	39	49	59	69	78	88
49	5	10	15	20	25	29	34	39	44	**99**	10	20	30	40	50	59	69	79	89
50	5	10	15	20	25	30	35	40	45	**100**	10	20	30	40	50	60	70	80	90

No.	a. Reciprocal	b. Square Root	c. Square	d. Cube	No.	a. Reciprocal	b. Square Root	c. Square	d. Cube
0	∞	0.00	0	0	50	.0200	7.07	2500	125000
1	1.000	1.00	1	1	51	196	7.14	2601	132651
2	0.500	1.41	4	8	52	192	7.21	2704	140608
3	333	1.73	9	27	53	189	7.28	2809	148877
4	250	2.00	16	64	54	185	7.35	2916	157464
5	0.200	2.24	25	125	55	.0182	7.42	3025	166375
6	167	2.45	36	216	56	179	7.48	3136	175616
7	143	2.65	49	343	57	175	7.55	3249	185193
8	125	2.83	64	512	58	172	7.62	3364	195112
9	111	3.00	81	729	59	169	7.68	3481	205379
10	0.100	3.16	100	1000	60	.0167	7.75	3600	216000
11	.0909	3.32	121	1331	61	164	7.81	3721	226981
12	833	3.46	144	1728	62	161	7.87	3844	238328
13	769	3.61	169	2197	63	159	7.94	3969	250047
14	714	3.74	196	2744	64	156	8.00	4096	262144
15	.0667	3.87	225	3375	65	.0154	8.06	4225	274625
16	625	4.00	256	4096	66	152	8.12	4356	287496
17	588	4.12	289	4913	67	149	8.19	4489	300763
18	556	4.24	324	5832	68	147	8.25	4624	314432
19	526	4.36	361	6859	69	145	8.31	4761	328509
20	.0500	4.47	400	8000	70	.0143	8.37	4900	343000
21	476	4.58	441	9261	71	141	8.43	5041	357911
22	455	4.69	484	10648	72	139	8.49	5184	373248
23	435	4.80	529	12167	73	137	8.54	5329	389017
24	417	4.90	576	13824	74	135	8.60	5476	405224
25	.0400	5.00	625	15625	75	0133	8.66	5625	421875
26	385	5.10	676	17576	76	132	8.72	5776	438976
27	370	5.20	729	19683	77	130	8.77	5929	456533
28	357	5.29	784	21952	78	128	8.83	6084	474552
29	345	5.39	841	24389	79	127	8.89	6241	493039
30	.0333	5.48	900	27000	80	.0125	8.94	6400	512000
31	323	5.57	961	29791	81	123	9.00	6561	531441
32	313	5.66	1024	32768	82	122	9.06	6724	551368
33	303	5.74	1089	35937	83	120	9.11	6889	571787
34	294	5.83	1156	39304	84	119	9.17	7056	592704
35	.0286	5.92	1225	42875	85	.0118	9.22	7225	614125
36	278	6.00	1296	46656	86	116	9.27	7396	636056
37	270	6.08	1369	50653	87	115	9.33	7569	658503
38	263	6.16	1444	54872	88	114	9.38	7744	681472
39	256	6.24	1521	59319	89	112	9.43	7921	704969
40	.0250	6.32	1600	64000	90	.0111	9.49	8100	729000
41	244	6.40	1681	68921	91	110	9.54	8281	753571
42	238	6.48	1764	74088	92	109	9.59	8464	778688
43	233	6.56	1849	79507	93	108	9.64	8649	804357
44	227	6.63	1936	85184	94	106	9.70	8836	830584
45	.0222	6.71	2025	91125	95	.0105	9.75	9025	857375
46	217	6.78	2116	97336	96	104	9.80	9216	884736
47	213	6.86	2209	103823	97	103	9.85	9409	912673
48	208	6.93	2304	110592	98	102	9.90	9604	941192
49	204	7.00	2401	117649	99	101	9.95	9801	970299
50	.0200	7.07	2500	125000	100	.0100	10.00	10000	1000000

Diameter	e. Logarithm	f. Circumference	g. Area of Circle	h. Volume of Sphere	Diameter	e. Logarithm	f. Circumference	g. Area of Circle	h. Volume of Sphere
.0	$-\infty$	0.00	0.00	.000	5.0	0.699	15.71	19.6	65
.1	$\bar{1}.000$	31	01	.001	5.1	708	16.02	20.4	69
.2	301	63	03	.004	5.2	716	16.34	21.2	74
.3	477	94	07	.014	5.3	724	16.65	22.1	78
.4	602	1.26	13	.034	5.4	732	16.96	22.9	82
.5	$\bar{1}.699$	1.57	0.20	.065	5.5	0.740	17.28	23.8	87
.6	778	1.88	28	.113	5.6	748	17.59	24.6	92
.7	845	2.20	38	.180	5.7	756	17.91	25.5	97
.8	903	2.51	50	.268	5.8	763	18.22	26.4	102
.9	954	2.83	64	.382	5.9	771	18.54	27.3	108
1.0	0.000	3.14	0.79	0.52	6.0	0.778	18.85	28.3	113
1.1	041	3.46	0.95	70	6.1	785	19.16	29.2	119
1.2	079	3.77	1.13	90	6.2	792	19.48	30.2	125
1.3	114	4.08	1.33	1.15	6.3	799	19.79	31.2	131
1.4	146	4.40	1.54	1.44	6.4	806	20.11	32.2	137
1.5	0.176	4.71	1.77	1.77	6.5	0.813	20.42	33.2	144
1.6	204	5.03	2.01	2.14	6.6	820	20.73	34.2	151
1.7	230	5.34	2.27	2.57	6.7	826	21.05	35.3	157
1.8	255	5.65	2.54	3.05	6.8	833	21.36	36.3	165
1.9	279	5.97	2.81	3.59	6.9	839	21.68	37.4	172
2.0	0.301	6.28	3.14	4.19	7.0	0.845	21.99	38.5	180
2.1	322	6.60	3.46	4.85	7.1	851	22.31	39.6	187
2.2	342	6.91	3.80	5.58	7.2	857	22.62	40.7	195
2.3	362	7.23	4.15	6.37	7.3	863	22.93	41.9	204
2.4	380	7.54	4.52	7.24	7.4	869	23.25	43.0	212
2.5	0.398	7.85	4.91	8.2	7.5	0.875	23.56	44.2	221
2.6	415	8.17	5.31	9.2	7.6	881	23.88	45.4	230
2.7	431	8.48	5.73	10.3	7.7	886	24.19	46.6	239
2.8	447	8.80	6.16	11.5	7.8	892	24.50	47.8	248
2.9	462	9.11	6.61	12.8	7.9	898	24.82	49.0	258
3.0	0.477	9.42	7.07	14.1	8.0	0.903	25.13	50.3	268
3.1	491	9.74	7.55	15.6	8.1	908	25.45	51.5	278
3.2	505	10.05	8.04	17.2	8.2	914	25.76	52.8	289
3.3	519	10.37	8.55	18.8	8.3	919	26.08	54.1	299
3.4	531	10.68	9.08	20.6	8.4	924	26.39	55.4	310
3.5	0.544	11.00	9.6	22.4	8.5	0.929	26.70	56.7	322
3.6	556	11.31	10.2	24.4	8.6	934	27.02	58.1	333
3.7	568	11.62	10.8	26.5	8.7	940	27.33	59.4	345
3.8	580	11.94	11.3	28.7	8.8	944	27.65	60.8	357
3.9	591	12.25	11.9	31.1	8.9	949	27.96	62.2	369
4.0	0.602	12.57	12.6	33.5	9.0	0.954	28.27	63.6	382
4.1	613	12.88	13.2	36.1	9.1	959	28.59	65.0	395
4.2	623	13.19	13.9	38.8	9.2	964	28.90	66.5	408
4.3	633	13.51	14.5	41.6	9.3	968	29.22	67.9	421
4.4	643	13.82	15.2	44.6	9.4	973	29.53	69.4	435
4.5	0.653	14.14	15.9	47.7	9.5	0.978	29.85	70.9	449
4.6	663	14.45	16.6	51.0	9.6	982	30.16	72.4	463
4.7	672	14.77	17.3	54.4	9.7	987	30.47	73.9	478
4.8	681	15.08	18.1	57.9	9.8	991	30.79	75.4	493
4.9	690	15.39	18.9	61.6	9.9	996	31.10	77.0	508
5.0	0.699	15.71	19.6	65.4	10.0	1.000	31.42	78.5	524

a. Angle.	*b.* Tangent.	*c.* Arc.	*d.* Chord.	*e.* Sine.	*f.* Cosine.	*g.* Rate of Vibration.	*h.* Secant.	*i.* Complement.
0°	0.000	0.000	0.000	0.000	1.000	1.00000	1.000	90°
1	017	017	017	017	1.000	1.00000	1.000	89
2	035	035	035	035	0.999	0.99998	1.001	88
3	052	052	052	052	999	99996	1.001	87
4	070	070	070	070	998	99992	1.002	86
5	0.087	0.087	0.087	0.087	0.996	0.99988	1.004	85
6	105	105	105	105	995	99983	1.006	84
7	123	122	122	122	993	99977	1.008	83
8	141	140	140	139	990	99970	1.010	82
9	158	157	157	156	988	99961	1.012	81
10	0.176	0.175	0.174	0.174	0.985	0.99952	1.015	80
11	194	192	192	191	982	99942	1.019	79
12	213	209	209	208	978	99931	1.022	78
13	231	227	226	225	974	99920	1.026	77
14	249	244	244	242	970	99907	1.031	76
15	0.268	0.262	0.261	0.259	0.966	0.99893	1.035	75
16	287	279	278	276	961	99878	1.040	74
17	306	297	296	292	956	99862	1.046	73
18	325	314	313	309	951	99846	1.051	72
19	344	332	330	326	946	99828	1.058	71
20	0.364	0.349	0.347	0.342	0.940	0.99810	1.064	70
21	384	367	364	358	934	99790	1.071	69
22	404	384	382	375	927	99770	1·079	68
23	424	401	399	391	921	99749	1.086	67
24	445	419	416	407	914	99726	1.095	66
25	0.466	0.436	0.433	0.423	0.906	0.99703	1.103	65
26	488	454	450	438	899	99678	1.113	64
27	510	471	467	454	891	99653	1.122	63
28	532	489	484	469	883	99627	1.133	62
29	554	506	501	485	875	99600	1.143	61
30	0.577	0.524	0.518	0.500	0.866	0.99572	1.155	60
31	601	541	534	515	857	99543	1.167	59
32	625	559	551	530	848	99513	1.179	58
33	649	576	568	545	839	99482	1.192	57
34	675	593	585	559	829	99450	1.206	56
35	0.700	0.611	0.601	0.574	0.819	0.99417	1.221	55
36	727	628	618	588	809	99384	1.236	54
37	754	646	635	602	799	99349	1.252	53
38	781	663	651	616	788	99314	1.269	52
39	810	681	668	629	777	99277	1.287	51
40	0.839	0.698	0.684	0.643	0.766	0.99239	1.305	50
41	869	716	700	656	755	99200	1.325	49
42	900	733	717	669	743	99161	1.346	48
43	933	750	733	682	731	99121	1.367	47
44	966	768	749	695	719	99079	1.390	46
45°	1.000	0.785	0.765	0.707	0.707	0.99037	1.414	45°

a. Angle.	b. Tangent.	c. Arc.	d. Chord.	e. Sine.	f. Cosine.	g. Coversine	h. Secant.	i. Complement.
45°	1.000	0.785	0.765	0.707	0.707	0.293	1.414	45°
46	1.036	0.803	781	719	695	281	1.440	44
47	1.072	0.820	797	731	682	269	1.466	43
48	1.111	0.838	813	743	669	257	1.494	42
49	1.150	0.855	829	755	656	245	1.524	41
50	1.192	0.873	0.845	0.766	0.643	0.234	1.556	40
51	1.235	0.890	861	777	629	223	1.589	39
52	1.280	0.908	877	788	616	212	1.624	38
53	1.327	0.925	892	799	602	201	1.662	37
54	1.376	0.942	908	809	588	191	1.701	36
55	1.428	0.960	0.923	0.819	0.574	0.181	1.743	35
56	1.483	0.977	939	829	559	171	1.788	34
57	1.540	0.995	954	839	545	161	1.836	33
58	1.600	1.012	970	848	530	152	1.887	32
59	1.664	1.030	985	857	515	143	1 942	31
60	1.732	1.047	1.000	0.866	0.500	0.134	2.000	30
61	1.804	1.065	1.015	875	485	125	2.063	29
62	1.881	1.082	1.030	883	469	117	2.130	28
63	1.963	1.100	1.045	891	454	109	2.203	27
64	2.050	1.117	1.060	899	438	101	2.281	26
65	2.145	1.134	1.075	0 906	0.423	0.094	2 366	25
66	2.246	1.152	1.089	914	407	086	2.459	24
67	2.356	1.169	1.104	921	391	079	2.559	23
68	2.475	1.187	1.118	927	375	073	2.669	22
69	2.605	1.204	1.133	934	358	066	2.790	21
70	2.747	1.222	1.147	0.940	0.342	0.060	2.924	20
71	2.904	1.239	1.161	946	326	054	3.072	19
72	3.078	1.257	1.176	951	309	049	3.236	18
73	3.271	1.274	1.190	956	292	044	3.420	17
74	3.487	1.292	1.204	961	276	039	3.628	16
75	3.732	1.309	1 218	0.966	0.259	0.034	3.864	15
76	4.011	1.326	1.231	970	242	030	4.134	14
77	4.331	1.344	1.245	974	225	026	4.445	13
78	4.705	1.361	1.259	978	208	022	4.810	12
79	5.145	1.379	1.272	982	191	018	5.241	11
80	5.671	1.396	1.286	0 985	0.174	0.0152	5.759	10
81	6.314	1.414	1 299	988	156	0123	6.392	9
82	7.115	1.431	1.312	990	139	0097	7.185	8
83	8.144	1.449	1.325	993	122	0075	8.206	7
84	9.514	1.466	1 338	995	105	0055	9.567	6
85	11.43	1.484	1.351	0.996	0.087	0.00381	11.47	5
86	14.30	1.501	1 364	998	070	00244	14.34	4
87	19.08	1.518	1.377	999	052	00137	19.11	3
88	28.64	1.536	1.389	999	035	00061	28.65	2
89	57.29	1.553	1 402	1.000	017	00015	57.30	1
90°	∞	1.571	1.414	1.000	0.000	0.00000	∞	0°

№	0	1	2	3	4	5	6	7	8	9	M.
1.0	1.0000	9901	9804	9709	9615	9524	9434	9346	9259	9174	92
1.1	0.9091	9009	8929	8850	8772	8696	8621	8547	8475	8403	76
1.2	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752	66
1.3	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194	55
1.4	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	48
1.5	0.6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	42
1.6	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	37
1.7	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	33
1.8	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	29
1.9	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	26
2.0	0.5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	24
2.1	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	22
2.2	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	20
2.3	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	18
2.4	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	17
2.5	0.4000	3984	3968	3953	3937	3922	3906	3891	3876	3861	15
2.6	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	14
2.7	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	13
2.8	3571	3559	3546	3534	3521	3509	3496	3484	3472	3460	12
2.9	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	12
3.0	0.3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	11
3.1	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	10
3.2	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	9
3.3	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	9
3.4	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	
3.5	0.2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	8
3.6	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	
3.7	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	7
3.8	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	
3.9	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	
4.0	0.2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	6
4.1	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	
4.2	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	
4.3	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	
4.4	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	5
4.5	0.2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	
4.6	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	
4.7	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	
4.8	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	
4.9	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	
5.0	0.2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	4
5.1	1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	
5.2	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	
5.3	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	
5.4	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	
5.5	0.1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	
5.6	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	
5.7	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	3
5.8	1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	
5.9	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	
6.0	0.1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	

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6.0	0.16667	16639	16611	16584	16556	16529	16502	16474	16447	16420	27
6.1	16393	16367	16340	16313	16287	16260	16234	16207	16181	16155	26
6.2	16129	16103	16077	16051	16026	16000	15974	15949	15924	15898	26
6.3	15873	15848	15823	15798	15773	15748	15723	15699	15674	15649	25
6.4	15625	15601	15576	15552	15528	15504	15480	15456	15432	15408	24
6.5	0.15385	15361	15337	15314	15291	15267	15244	15221	15198	15175	23
6.6	15152	15129	15106	15083	15060	15038	15015	14992	14970	14948	23
6.7	14925	14903	14881	14859	14837	14815	14793	14771	14749	14728	22
6.8	14706	14684	14663	14641	14620	14599	14577	14556	14535	14514	21
6.9	14493	14472	14451	14430	14409	14388	14368	14347	14327	14306	21
7.0	0.14286	14265	14245	14225	14205	14184	14164	14144	14124	14104	20
7.1	14085	14065	14045	14025	14006	13986	13966	13947	13928	13908	
7.2	13889	13870	13850	13831	13812	13793	13774	13755	13736	13717	19
7.3	13699	13680	13661	13643	13624	13605	13587	13569	13550	13532	
7.4	13514	13495	13477	13459	13441	13423	13405	13387	13369	13351	18
7.5	0.13333	13316	13298	13280	13263	13245	13228	13210	13193	13175	
7.6	13158	13141	13123	13106	13089	13072	13055	13038	13021	13004	17
7.7	12987	12970	12953	12937	12920	12903	12887	12870	12853	12837	
7.8	12821	12804	12788	12771	12755	12739	12723	12706	12690	12674	16
7.9	12658	12642	12626	12610	12594	12579	12563	12547	12531	12516	
8.0	0.12500	12484	12469	12453	12438	12422	12407	12392	12376	12361	
8.1	12346	12330	12315	12300	12285	12270	12255	12240	12225	12210	15
8.2	12195	12180	12165	12151	12136	12121	12107	12092	12077	12063	
8.3	12048	12034	12019	12005	11990	11976	11962	11947	11933	11919	
8.4	11905	11891	11876	11862	11848	11834	11820	11806	11792	11779	14
8.5	0.11765	11751	11737	11723	11710	11696	11682	11669	11655	11641	
8.6	11628	11614	11601	11587	11574	11561	11547	11534	11521	11507	
8.7	11494	11481	11468	11455	11442	11429	11416	11403	11390	11377	13
8.8	11364	11351	11338	11325	11312	11299	11287	11274	11261	11249	
8.9	11236	11223	11211	11198	11186	11173	11161	11148	11136	11123	
9.0	0.11111	11099	11086	11074	11062	11050	11038	11025	11013	11001	
9.1	10989	10977	10965	10953	10941	10929	10917	10905	10893	10881	12
9.2	10870	10858	10846	10834	10823	10811	10799	10787	10776	10764	
9.3	10753	10741	10730	10718	10707	10695	10684	10672	10661	10650	
9.4	10638	10627	10616	10604	10593	10582	10571	10560	10549	10537	
9.5	0.10526	10515	10504	10493	10482	10471	10460	10449	10438	10428	11
9.6	10417	10406	10395	10384	10373	10363	10352	10341	10331	10320	
9.7	10309	10299	10288	10277	10267	10256	10246	10235	10225	10215	
9.8	10204	10194	10183	10173	10163	10152	10142	10132	10121	10111	
9.9	10101	10091	10081	10070	10060	10050	10040	10030	10020	10010	
10.0	0.10000	.09990	9980	9970	9960	9950	9940	9930	9921	9911	10
10.1	.09901	9891	9881	9872	9862	9852	9843	9833	9823	9814	
10.2	9804	9794	9785	9775	9766	9756	9747	9737	9728	9718	
10.3	9709	9699	9690	9681	9671	9662	9653	9643	9634	9625	
10.4	9615	9606	9597	9588	9579	9569	9560	9551	9542	9533	
10.5	0.09524	9515	9506	9497	9488	9479	9470	9461	9452	9443	9
10.6	9434	9425	9416	9407	9398	9390	9381	9372	9363	9355	
10.7	9346	9337	9328	9320	9311	9302	9294	9285	9276	9268	
10.8	9259	9251	9242	9234	9225	9217	9208	9200	9191	9183	
10.9	9174	9166	9158	9149	9141	9132	9124	9116	9107	9099	
11.0	0.09091	9083	9074	9066	9058	9050	9042	9033	9025	9017	8

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1.0	1.000	1.020	1.040	1.061	1.082	1.103	1.124	1.145	1.166	1.188	21
1.1	1.210	1.232	1.254	1.277	1.300	1.323	1.346	1.369	1.392	1.416	23
1.2	1.440	1.464	1.488	1.513	1.538	1.563	1.588	1.613	1.638	1.664	25
1.3	1.690	1.716	1.742	1.769	1.796	1.823	1.850	1.877	1.904	1.932	27
1.4	1.960	1.988	2.016	2.045	2.074	2.103	2.132	2.161	2.190	2.220	29
1.5	2.250	2.280	2.310	2.341	2.372	2.403	2.434	2.465	2.496	2.528	31
1.6	2.560	2.592	2.624	2.657	2.690	2.723	2.756	2.789	2.822	2.856	33
1.7	2.890	2.924	2.958	2.993	3.028	3.063	3.098	3.133	3.168	3.204	35
1.8	3.240	3.276	3.312	3.349	3.386	3.423	3.460	3.497	3.534	3.572	37
1.9	3.610	3.648	3.686	3.725	3.764	3.803	3.842	3.881	3.920	3.960	39
2.0	4.000	4.040	4.080	4.121	4.162	4.203	4.244	4.285	4.326	4.368	41
2.1	4.410	4.452	4.494	4.537	4.580	4.623	4.666	4.709	4.752	4.796	43
2.2	4.840	4.884	4.928	4.973	5.018	5.063	5.108	5.153	5.198	5.244	45
2.3	5.290	5.336	5.382	5.429	5.476	5.523	5.570	5.617	5.664	5.712	47
2.4	5.760	5.808	5.856	5.905	5.954	6.003	6.052	6.101	6.150	6.200	49
2.5	6.250	6.300	6.350	6.401	6.452	6.503	6.554	6.605	6.656	6.708	51
2.6	6.760	6.812	6.864	6.917	6.970	7.023	7.076	7.129	7.182	7.236	53
2.7	7.290	7.344	7.398	7.453	7.508	7.563	7.618	7.673	7.728	7.784	55
2.8	7.840	7.896	7.952	8.009	8.066	8.123	8.180	8.237	8.294	8.352	57
2.9	8.410	8.468	8.526	8.585	8.644	8.703	8.762	8.821	8.880	8.940	59
3.0	9.000	9.060	9.120	9.181	9.242	9.303	9.364	9.425	9.486	9.548	61
3.1	9.610	9.672	9.734	9.797	9.860	9.923	9.986	10.05	10.11	10.18	—
3.2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82	
3.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	
3.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	7
3.6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62	
3.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	
3.8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13	
3.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73	8
4.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	
4.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40	
4.3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27	
4.4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16	
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	9
4.6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	
4.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.94	
4.8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	
4.9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.90	
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.91	10
5.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	
5.2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.98	
5.3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05	
5.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14	
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25	11
5.6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38	
5.7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52	
5.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	
5.9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88	
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	

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6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	1
6.1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32	
6.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56	
6.3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83	
6.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12	
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43	13
6.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76	
6.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10	
6.8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.33	47.47	
6.9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86	
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	50.13	50.27	14
7.1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	51.55	51.70	
7.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	53.00	53.14	
7.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61	
7.4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.95	56.10	
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.46	57.61	15
7.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14	
7.7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.53	60.68	
7.8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	62.09	62.25	
7.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84	
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45	16
8.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08	
8.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72	
8.3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39	
8.4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08	
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79	17
8.6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52	
8.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26	
8.8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03	
8.9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.64	80.82	
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63	18
9.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46	
9.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	85.93	86.12	86.30	
9.3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.98	88.17	
9.4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06	
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97	19
9.6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90	
9.7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45	95.65	95.84	
9.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81	
9.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80	
10.0	100.00	100.20	100.40	100.60	100.80	101.00	101.20	101.40	101.61	101.81	20
10.1	102.01	102.21	102.41	102.62	102.82	103.02	103.23	103.43	103.63	103.84	
10.2	104.04	104.24	104.45	104.65	104.86	105.06	105.27	105.47	105.68	105.88	
10.3	106.09	106.30	106.50	106.71	106.92	107.12	107.33	107.54	107.74	107.95	
10.4	108.16	108.37	108.58	108.78	108.99	109.20	109.41	109.62	109.83	110.04	
10.5	110.25	110.46	110.67	110.88	111.09	111.30	111.51	111.72	111.94	112.15	21
10.6	112.36	112.57	112.78	113.00	113.21	113.42	113.64	113.85	114.06	114.28	
10.7	114.49	114.70	114.92	115.13	115.35	115.56	115.78	115.99	116.21	116.42	
10.8	116.64	116.86	117.07	117.29	117.51	117.72	117.94	118.16	118.37	118.59	
10.9	118.81	119.03	119.25	119.46	119.68	119.90	120.12	120.34	120.56	120.78	
11.0	121.00	121.22	121.44	121.66	121.88	122.10	122.32	122.54	122.77	122.99	22

№	0	1	2	3	4	5	6	7	8	9	Dif.
1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	33
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685	39
1.2	1.728	1.772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147	47
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686	55
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308	63
1.5	3.375	3.443	3.512	3.582	3.652	3.724	3.796	3.870	3.944	4.020	72
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827	82
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735	92
1.8	5.832	5.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751	101
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881	114
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	8.870	8.999	9.129	126
2.1	9.261	9.394	9.528	9.664	9.800	9.938	10.08	10.22	10.36	10.50	—
2.2	10.65	10.79	10.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01	15
2.3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	13.48	13.65	16
2.4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44	18
2.5	15.63	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37	19
2.6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47	21
2.7	19.68	19.90	20.12	20.35	20.57	20.80	21.02	21.25	21.48	21.72	22
2.8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14	24
2.9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	26.46	26.73	26
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50	28
3.1	29.79	30.08	30.37	30.66	30.96	31.26	31.55	31.86	32.16	32.46	30
3.2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61	32
3.3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96	34
3.4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51	36
3.5	42.88	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27	38
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24	40
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44	42
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86	44
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52	47
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42	49
4.1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56	52
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95	54
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60	57
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52	60
4.5	91.13	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70	62
4.6	97.34	97.97	98.61	99.25	99.90	100.6	101.2	101.8	102.5	103.2	—
4.7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9	7
4.8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9	7
4.9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3	7
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9	8
5.1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8	8
5.2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0	8
5.3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.9	155.7	156.6	9
5.4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5	9
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7	9
5.6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2	10
5.7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1	10
5.8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3	10
5.9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9	11
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9	11

	0	1	2	3	4	5	6	7	8	9	DH.
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9	11
6.1	227 0	228.1	229.2	230.3	231.5	232.6	233.7	234.9	236.0	237.2	11
6.2	238.3	239.5	240.6	241.8	243.0	244.1	245.3	246.5	247.7	248.9	12
6.3	250.0	251.2	252.4	253.6	254.8	256.0	257.3	258.5	259.7	260.9	12
6.4	262.1	263.4	264.6	265.8	267.1	268.3	269.6	270.8	272.1	273.4	12
6.5	274.6	275.9	277.2	278.4	279.7	281.0	282.3	283.6	284.9	286.2	13
6.6	287.5	288.8	290 1	291.4	292.8	294.1	295.4	296.7	298.1	299.4	13
6.7	300.8	302.1	303.5	304.8	306.2	307.5	308.9	310.3	311.7	313.0	14
6.8	314.4	315.8	317.2	318.6	320.0	321.4	322.8	324.2	325.7	327.1	14
6.9	328.5	329.9	331.4	332.8	334.3	335.7	337.2	338.6	340.1	341.5	15
7.0	343.0	344.5	345.9	347.4	348.9	350.4	351.9	353.4	354.9	356.4	15
7.1	357.9	359.4	360.9	362.5	364.0	365.5	367.1	368.6	370.1	371.7	15
7.2	373.2	374.8	376.4	377.9	379.5	381.1	382.7	384.2	385.8	387.4	16
7.3	389.0	390.6	392.2	393.8	395.4	397.1	398.7	400.3	401.9	403.6	16
7.4	405.2	406.9	408.5	410.2	411.8	413.5	415.2	416.8	418.5	420.2	16
7.5	421.9	423.6	425.3	427.0	428.7	430.4	432.1	433.8	435.5	437.2	17
7.6	439.0	440.7	442.5	444.2	445.9	447.7	449.5	451.2	453.0	454.8	17
7.7	456 5	458.3	460.1	461.9	463.7	465.5	467.3	469.1	470.9	472.7	18
7.8	474.6	476.4	478.2	480.0	481.9	483.7	485.6	487.4	489.3	491.2	18
7.9	493.0	494.9	496.8	498.7	500.6	502.5	504.4	506.3	508.2	510.1	19
8.0	512.0	513.9	515.8	517.8	519.7	521.7	523.6	525.6	527.5	529.5	19
8.1	531.4	533.4	535.4	537.4	539.4	541.3	543.3	545.3	547.3	549.4	20
8.2	551.4	553.4	555.4	557.4	559.5	561.5	563.6	565.6	567.7	569 7	20
8.3	571.8	573.9	575.9	578.0	580.1	582.2	584.3	586.4	588.5	590.6	21
8.4	592.7	594.8	596.9	599.1	601.2	603.4	605.5	607.6	609.8	612.0	21
8.5	614.1	616.3	618.5	620.7	622.8	625.0	627.2	629.4	631.6	633.8	22
8.6	636.1	638.3	640.5	642.7	645 0	647.2	649.5	651.7	654.0	656.2	22
8.7	658 5	660.8	663.1	665.3	667.6	669.9	672.2	674.5	676.8	679.2	23
8.8	681.5	683.8	686.1	688.5	690.8	693.2	695.5	697.9	700.2	702.6	23
8.9	705.0	707.3	709.7	712.1	714.5	716.9	719.3	721.7	724.2	726.6	24
9.0	729.0	731.4	733.9	736.3	738.8	741.2	743.7	746.1	748.6	751.1	25
9.1	753.6	756.1	758.6	761.0	763.6	766 1	768.6	771.1	773.6	776.2	25
9.2	778.7	781.2	783.8	786.3	788.9	791.5	794.0	796.6	799.2	801.8	26
9.3	804.4	807.0	809.6	812.2	814.8	817.4	820.0	822.7	825.3	827.9	26
9.4	830.6	833.2	835.9	838.6	841.2	843.9	846.6	849.3	852.0	854.7	27
9.5	857.4	860.1	862.8	865.5	868.3	871.0	873.7	876.5	879.2	882.0	27
9.6	884.7	887.5	890.3	893.1	895.8	898.6	901.4	904.2	907.0	909.9	28
9.7	912.7	915.5	918.3	921.2	924.0	926.9	929.7	932.6	935.4	938.3	28
9.8	941.2	944.1	947.0	949.9	952.8	955.7	958.6	961.5	964.4	967.4	29
9.9	970.3	973.2	976.2	979.1	982.1	985.1	988.0	991.0	994.0	997.0	30
10.0	1000.0	1003.0	1006.0	1009.0	1012.0	1015.1	1018.1	1021.1	1024.2	1027.2	30
10.1	1030.3	1033.4	1036.4	1039.5	1042.6	1045.7	1048.8	1051.9	1055.0	1058.1	31
10.2	1061.2	1064.3	1067.5	1070.6	1073.7	107.69	1080.0	1083.2	1086.4	1089.5	31
10.3	1092.7	1095.9	1099.1	1102.3	1105.5	1108.7	1111.9	1115.2	1118.4	1121.6	32
10.4	1124.9	1128.1	1131.4	1134.6	1137.9	1141.2	1144.4	1147.7	1151.0	1154.3	33
10.5	1157.6	1160.9	1164.3	1167.6	1170.9	1174.2	1177.6	1180.9	1184.3	1187.6	33
10.6	1191.0	1194.4	1197.8	1201.2	1204.6	1207.9	1211.4	1214.8	1218.2	1221.6	34
10.7	1225.0	1228.5	1231.9	1235.4	1238.8	1242.3	1245.8	1249.2	1252.7	1256.2	35
10.8	1259.7	1263.2	1266.7	1270.2	1273.8	1277.3	1280.8	1284.4	1287.9	1291.5	35
10.9	1295.0	1298.6	1302.2	1305.8	1309.3	1312.9	1316.5	1320.1	1323.8	1327.4	36
11.0	1331.0	1334.6	1338.3	1341.9	1345.6	1349.2	1352.9	1356.6	1360.3	1363.9	37

Diam.	00.	10.	20.	30.	40.	50.	60.	70.	80.	90.
10	3142	3173	3204	3236	3267	3299	3330	3362	3393	3424
11	3456	3487	3519	3550	3581	3613	3644	3676	3707	3738
12	3770	3801	3833	3864	3896	3927	3958	3990	4021	4053
13	4084	4115	4147	4178	4210	4241	4273	4304	4335	4367
14	4398	4430	4461	4492	4524	4555	4587	4618	4650	4681
15	4712	4744	4775	4807	4838	4869	4901	4932	4964	4995
16	5027	5058	5089	5121	5152	5184	5215	5246	5278	5309
17	5341	5372	5404	5435	5466	5498	5529	5561	5592	5623
18	5655	5686	5718	5749	5781	5812	5843	5875	5906	5938
19	5969	6000	6032	6063	6095	6126	6158	6189	6220	6252
20	6283	6315	6346	6377	6409	6440	6472	6503	6535	6566
21	6597	6629	6660	6692	6723	6754	6786	6817	6849	6880
22	6912	6943	6974	7006	7037	7069	7100	7131	7163	7194
23	7226	7257	7288	7320	7351	7383	7414	7446	7477	7508
24	7540	7571	7603	7634	7665	7697	7728	7760	7791	7823
25	7854	7885	7917	7948	7980	8011	8042	8074	8105	8137
26	8168	8200	8231	8262	8294	8325	8357	8388	8419	8451
27	8482	8514	8545	8577	8608	8639	8671	8702	8734	8765
28	8796	8828	8859	8891	8922	8954	8985	9016	9048	9079
29	9111	9142	9173	9205	9236	9268	9299	9331	9362	9393
30	9425	9456	9488	9519	9550	9582	9613	9645	9676	9708
31	9739	9770	9802	9833	9865	9896	9927	9959	9990	10022
32	10053	10085	10116	10147	10179	10210	10242	10273	10304	10336
33	10367	10399	10430	10462	10493	10524	10556	10587	10619	10650
34	10681	10713	10744	10776	10807	10838	10870	10901	10933	10964
35	10996	11027	11058	11090	11121	11153	11184	11215	11247	11278
36	11310	11341	11373	11404	11435	11467	11498	11530	11561	11592
37	11624	11655	11687	11718	11750	11781	11812	11844	11875	11907
38	11938	11969	12001	12032	12064	12095	12127	12158	12189	12221
39	12252	12284	12315	12346	12378	12409	12441	12472	12504	12535
40	12566	12598	12629	12661	12692	12723	12755	12786	12818	12849
41	12881	12912	12943	12975	13006	13038	13069	13100	13132	13163
42	13195	13226	13258	13289	13320	13352	13383	13415	13446	13477
43	13509	13540	13572	13603	13635	13666	13697	13729	13760	13792
44	13823	13854	13886	13917	13949	13980	14012	14043	14074	14106
45	14137	14169	14200	14231	14263	14294	14326	14357	14388	14420
46	14451	14483	14514	14546	14577	14608	14640	14671	14703	14734
47	14765	14797	14828	14860	14891	14923	14954	14985	15017	15048
48	15080	15111	15142	15174	15205	15237	15268	15300	15331	15362
49	15394	15425	15457	15488	15519	15551	15582	15614	15645	15677
50	15708	15739	15771	15802	15834	15865	15896	15928	15959	15991
51	16022	16054	16085	16116	16148	16179	16211	16242	16273	16305
52	16336	16368	16399	16431	16462	16493	16525	16556	16588	16619
53	16650	16682	16713	16745	16776	16808	16839	16870	16902	16933
54	16965	16996	17027	17059	17090	17122	17153	17185	17216	17247
55	17279	17310	17342	17373	17404	17436	17467	17499	17530	17562
Dif.	(Mean)	(1) 3	(2) 6	(3) 9	(4) 13	(5) 16	(6) 19	(7) 22	(8) 25	(9) 28

Diam.	00.	10.	20.	30.	40.	50.	60.	70.	80.	90.
55	17279	17310	17342	17373	17404	17436	17467	17499	17530	17562
56	17593	17624	17656	17687	17719	17750	17781	17813	17844	17876
57	17907	17938	17970	18001	18033	18064	18096	18127	18158	18190
58	18221	18253	18284	18315	18347	18378	18410	18441	18473	18504
59	18535	18567	18598	18630	18661	18692	18724	18755	18787	18818
60	18850	18881	18912	18944	18975	19007	19038	19069	19101	19132
61	19164	19195	19227	19258	19289	19321	19352	19384	19415	19446
62	19478	19509	19541	19572	19604	19635	19666	19698	19729	19761
63	19792	19823	19855	19886	19918	19949	19981	20012	20043	20075
64	20106	20138	20169	20200	20232	20263	20295	20326	20358	20389
65	20420	20452	20483	20515	20546	20577	20609	20640	20672	20703
66	20735	20766	20797	20829	20860	20892	20923	20954	20986	21017
67	21049	21080	21112	21143	21174	21206	21237	21269	21300	21331
68	21363	21394	21426	21457	21488	21520	21551	21583	21614	21646
69	21677	21708	21740	21771	21803	21834	21865	21897	21928	21960
70	21991	22023	22054	22085	22117	22148	22180	22211	22242	22274
71	22305	22337	22368	22400	22431	22462	22494	22525	22557	22588
72	22619	22651	22682	22714	22745	22777	22808	22839	22871	22902
73	22934	22965	22996	23028	23059	23091	23122	23154	23185	23216
74	23248	23279	23311	23342	23373	23405	23436	23468	23499	23531
75	23562	23593	23625	23656	23688	23719	23750	23782	23813	23845
76	23876	23908	23939	23970	24002	24033	24065	24096	24127	24159
77	24190	24222	24253	24285	24316	24347	24379	24410	24442	24473
78	24504	24536	24567	24599	24630	24662	24693	24724	24756	24787
79	24819	24850	24881	24913	24944	24976	25007	25038	25070	25101
80	25133	25164	25196	25227	25258	25290	25321	25353	25384	25415
81	25447	25478	25510	25541	25573	25604	25635	25667	25698	25730
82	25761	25792	25824	25855	25887	25918	25950	25981	26012	26044
83	26075	26107	26138	26169	26201	26232	26264	26295	26327	26358
84	26389	26421	26452	26484	26515	26546	26578	26609	26641	26672
85	26704	26735	26766	26798	26829	26861	26892	26923	26955	26986
86	27018	27049	27081	27112	27143	27175	27206	27238	27269	27300
87	27332	27363	27395	27426	27458	27489	27520	27552	27583	27615
88	27646	27677	27709	27740	27772	27803	27835	27866	27897	27929
89	27960	27992	28023	28054	28086	28117	28149	28180	28212	28243
90	28274	28306	28337	28369	28400	28431	28463	28494	28526	28557
91	28588	28620	28651	28683	28714	28746	28777	28808	28840	28871
92	28903	28934	28965	28997	29028	29060	29091	29123	29154	29185
93	29217	29248	29280	29311	29342	29374	29405	29437	29468	29500
94	29531	29562	29594	29625	29657	29688	29719	29751	29782	29814
95	29845	29877	29908	29939	29971	30002	30034	30065	30096	30128
96	30159	30191	30222	30254	30285	30316	30348	30379	30411	30442
97	30473	30505	30536	30568	30599	30631	30662	30693	30725	30756
98	30788	30819	30850	30882	30913	30945	30976	31008	31039	31070
99	31102	31133	31165	31196	31227	31259	31290	31322	31353	31385
100	31416	31447	31479	31510	31542	31573	31604	31636	31667	31699
Dif.	(Mean)	(1) 3	(2) 6	(3) 9	(4) 13	(5) 16	(6) 19	(7) 22	(8) 25	(9) 28

Diam.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
10	78.5	80.1	81.7	83.3	84.9	86.6	88.2	89.9	91.6	93.3	17
11	95.0	96.8	98.5	100.3	102.1	103.9	105.7	107.5	109.4	111.2	18
12	113.1	115.0	116.9	118.8	120.8	122.7	124.7	126.7	128.7	130.7	20
13	132.7	134.8	136.8	138.9	141.0	143.1	145.3	147.4	149.6	151.7	21
14	153.9	156.1	158.4	160.6	162.9	165.1	167.4	169.7	172.0	174.4	23
15	176.7	179.1	181.5	183.9	186.3	188.7	191.1	193.6	196.1	198.6	24
16	201.1	203.6	206.1	208.7	211.2	213.8	216.4	219.0	221.7	224.3	26
17	227.0	229.7	232.4	235.1	237.8	240.5	243.3	246.1	248.8	251.6	28
18	254.5	257.3	260.2	263.0	265.9	268.8	271.7	274.6	277.6	280.6	29
19	283.5	286.5	289.5	292.6	295.6	298.6	301.7	304.8	307.9	311.0	31
20	314.2	317.3	320.5	323.7	326.9	330.1	333.3	336.5	339.8	343.1	32
21	346.4	349.7	353.0	356.3	359.7	363.1	366.4	369.8	373.3	376.7	34
22	380.1	383.6	387.1	390.6	394.1	397.6	401.1	404.7	408.3	411.9	35
23	415.5	419.1	422.7	426.4	430.1	433.7	437.4	441.2	444.9	448.6	37
24	452.4	456.2	460.0	463.8	467.6	471.4	475.3	479.2	483.1	487.0	38
25	490.9	494.8	498.8	502.7	506.7	510.7	514.7	518.7	522.8	526.9	40
26	530.9	535.0	539.1	543.3	547.4	551.5	555.7	559.9	564.1	568.3	42
27	572.6	576.8	581.1	585.3	589.6	594.0	598.3	602.6	607.0	611.4	43
28	615.8	620.2	624.6	629.0	633.5	637.9	642.4	646.9	651.4	656.0	45
29	660.5	665.1	669.7	674.3	678.9	683.5	688.1	692.8	697.5	702.2	46
30	706.9	711.6	716.3	721.1	725.8	730.6	735.4	740.2	745.1	749.9	48
31	754.8	759.6	764.5	769.4	774.4	779.3	784.3	789.2	794.2	799.2	50
32	804.2	809.3	814.3	819.4	824.5	829.6	834.7	839.8	845.0	850.1	51
33	855.3	860.5	865.7	870.9	876.2	881.4	886.7	892.0	897.3	902.6	53
34	907.9	913.3	918.6	924.0	929.4	934.8	940.2	945.7	951.1	956.6	54
35	962	968	973	979	984	990	995	1001	1007	1012	—
36	1018	1024	1029	1035	1041	1046	1052	1058	1064	1069	
37	1075	1081	1087	1093	1099	1104	1110	1116	1122	1128	
38	1134	1140	1146	1152	1158	1164	1170	1176	1182	1188	6
39	1195	1201	1207	1213	1219	1225	1232	1238	1244	1250	
40	1257	1263	1269	1276	1282	1288	1295	1301	1307	1314	
41	1320	1327	1333	1340	1346	1353	1359	1366	1372	1379	
42	1385	1392	1399	1405	1412	1419	1425	1432	1439	1445	
43	1452	1459	1466	1473	1479	1486	1493	1500	1507	1514	
44	1521	1527	1534	1541	1548	1555	1562	1569	1576	1583	
45	1590	1598	1605	1612	1619	1626	1633	1640	1647	1655	7
46	1662	1669	1676	1684	1691	1698	1706	1713	1720	1728	
47	1735	1742	1750	1757	1765	1772	1780	1787	1795	1802	
48	1810	1817	1825	1832	1840	1847	1855	1863	1870	1878	
49	1886	1893	1901	1909	1917	1924	1932	1940	1948	1956	
50	1963	1971	1979	1987	1995	2003	2011	2019	2027	2035	
51	2043	2051	2059	2067	2075	2083	2091	2099	2107	2116	8
52	2124	2132	2140	2148	2157	2165	2173	2181	2190	2198	
53	2206	2215	2223	2231	2240	2248	2256	2265	2273	2282	
54	2290	2299	2307	2316	2324	2333	2341	2350	2359	2367	
55	2376	2384	2393	2402	2411	2419	2428	2437	2445	2454	

Diam.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
55	2376	2384	2393	2402	2411	2419	2428	2437	2445	2454	
56	2463	2472	2481	2489	2498	2507	2516	2525	2534	2543	
57	2552	2561	2570	2579	2588	2597	2606	2615	2624	2633	9
58	2642	2651	2660	2669	2679	2688	2697	2706	2715	2725	
59	2734	2743	2753	2762	2771	2781	2790	2799	2809	2818	
60	2827	2837	2846	2856	2865	2875	2884	2894	2903	2913	
61	2922	2932	2942	2951	2961	2971	2980	2990	3000	3009	
62	3019	3029	3039	3048	3058	3068	3078	3088	3097	3107	
63	3117	3127	3137	3147	3157	3167	3177	3187	3197	3207	
64	3217	3227	3237	3247	3257	3267	3278	3288	3298	3308	10
65	3318	3329	3339	3349	3359	3370	3380	3390	3400	3411	
66	3421	3432	3442	3452	3463	3473	3484	3494	3505	3515	
67	3526	3536	3547	3557	3568	3578	3589	3600	3610	3621	
68	3632	3642	3653	3664	3675	3685	3696	3707	3718	3728	
69	3739	3750	3761	3772	3783	3794	3805	3816	3826	3837	
70	3848	3859	3871	3882	3893	3904	3915	3926	3937	3948	11
71	3959	3970	3982	3993	4004	4015	4026	4038	4049	4060	
72	4072	4083	4094	4106	4117	4128	4140	4151	4162	4174	
73	4185	4197	4208	4220	4231	4243	4254	4266	4278	4289	
74	4301	4312	4324	4336	4347	4359	4371	4383	4394	4406	
75	4418	4430	4441	4453	4465	4477	4489	4501	4513	4524	
76	4536	4548	4560	4572	4584	4596	4608	4620	4632	4645	12
77	4657	4669	4681	4693	4705	4717	4729	4742	4754	4766	
78	4778	4791	4803	4815	4827	4840	4852	4865	4877	4889	
79	4902	4914	4927	4939	4951	4964	4976	4989	5001	5014	
80	5027	5039	5052	5064	5077	5090	5102	5115	5128	5140	
81	5153	5166	5178	5191	5204	5217	5230	5242	5255	5268	
82	5281	5294	5307	5320	5333	5346	5359	5372	5385	5398	13
83	5411	5424	5437	5450	5463	5476	5489	5502	5515	5529	
84	5542	5555	5568	5581	5595	5608	5621	5635	5648	5661	
85	5675	5638	5701	5715	5728	5741	5755	5768	5782	5795	
86	5809	5822	5836	5849	5863	5877	5890	5904	5917	5931	
87	5945	5958	5972	5986	5999	6013	6027	6041	6055	6068	
88	6082	6096	6110	6124	6138	6151	6165	6179	6193	6207	
89	6221	6235	6249	6263	6277	6291	6305	6319	6333	6348	14
90	6362	6376	6390	6404	6418	6433	6447	6461	6475	6490	
91	6504	6518	6533	6547	6561	6576	6590	6604	6619	6633	
92	6648	6662	6677	6691	6706	6720	6735	6749	6764	6778	
93	6793	6808	6822	6837	6851	6866	6881	6896	6910	6925	
94	6940	6955	6969	6984	6999	7014	7029	7044	7058	7073	
95	7088	7103	7118	7133	7148	7163	7178	7193	7208	7223	15
96	7238	7253	7268	7284	7299	7314	7329	7344	7359	7375	
97	7390	7405	7420	7436	7451	7466	7482	7497	7512	7528	
98	7543	7558	7574	7589	7605	7620	7636	7651	7667	7682	
99	7698	7713	7729	7744	7760	7776	7791	7807	7823	7838	
100	7854	7870	7885	7901	7917	7933	7949	7964	7980	7996	16

Diam.	0	1	2	3	4	5	6	7	8	9	Dif.
1.0	.524	.539	.556	.572	.589	.606	.624	.641	.660	.678	17
1.1	.697	.716	.736	.755	.776	.796	.817	.839	.860	.882	21
1.2	.905	.928	.951	.974	.998	1.023	1.047	1.073	1.098	1.124	25
1.3	1.150	1.177	1.204	1.232	1.260	1.288	1.317	1.346	1.376	1.406	29
1.4	1.437	1.468	1.499	1.531	1.563	1.596	1.630	1.663	1.697	1.732	33
1.5	1.767	1.803	1.839	1.875	1.912	1.950	1.988	2.026	2.065	2.105	38
1.6	2.145	2.185	2.226	2.268	2.310	2.352	2.395	2.439	2.483	2.527	43
1.7	2.572	2.618	2.664	2.711	2.758	2.806	2.855	2.903	2.953	3.003	48
1.8	3.054	3.105	3.157	3.209	3.262	3.315	3.369	3.424	3.479	3.535	54
1.9	3.591	3.648	3.706	3.764	3.823	3.882	3.942	4.003	4.064	4.126	60
2.0	4.189	4.252	4.316	4.380	4.445	4.511	4.577	4.644	4.712	4.780	66
2.1	4.849	4.919	4.989	5.060	5.131	5.204	5.277	5.350	5.425	5.500	73
2.2	5.575	5.652	5.729	5.806	5.885	5.964	6.044	6.125	6.206	6.288	80
2.3	6.371	6.451	6.538	6.623	6.709	6.795	6.882	6.970	7.059	7.148	87
2.4	7.238	7.329	7.421	7.513	7.606	7.700	7.795	7.890	7.986	8.083	94
2.5	8.18	8.28	8.38	8.48	8.58	8.68	8.78	8.89	8.99	9.10	10
2.6	9.20	9.31	9.42	9.53	9.63	9.74	9.85	9.97	10.08	10.19	11
2.7	10.31	10.42	10.54	10.65	10.77	10.89	11.01	11.13	11.25	11.37	12
2.8	11.49	11.62	11.74	11.87	11.99	12.12	12.25	12.38	12.51	12.64	13
2.9	12.77	12.90	13.04	13.17	13.31	13.44	13.58	13.72	13.86	14.00	14
3.0	14.14	14.28	14.42	14.57	14.71	14.86	15.00	15.15	15.30	15.45	15
3.1	15.60	15.75	15.90	16.06	16.21	16.37	16.52	16.68	16.84	17.00	16
3.2	17.16	17.32	17.48	17.64	17.81	17.97	18.14	18.31	18.48	18.65	17
3.3	18.82	18.99	19.16	19.33	19.51	19.68	19.86	20.04	20.22	20.40	18
3.4	20.58	20.76	20.94	21.13	21.31	21.50	21.69	21.88	22.07	22.26	19
3.5	22.45	22.64	22.84	23.03	23.23	23.43	23.62	23.82	24.02	24.23	20
3.6	24.43	24.63	24.84	25.04	25.25	25.46	25.67	25.88	26.09	26.31	21
3.7	26.52	26.74	26.95	27.17	27.39	27.61	27.83	28.06	28.28	28.50	22
3.8	28.73	28.96	29.19	29.42	29.65	29.88	30.11	30.35	30.58	30.82	23
3.9	31.06	31.30	31.54	31.78	32.02	32.27	32.52	32.76	33.01	33.26	24
4.0	33.51	33.76	34.02	34.27	34.53	34.78	35.04	35.30	35.56	35.82	26
4.1	36.09	36.35	36.62	36.88	37.15	37.42	37.69	37.97	38.24	38.52	27
4.2	38.79	39.07	39.35	39.63	39.91	40.19	40.48	40.76	41.05	41.34	29
4.3	41.63	41.92	42.21	42.51	42.80	43.10	43.40	43.70	44.00	44.30	30
4.4	44.60	44.91	45.21	45.52	45.83	46.14	46.45	46.77	47.08	47.40	31
4.5	47.71	48.03	48.35	48.67	49.00	49.32	49.65	49.97	50.30	50.63	33
4.6	50.97	51.30	51.63	51.97	52.31	52.65	52.99	53.33	53.67	54.02	34
4.7	54.36	54.71	55.06	55.41	55.76	56.12	56.47	56.83	57.18	57.54	36
4.8	57.91	58.27	58.63	59.00	59.37	59.73	60.10	60.48	60.85	61.22	37
4.9	61.60	61.98	62.36	62.74	63.12	63.51	63.89	64.28	64.67	65.06	38
5.0	65.45	65.84	66.24	66.64	67.03	67.43	67.83	68.24	68.64	69.05	40
5.1	69.46	69.87	70.28	70.69	71.10	71.52	71.94	72.36	72.78	73.20	42
5.2	73.62	74.05	74.47	74.90	75.33	75.77	76.20	76.64	77.07	77.51	43
5.3	77.95	78.39	78.84	79.28	79.73	80.18	80.63	81.08	81.54	81.99	45
5.4	82.45	82.91	83.37	83.83	84.29	84.76	85.23	85.70	86.17	86.64	47
5.5	87.11	87.59	88.07	88.55	89.03	89.51	90.00	90.48	90.97	91.46	48

Diam.	0	1	2	3	4	5	6	7	8	9	Diff.
5.5	87.1	87.6	88.1	88.5	89.0	89.5	90.0	90.5	91.0	91.5	
5.6	92.0	92.4	92.9	93.4	93.9	94.4	94.9	95.4	95.9	96.5	5
5.7	97.0	97.5	98.0	98.5	99.0	99.5	100.1	100.6	101.1	101.6	
5.8	102.2	102.7	103.2	103.8	104.3	104.8	105.4	105.9	106.4	107.0	
5.9	107.5	108.1	108.6	109.2	109.7	110.3	110.9	111.4	112.0	112.5	
6.0	113.1	113.7	114.2	114.8	115.4	115.9	116.5	117.1	117.7	118.3	
6.1	118.8	119.4	120.0	120.6	121.2	121.8	122.4	123.0	123.6	124.2	6
6.2	124.8	125.4	126.0	126.6	127.2	127.8	128.4	129.1	129.7	130.3	
6.3	130.9	131.5	132.2	132.8	133.4	134.1	134.7	135.3	136.0	136.6	
6.4	137.3	137.9	138.5	139.2	139.8	140.5	141.2	141.8	142.5	143.1	
6.5	143.8	144.5	145.1	145.8	146.5	147.1	147.8	148.5	149.2	149.8	
6.6	150.5	151.2	151.9	152.6	153.3	154.0	154.7	155.4	156.1	156.8	7
6.7	157.5	158.2	158 9	159.6	160.3	161.0	161.7	162.5	163.2	163.9	
6.8	164.6	165.4	166.1	166.8	167.6	168.3	169.0	169.8	170.5	171.3	
6.9	172.0	172.8	173.5	174.3	175.0	175.8	176.5	177.3	178.1	178.8	
7.0	179.6	180.4	181.1	181.9	182.7	183.5	184.3	185.0	185.8	186.6	
7.1	187.4	188.2	189.0	189.8	190.6	191.4	192.2	193.0	193.8	194.6	8
7.2	195.4	196.2	197.1	197.9	198.7	199.5	200.4	201.2	202.0	202.9	
7.3	203.7	204.5	205.4	206.2	207.1	207.9	208.8	209.6	210.5	211.3	
7.4	212.2	213.0	213.9	214.8	215.6	216.5	217.4	218.3	219.1	220.0	
7.5	220.9	221.8	222.7	223.6	224.4	225.3	226.2	227.1	228.0	228.9	9
7.6	229.8	230.8	231.7	232.6	233.5	234.4	235.3	236.3	237.2	238.1	
7.7	239.0	240.0	240.9	241.8	242.8	243.7	244.7	245.6	246.6	247.5	
7.8	248.5	249.4	250.4	251.4	252.3	253.3	254.3	255.2	256.2	257.2	
7.9	258.2	259.1	260.1	261.1	262.1	263.1	264.1	265.1	266.1	267.1	10
8.0	268.1	269.1	270.1	271.1	272.1	273.1	274.2	275.2	276.2	277.2	
8.1	278.3	279.3	280.3	281.4	282.4	283.4	284.5	285.5	286.6	287.6	
8.2	288.7	289.8	290.8	291.9	292·9	294.0	295.1	296.2	297.2	298.3	
8.3	299.4	300.5	301.6	302.6	303.7	304.8	305.9	307.0	308.1	309.2	11
8.4	310.3	311.4	312.6	313.7	314.8	315.9	317.0	318.2	319.3	320.4	
8.5	321.6	322.7	323.8	325.0	326.1	327.3	328.4	329.6	330.7	331.9	
8.6	333.0	334.2	335.4	336.5	337.7	338.9	340.1	341.2	342.4	343.6	
8.7	344.8	346.0	347.2	348.4	349.6	350.8	352.0	353.2	354.4	355.6	12
8.8	356.8	358.0	359.3	360.5	361.7	362.9	364.2	365.4	366.6	367.9	
8.9	369.1	370.4	371.6	372.9	374.1	375.4	376.6	377.9	379.2	380.4	
9 0	381.7	383.0	384.3	385.5	386.8	388.1	389.4	390.7	392.0	393.3	13
9.1	394.6	395.9	397.2	398.5	399.8	401.1	402.4	403.7	405.1	406.4	
9.2	407.7	409.1	410.4	411.7	413.1	414.4	415.7	417.1	418.4	419.8	
9.3	421.2	422.5	423.9	425.2	426.6	428.0	429.4	430.7	432.1	433.5	
9.4	434.9	436.3	437.7	439.1	440.5	441.9	443.3	444.7	446.1	447.5	14
9.5	448.9	450.3	451.8	453.2	454.6	456.0	457.5	458 9	460.4	461.8	
9.6	463.2	464.7	466.1	467.6	469.1	470.5	472.0	473.5	474.9	476.4	
9.7	477.9	479.4	480.8	482.3	483.8	485.3	486.8	488.3	489.8	491.3	15
9.8	492.8	494.3	495.8	497.3	498.9	500.4	501.9	503.4	505.0	506.5	
9.9	508.0	509.6	511.1	512.7	514.2	515.8	517.3	518.9	520.5	522.0	
10.0	523.6	525.2	526.7	528.3	529.9	531.5	533.1	534.7	536.3	537.9	16

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	An.
0°	0 0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89°
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	0523	87
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	0698	86
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	0872	85
5	0.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	1045	84
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	1219	83
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	1392	82
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	1564	81
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	1736	80
10	0.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	1908	79
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2079	78
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2250	77
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2419	76
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2588	75
15	0.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2756	74
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2924	73
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3090	72
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3256	71
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3420	70
20	0.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3584	69
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3746	68
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3907	67
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	4067	66
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	4226	65
25	0.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	4384	64
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	4540	63
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	4695	62
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	4848	61
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	5000	60
30	0.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	5150	59
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	5299	58
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	5446	57
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	5592	56
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	5736	55
35	0.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	5878	54
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	6018	53
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	6157	52
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	6293	51
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	6428	50
40	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	6561	49
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	6691	48
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	6820	47
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	6947	46
44°	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	7071	45°
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Natural Cosines.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	av.
45°	0.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	7193 44°	
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	7314 43	12
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	7431 42	
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	7547 41	
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	7660 40	
50	0.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	7771 39	
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	7880 38	11
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	7986 37	
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	8090 36	
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	8192 35	
55	0.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	8290 34	10
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	8387 33	
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	8480 32	
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	8572 31	
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	8660 30	9
60	0.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	8746 29	
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	8829 28	
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	8910 27	8
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	8988 26	
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	9063 25	
65	0.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	9135 24	
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	9205 23	7
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	9272 22	
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	9336 21	
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	9397 20	6
70	0.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	9455 19	
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	9511 18	
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	9563 17	
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	9613 16	5
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	9659 15	
75	0.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	9703 14	
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	9744 13	4
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	9781 12	
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	9816 11	
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	9848 10	
80	0.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	9877 9	3
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	9903 8	
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	9925 7	
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	9945 6	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	9962 5	
85	0.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	9976 4	
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	9986 3	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	9994 2	
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	9998 1	
89°	9998	9999	9999	9999	9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000 0°	0
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Natural Cosines.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	dif.
0°	$-\infty$	$\bar{3}.2419$	$\bar{3}.5429$	$\bar{3}.7190$	$\bar{3}.8439$	$\bar{3}.9408$	$\bar{2}.0200$	$\bar{2}.0870$	$\bar{2}.1450$	$\bar{2}.1961$	$\bar{2}.2419$	89°	—
1	$\bar{2}.2419$	$\bar{2}.2832$	$\bar{2}.3210$	$\bar{2}.3558$	$\bar{2}.3880$	$\bar{2}.4179$	$\bar{2}.4459$	$\bar{2}.4723$	$\bar{2}.4971$	$\bar{2}.5206$	$\bar{2}.5428$	88	—
2	5428	5640	5842	6035	6220	6397	6567	6731	6889	7041	7188	87	—
3	7188	7330	7468	7602	7731	7857	7979	8098	8213	8326	8436	86	—
4	8436	8543	8647	8749	8849	8946	9042	9135	9226	9315	9403	85	—
5	$\bar{2}.9403$	$\bar{2}.9489$	$\bar{2}.9573$	$\bar{2}.9655$	$\bar{2}.9736$	$\bar{2}.9816$	$\bar{2}.9894$	$\bar{2}.9970$	$\bar{1}.0046$	$\bar{1}.0120$	$\bar{1}.0192$	84	79
6	$\bar{1}.0192$	$\bar{1}.0264$	$\bar{1}.0334$	$\bar{1}.0403$	$\bar{1}.0472$	$\bar{1}.0539$	$\bar{1}.0605$	$\bar{1}.0670$	$\bar{1}.0734$	$\bar{1}.0797$	$\bar{1}.0859$	83	67
7	0859	0920	0981	1040	1099	1157	1214	1271	1326	1381	1436	82	58
8	1436	1489	1542	1594	1646	1697	1747	1797	1847	1895	1943	81	51
9	1943	1991	2038	2085	2131	2176	2221	2266	2310	2353	2397	80	45
10	$\bar{1}.2397$	$\bar{1}.2439$	$\bar{1}.2482$	$\bar{1}.2524$	$\bar{1}.2565$	$\bar{1}.2606$	$\bar{1}.2647$	$\bar{1}.2687$	$\bar{1}.2727$	$\bar{1}.2767$	$\bar{1}.2806$	79	41
11	2806	2845	2883	2921	2959	2997	3034	3070	3107	3143	3179	78	37
12	3179	3214	3249	3284	3319	3353	3387	3421	3455	3488	3521	77	34
13	3521	3554	3586	3618	3650	3682	3713	3745	3775	3806	3837	76	31
14	3837	3867	3897	3927	3957	3986	4015	4044	4073	4102	4130	75	29
15	$\bar{1}.4130$	$\bar{1}.4158$	$\bar{1}.4186$	$\bar{1}.4214$	$\bar{1}.4242$	$\bar{1}.4269$	$\bar{1}.4296$	$\bar{1}.4323$	$\bar{1}.4350$	$\bar{1}.4377$	$\bar{1}.4403$	74	27
16	4403	4430	4456	4482	4508	4533	4559	4584	4609	4634	4659	73	26
17	4659	4684	4709	4733	4757	4781	4805	4829	4853	4876	4900	72	24
18	4900	4923	4946	4969	4992	5015	5037	5060	5082	5104	5126	71	23
19	5126	5148	5170	5192	5213	5235	5256	5278	5299	5320	5341	70	22
20	$\bar{1}.5341$	$\bar{1}.5361$	$\bar{1}.5382$	$\bar{1}.5402$	$\bar{1}.5423$	$\bar{1}.5443$	$\bar{1}.5463$	$\bar{1}.5484$	$\bar{1}.5504$	$\bar{1}.5523$	$\bar{1}.5543$	69	20
21	5543	5563	5583	5602	5621	5641	5660	5679	5698	5717	5736	68	19
22	5736	5754	5773	5792	5810	5828	5847	5865	5883	5901	5919	67	18
23	5919	5937	5954	5972	5990	6007	6024	6042	6059	6076	6093	66	18
24	6093	6110	6127	6144	6161	6177	6194	6210	6227	6243	6259	65	17
25	$\bar{1}.6259$	$\bar{1}.6276$	$\bar{1}.6292$	$\bar{1}.6308$	$\bar{1}.6324$	$\bar{1}.6340$	$\bar{1}.6356$	$\bar{1}.6371$	$\bar{1}.6387$	$\bar{1}.6403$	$\bar{1}.6418$	64	16
26	6418	6434	6449	6465	6480	6495	6510	6526	6541	6556	6570	63	16
27	6570	6585	6600	6615	6629	6644	6659	6673	6687	6702	6716	62	15
28	6716	6730	6744	6759	6773	6787	6801	6814	6828	6842	6856	61	14
29	6856	6869	6883	6896	6910	6923	6937	6950	6963	6977	6990	60	
30	$\bar{1}.6990$	$\bar{1}.7003$	$\bar{1}.7016$	$\bar{1}.7029$	$\bar{1}.7042$	$\bar{1}.7055$	$\bar{1}.7068$	$\bar{1}.7080$	$\bar{1}.7093$	$\bar{1}.7106$	$\bar{1}.7118$	59	13
31	7118	7131	7144	7156	7168	7181	7193	7205	7218	7230	7242	58	
32	7242	7254	7266	7278	7290	7302	7314	7326	7338	7349	7361	57	12
33	7361	7373	7384	7396	7407	7419	7430	7442	7453	7464	7476	56	
34	7476	7487	7498	7509	7520	7531	7542	7553	7564	7575	7586	55	11
35	$\bar{1}.7586$	$\bar{1}.7597$	$\bar{1}.7607$	$\bar{1}.7618$	$\bar{1}.7629$	$\bar{1}.7640$	$\bar{1}.7650$	$\bar{1}.7661$	$\bar{1}.7671$	$\bar{1}.7682$	$\bar{1}.7692$	54	
36	7692	7703	7713	7723	7734	7744	7754	7764	7774	7785	7795	53	
37	7795	7805	7815	7825	7835	7844	7854	7864	7874	7884	7893	52	10
38	7893	7903	7913	7922	7932	7941	7951	7960	7970	7979	7989	51	
39	7989	7998	8007	8017	8026	8035	8044	8053	8063	8072	8081	50	
40	$\bar{1}.8081$	$\bar{1}.8090$	$\bar{1}.8099$	$\bar{1}.8108$	$\bar{1}.8117$	$\bar{1}.8125$	$\bar{1}.8134$	$\bar{1}.8143$	$\bar{1}.8152$	$\bar{1}.8161$	$\bar{1}.8169$	49	9
41	8169	8178	8187	8195	8204	8213	8221	8230	8238	8247	8255	48	
42	8255	8264	8272	8280	8289	8297	8305	8313	8322	8330	8338	47	
43	8338	8346	8354	8362	8370	8378	8386	8394	8402	8410	8418	46	
44°	8418	8426	8433	8441	8449	8457	8464	8472	8480	8487	8495	45°	8
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle		

Logarithmic Cosines.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement		
45°	$\bar{1}$.8495	$\bar{1}$.8502	$\bar{1}$.8510	$\bar{1}$.8517	$\bar{1}$.8525	$\bar{1}$.8532	$\bar{1}$.8540	$\bar{1}$.8547	$\bar{1}$.8555	$\bar{1}$.8562	$\bar{1}$.8569	44°	
46	8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	8641	43	
47	8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	8711	42	7
48	8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	8778	41	
49	8778	8784	8791	8797	8804	8810	8317	8823	8830	8836	8843	40	
50	$\bar{1}$.8843	$\bar{1}$.8849	$\bar{1}$.8855	$\bar{1}$.8862	$\bar{1}$.8868	$\bar{1}$.8874	$\bar{1}$.8880	$\bar{1}$.8887	$\bar{1}$.8893	$\bar{1}$.8899	$\bar{1}$.8905	39	
51	8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	8965	38	6
52	8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	9023	37	
53	9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	9080	36	
54	9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	9134	35	
55	$\bar{1}$.9134	$\bar{1}$.9139	$\bar{1}$.9144	$\bar{1}$.9149	$\bar{1}$.9155	$\bar{1}$.9160	$\bar{1}$.9165	$\bar{1}$.9170	$\bar{1}$.9175	$\bar{1}$.9181	$\bar{1}$.9186	34	
56	9186	9191	9196	9201	9206	9211	9216	9221	9226	9231	9236	33	5
57	9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	9284	32	
58	9284	9289	9294	9298	9303	9308	9312	9317	9322	9326	9331	31	
59	9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	9375	30	
60	$\bar{1}$.9375	$\bar{1}$.9380	$\bar{1}$.9384	$\bar{1}$.9388	$\bar{1}$.9393	$\bar{1}$.9397	$\bar{1}$.9401	$\bar{1}$.9406	$\bar{1}$.9410	$\bar{1}$.9414	$\bar{1}$.9418	29	
61	9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	9459	28	
62	9459	9463	9467	9471	9475	9479	9483	9487	9491	9495	9499	27	4
63	9499	9503	9506	9510	9514	9518	9522	9525	9529	9533	9537	26	
64	9537	9540	9544	9548	9551	9555	9558	9562	9566	9569	9573	25	
65	$\bar{1}$.9573	$\bar{1}$.9576	$\bar{1}$.9580	$\bar{1}$.9583	$\bar{1}$.9587	$\bar{1}$.9590	$\bar{1}$.9594	$\bar{1}$.9597	$\bar{1}$.9601	$\bar{1}$.9604	$\bar{1}$.9607	24	
66	9607	9611	9614	9617	9621	9624	9627	9631	9634	9637	9640	23	
67	9640	9643	9647	9650	9653	9656	9659	9662	9665	9669	9672	22	
68	9672	9675	9678	9681	9684	9687	9690	9693	9696	9699	9702	21	3
69	9702	9704	9707	9710	9713	9716	9719	9722	9724	9727	9730	20	
70	$\bar{1}$.9730	$\bar{1}$.9733	$\bar{1}$.9735	$\bar{1}$.9738	$\bar{1}$.9741	$\bar{1}$.9743	$\bar{1}$.9746	$\bar{1}$.9749	$\bar{1}$.9751	$\bar{1}$.9754	$\bar{1}$.9757	19	
71	9757	9759	9762	9764	9767	9770	9772	9775	9777	9780	9782	18	
72	9782	9785	9787	9789	9792	9794	9797	9799	9801	9804	9806	17	
73	9806	9808	9811	9813	9815	9817	9820	9822	9824	9826	9828	16	
74	9828	9831	9833	9835	9837	9839	9841	9843	9845	9847	9849	15	
75	$\bar{1}$.9849	$\bar{1}$.9851	$\bar{1}$.9853	$\bar{1}$.9855	$\bar{1}$.9857	$\bar{1}$.9859	$\bar{1}$.9861	$\bar{1}$.9863	$\bar{1}$.9865	$\bar{1}$.9867	$\bar{1}$.9869	14	2
76	9869	9871	9873	9875	9876	9878	9880	9882	9884	9885	9887	13	
77	9887	9889	9891	9892	9894	9896	9897	9899	9901	9902	9904	12	
78	9904	9906	9907	9909	9910	9912	9913	9915	9916	9918	9919	11	
79	9919	9921	9922	9924	9925	9927	9928	9929	9931	9932	9934	10	
80	$\bar{1}$.9934	$\bar{1}$.9935	$\bar{1}$.9936	$\bar{1}$.9937	$\bar{1}$.9939	$\bar{1}$.9940	$\bar{1}$.9941	$\bar{1}$.9943	$\bar{1}$.9944	$\bar{1}$.9945	$\bar{1}$.9946	9	
81	9946	9947	9949	9950	9951	9952	9953	9954	9955	9956	9958	8	
82	9958	9959	9960	9961	9962	9963	9964	9965	9966	9967	9968	7	1
83	9968	9968	9969	9970	9971	9972	9973	9974	9975	9975	9676	6	
84	9976	9977	9978	9978	9979	9980	9981	9981	9982	9983	9983	5	
85	$\bar{1}$.9983	$\bar{1}$.9984	$\bar{1}$.9985	$\bar{1}$.9985	$\bar{1}$.9986	$\bar{1}$.9987	$\bar{1}$.9987	$\bar{1}$.9988	$\bar{1}$.9988	$\bar{1}$.9989	$\bar{1}$.9989	4	
86	9989	9990	9990	9991	9991	9992	9992	9993	9993	9994	9994	3	
87	9994	9994	9995	9995	9996	9996	9996	9996	9997	9997	9997	2	
88	9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	9999	1	
89°	9999	9999	0000	0000	0000	0000	0000	0000	0000	0000	0000	0°	0
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle		

Logarithmic Cosines.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	SH.
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175 89°	
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349 88	
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	0524 87	
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	0699 86	
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	0875 85	
5	0.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	1051 84	
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	1228 83	
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	1405 82	
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	1584 81	
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	1763 80	
10	0.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	1944 79	18
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	2126 78	
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	2309 77	
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	2493 76	
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	2679 75	
15	0.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	2867 74	
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3057 73	19
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3249 72	
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3443 71	
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3640 70	
20	0.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3839 69	
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	4040 68	20
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	4245 67	
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	4452 66	
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4663 65	21
25	0 4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4877 64	
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	5095 63	
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	5317 62	22
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	5543 61	
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	5774 60	23
30	0.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	6009 59	
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	6249 58	24
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	6494 57	
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	6745 56	25
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	7002 55	
35	0.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	7265 54	26
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	7536 53	27
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	7813 52	28
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	8098 51	28
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	8391 50	29
40	0.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	8693 49	30
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	9004 48	31
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	9325 47	32
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	9657 46	33
44°	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	1.0000 45°	34
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Natural Cotangents.

Angle.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
45°	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	36
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	37
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	38
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1 1463	40
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	41
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	43
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	45
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	47
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	49
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	52
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	54
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	57
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	60
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	64
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	68
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	72
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1 8495	1.8572	1.8650	1 8728	77
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1 9542	82
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	88
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	94
65	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	10
66	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	11
67	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	12
68	2.475	2.488	2.500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	13
69	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	14
70	2.747	2.762	2.778	2.793	2.808	2.824	2.840	2.856	2.872	2.888	16
71	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.060	17
72	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3 250	19
73	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	22
74	3.487	3.511	3.534	3.558	3 582	3 606	3.630	3.655	3.681	3.706	25
75	3.732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	28
76	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	32
77	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	37
78	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	44
79	5.145	5.193	5.242	5 292	5.343	5.396	5.449	5.503	5.558	5 614	53
80	5.67	5.73	5.79	5.85	5.91	5.98	6.04	6.11	6.17	6.24	7
81	6.31	6 39	6.46	6.54	6.61	6.69	6.77	6.85	6.94	7.03	8
82	7.12	7.21	7.30	7.40	7.49	7.60	7.70	7.81	7.92	8.03	10
83	8.14	8 26	8.39	8.51	8.64	8.78	8.92	9.06	9.21	9.36	14
84	9.51	9.68	9.84	10.0	10.2	10.4	10.6	10.8	11.0	11.2	
85	11.4	11.7	11.9	12.2	12.4	12.7	13.0	13.3	13.6	14.0	3
86	14.3	14.7	15.1	15.5	15.9	16.3	16.8	17.3	17.9	18.5	6
87	19.1	19.7	20.4	21.2	22.0	22.9	23.9	24.9	26.0	27.3	
88	28.6	30.1	31.8	33.7	35.8	38.2	40.9	44 1	47.7	52.1	
89°	57.	64.	72.	82.	95.	115.	143.	191.	286.	573.	
Angle.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	Dif.
0°	−∞	$\bar{3}$.2419	$\bar{3}$.5429	$\bar{3}$.7190	$\bar{3}$.8439	$\bar{3}$.9409	$\bar{2}$.0200	$\bar{2}$.0870	$\bar{2}$.1450	$\bar{2}$.1962	$\bar{2}$.2419	89°	—
1	$\bar{2}$.2419	$\bar{2}$.2833	$\bar{2}$.3211	$\bar{2}$.3559	$\bar{2}$.3881	$\bar{2}$.4181	$\bar{2}$.4461	$\bar{2}$.4725	$\bar{2}$.4973	$\bar{2}$.5208	$\bar{2}$.5431	88	—
2	5431	5643	5845	6038	6223	6401	6571	6736	6894	7046	7194	87	—
3	7194	7337	7475	7609	7739	7865	7988	8107	8223	8336	8446	86	—
4	8446	8554	8659	8762	8862	8960	9056	9150	9241	9331	9420	85	—
5	$\bar{2}$.9420	$\bar{2}$.9506	$\bar{2}$.9591	$\bar{2}$.9674	$\bar{2}$.9756	$\bar{2}$.9836	$\bar{2}$.9915	$\bar{2}$.9992	$\bar{1}$.0068	$\bar{1}$.0143	$\bar{1}$.0216	84	80
6	$\bar{1}$.0216	$\bar{1}$.0289	$\bar{1}$.0360	$\bar{1}$.0430	$\bar{1}$.0499	$\bar{1}$.0567	$\bar{1}$.0633	$\bar{1}$.0699	$\bar{1}$.0764	$\bar{1}$.0828	$\bar{1}$.0891	83	68
7	0891	0954	1015	1076	1135	1194	1252	1310	1367	1423	1478	82	59
8	1478	1533	1587	1640	1693	1745	1797	1848	1898	1948	1997	81	52
9	1997	2046	2094	2142	2189	2236	2282	2328	2374	2419	2463	80	46
10	$\bar{1}$.2643	$\bar{1}$.2507	$\bar{1}$.2551	$\bar{1}$.2594	$\bar{1}$.2637	$\bar{1}$.2680	$\bar{1}$.2722	$\bar{1}$.2764	$\bar{1}$.2805	$\bar{1}$.2846	$\bar{1}$.2887	79	42
11	2887	2927	2967	3006	3046	3085	3123	3162	3200	3237	3275	78	39
12	3275	3312	3349	3385	3422	3458	3493	3529	3564	3599	3634	77	39
13	3634	3668	3702	3736	3770	3804	3837	3870	3903	3935	3968	76	33
14	3968	4000	4032	4064	4095	4127	4158	4189	4220	4250	4281	75	31
15	$\bar{1}$.4281	$\bar{1}$.4311	$\bar{1}$.4341	$\bar{1}$.4371	$\bar{1}$.4400	$\bar{1}$.4430	$\bar{1}$.4459	$\bar{1}$.4488	$\bar{1}$.4517	$\bar{1}$.4546	$\bar{1}$.4575	74	30
16	4575	4603	4632	4660	4688	4716	4744	4771	4799	4826	4853	73	28
17	4853	4880	4907	4934	4961	4987	5014	5040	5066	5092	5118	72	27
18	5118	5143	5169	5195	5220	5245	5270	5295	5320	5345	5370	71	25
19	5370	5394	5419	5443	5467	5491	5516	5539	5563	5587	5611	70	24
20	$\bar{1}$.5611	$\bar{1}$.5634	$\bar{1}$.5658	$\bar{1}$.5681	$\bar{1}$.5704	$\bar{1}$.5727	$\bar{1}$.5750	$\bar{1}$.5773	$\bar{1}$,5796	$\bar{1}$.5819	$\bar{1}$.5842	69	23
21	5842	5864	5887	5909	5932	5954	5976	5998	6020	6042	6064	68	22
22	6064	6086	6108	6129	6151	6172	6194	6215	6236	6257	6279	67	21
23	6279	6300	6321	6341	6362	6383	6404	6424	6445	6465	6486	66	21
24	6486	6506	6527	6547	6567	6587	6607	6627	6647	6667	6687	65	20
25	$\bar{1}$.6687	$\bar{1}$.6706	$\bar{1}$.6726	$\bar{1}$.6746	$\bar{1}$.6765	$\bar{1}$.6785	$\bar{1}$.6804	$\bar{1}$.6824	$\bar{1}$.6843	$\bar{1}$.6863	$\bar{1}$.6882	64	
26	6882	6901	6920	6939	6958	6977	6996	7015	7034	7053	7072	63	19
27	7072	7090	7109	7128	7146	7165	7183	7202	7220	7238	7257	62	
28	7257	7275	7293	7311	7330	7348	7366	7384	7402	7420	7438	61	18
29	7438	7455	7473	7491	7509	7526	7544	7562	7579	7597	7614	60	
30	$\bar{1}$.7614	$\bar{1}$.7632	$\bar{1}$.7649	$\bar{1}$.7667	$\bar{1}$.7684	$\bar{1}$.7701	$\bar{1}$.7719	$\bar{1}$.7736	$\bar{1}$.7753	$\bar{1}$.7771	$\bar{1}$.7788	59	
31	7788	7805	7822	7839	7856	7873	7890	7907	7924	7941	7958	58	17
32	7958	7975	7992	8008	8025	8042	8059	8075	8092	8109	8125	57	
33	8125	8142	8158	8175	8191	8208	8224	8241	8257	8274	8290	56	
34	8290	8306	8323	8339	8355	8371	8388	8404	8420	8436	8452	55	
35	$\bar{1}$.8452	$\bar{1}$.8468	$\bar{1}$.8484	$\bar{1}$.8501	$\bar{1}$.8517	$\bar{1}$.8533	$\bar{1}$.8549	$\bar{1}$.8565	$\bar{1}$.8581	$\bar{1}$.8597	$\bar{1}$.8613	54	16
36	8613	8629	8644	8660	8676	8692	8708	8724	8740	8755	8771	53	
37	8771	8787	8803	8818	8834	8850	8865	8881	8897	8912	8928	52	
38	8928	8944	8959	8975	8990	9006	9022	9037	9053	9068	9084	51	
39	9084	9099	9115	9130	9146	9161	9176	9192	9207	9223	9238	50	
40	$\bar{1}$.9238	$\bar{1}$.9254	$\bar{1}$.9269	$\bar{1}$.9284	$\bar{1}$.9300	$\bar{9}$.9315	$\bar{1}$.9330	$\bar{1}$.9346	$\bar{1}$.9361	$\bar{1}$.9376	$\bar{1}$.9392	49	
41	9392	9407	9422	9438	9453	9468	9483	9499	9514	9529	9544	48	
42	9544	9560	9575	9590	9605	9621	9636	9651	9666	9681	9697	47	
43	9697	9712	9727	9742	9757	9772	9788	9803	9818	9833	9848	46	
44°	9848	9864	9879	9894	9909	9924	9939	9955	9970	9985	0000	45°	15
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Logarithmic Cotangents.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	Dif.
45°	0.0000	0.0015	0.0030	0.0045	0.0061	0.0076	0.0091	0.0106	0.0121	0.0136	0.0152	44°	15
46	0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	0303	43	
47	0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	0456	42	
48	0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	0608	41	
49	0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	0762	40	
50	0.0762	0.0777	0.0793	0.0808	0.0824	0.0839	0.0854	0.0870	0.0885	0.0901	0.0916	39	
51	0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	1072	38	
52	1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	1229	37	
53	1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	1387	36	
54	1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	1548	35	16
55	0.1548	0.1564	0.1580	0.1596	0.1612	0.1629	0.1645	0.1661	0.1677	0.1694	0.1710	34	
56	1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	1875	33	
57	1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	2042	32	
58	2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	2212	31	17
59	2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	2386	30	
60	0.2386	0.2403	0.2421	0.2438	0.2456	0.2474	0.2491	0.2509	0.2527	0.2545	0.2562	29	
61	2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	2743	28	18
62	2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	2928	27	
63	2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3118	26	19
64	3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3313	25	
65	0.3313	0.3333	0.3353	0.3373	0.3393	0.3413	0.3433	0.3453	0.3473	0.3494	0.3514	24	20
66	3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3721	23	21
67	3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	3936	22	22
68	3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4158	21	22
69	4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4389	20	23
70	0.4389	0.4413	0.4437	0.4461	0.4484	0.4509	0.4533	0.4557	0.4581	0.4606	0.4630	19	24
71	4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4882	18	25
72	4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	5147	17	27
73	5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5425	16	28
74	5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5719	15	29
75	0.5719	0.5750	0.5780	0.5811	0.5842	0.5873	0.5905	0.5936	0.5968	0.6000	0.6032	14	31
76	6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6366	13	33
77	6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6725	12	36
78	6725	6763	6800	6838	6877	6915	6954	6994	7033	7073	7113	11	39
79	7113	7154	7195	7236	7278	7320	7363	7406	7449	7493	7537	10	42
80	0.7537	0.7581	0.7626	0.7672	0.7718	0.7764	0.7811	0.7858	0.7906	0.7954	0.8003	9	47
81	8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	8522	8	52
82	8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	9109	7	59
83	9109	9172	9236	9301	9367	9433	9501	9570	9640	9711	9784	6	68
84	9784	9857	9932	1.0008	1.0085	1.0164	1.0244	1.0326	1.0409	1.0494	1.0580	5	80
85	1.0580	1.0669	1.0759	1.0850	1.0944	1.1040	1.1138	1.1238	1.1341	1.1446	1.1554	4	—
86	1554	1664	1777	1893	2012	2135	2261	2391	2525	2663	2806	3	—
87	2806	2954	3106	3264	3429	3599	3777	3962	4155	4357	4569	2	—
88	4569	4792	5027	5275	5539	5819	6119	6441	6789	7167	7581	1	—
89°	7581	8038	8550	9130	9800	2.0591	2.1561	2.2810	2.4571	2.7581	∞	0°	—
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Logarithmic Cotangents.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389	43; 1 4
101	00432	00475	00518	00561	00604	00647	00689	00732	00775	00817	2 9; 3 13
102	00860	00903	00945	00988	01030	01072	01115	01157	01199	01242	4 17; 5 22
103	01284	01326	01368	01410	01452	01494	01536	01578	01620	01662	6 26; 7 30
104	01703	01745	01787	01828	01870	01912	01953	01995	02036	02078	8 34; 9 39
105	02119	02160	02202	02243	02284	02325	02366	02407	02449	02490	41; 1 4
106	02531	02572	02612	02653	02694	02735	02776	02816	02857	02898	2 8; 3 12
107	02938	02979	03019	03060	03100	03141	03181	03222	03262	03302	4 16; 5 21
108	03342	03383	03423	03463	03503	03543	03583	03623	03663	03703	6 25; 7 29
109	03743	03782	03822	03862	03902	03941	03981	04021	04060	04100	8 33; 9 37
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493	39; 1 4
111	04532	04571	04610	04650	04689	04727	04766	04805	04844	04883	2 8; 3 12
112	04922	04961	04999	05038	05077	05115	05154	05192	05231	05269	4 16; 5 20
113	05308	05346	05385	05423	05461	05500	05538	05576	05614	05652	6 23; 7 27
114	05690	05729	05767	05805	05843	05881	05918	05956	05994	06032	8 31; 9 35
115	06070	06108	06145	06183	06221	06258	06296	06333	06371	06408	37; 1 4
116	06446	06483	06521	06558	06595	06633	06670	06707	06744	06781	2 7; 3 11
117	06819	06856	06893	06930	06967	07004	07041	07078	07115	07151	4 15; 5 19
118	07188	07225	07262	07298	07335	07372	07408	07445	07482	07518	6 22; 7 26
119	07555	07591	07628	07664	07700	07737	07773	07809	07846	07882	8 30; 9 33
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243	35; 1 4
121	08279	08314	08350	08386	08422	08458	08493	08529	08565	08600	2 7; 3 11
122	08636	08672	08707	08743	08778	08814	08849	08884	08920	08955	4 14; 5 18
123	08991	09026	09061	09096	09132	09167	09202	09237	09272	09307	6 21; 7 25
124	09342	09377	09412	09447	09482	09517	09552	09587	09621	09656	8 28; 9 32
125	09691	09726	09760	09795	09830	09864	09899	09934	09968	10003	34; 1 3
126	10037	10072	10106	10140	10175	10209	10243	10278	10312	10346	2 7; 3 10
127	10380	10415	10449	10483	10517	10551	10585	10619	10653	10687	4 14; 5 17
128	10721	10755	10789	10823	10857	10890	10924	10958	10992	11025	6 20; 7 24
129	11059	11093	11126	11160	11193	11227	11261	11294	11327	11361	8 27; 9 31
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694	33; 1 3
131	11727	11760	11793	11826	11860	11893	11926	11959	11992	12024	2 7; 3 10
132	12057	12090	12123	12156	12189	12222	12254	12287	12320	12352	4 13; 5 17
133	12385	12418	12450	12483	12516	12548	12581	12613	12646	12678	6 20; 7 23
134	12710	12743	12775	12808	12840	12872	12905	12937	12969	13001	8 26; 9 30
135	13033	13066	13098	13130	13162	13194	13226	13258	13290	13322	32; 1 3
136	13354	13386	13418	13450	13481	13513	13545	13577	13609	13640	2 6; 3 10
137	13672	13704	13735	13767	13799	13830	13862	13893	13925	13956	4 13; 5 16
138	13988	14019	14051	14082	14114	14145	14176	14208	14239	14270	6 19; 7 22
139	14301	14333	14364	14395	14426	14457	14489	14520	14551	14582	8 26; 9 29
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891	31; 1 3
141	14922	14953	14983	15014	15045	15076	15106	15137	15168	15198	2 6; 3 9
142	15229	15259	15290	15320	15351	15381	15412	15442	15473	15503	4 12; 5 16
143	15534	15564	15594	15625	15655	15685	15715	15746	15776	15806	6 19; 7 22
144	15836	15866	15897	15927	15957	15987	16017	16047	16077	16107	8 25; 9 28
145	16137	16167	16197	16227	16256	16286	16316	16346	16376	16406	30; 1 3
146	16435	16465	16495	16524	16554	16584	16613	16643	16673	16702	2 6; 3 9
147	16732	16761	16791	16820	16850	16879	16909	16938	16967	16997	4 12; 5 15
148	17026	17056	17085	17114	17143	17173	17202	17231	17260	17289	6 18; 7 21
149	17319	17348	17377	17406	17435	17464	17493	17522	17551	17580	8 24; 9 27
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869	

No.	0	1	2	3	4	5	6	7	8	9	Dif.
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869	29: 1 3, 2 6, 3 9, 4 12, 5 15, 6 17, 7 20, 8 23, 9 26
151	17898	17926	17955	17984	18013	18041	18070	18099	18127	18156	
152	18184	18213	18241	18270	18298	18327	18355	18384	18412	18441	
153	18469	18498	18526	18554	18583	18611	18639	18667	18696	18724	
154	18752	18780	18808	18837	18865	18893	18921	18949	18977	19005	
155	19033	19061	19089	19117	19145	19173	19201	19229	19257	19285	28: 1 3, 2 6, 3 8, 4 11, 5 14, 6 17, 7 20, 8 22, 9 25
156	19312	19340	19368	19396	19424	19451	19479	19507	19535	19562	
157	19590	19618	19645	19673	19700	19728	19756	19783	19811	19838	
158	19866	19893	19921	19948	19976	20003	20030	20058	20085	20112	
159	20140	20167	20194	20222	20249	20276	20303	20330	20358	20385	
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656	27: 1 3, 2 5, 3 8, 4 11, 5 14, 6 16, 7 19, 8 22, 9 24
161	20683	20710	20737	20763	20790	20817	20844	20871	20898	20925	
162	20952	20978	21005	21032	21059	21085	21112	21139	21165	21192	
163	21219	21245	21272	21299	21325	21352	21378	21405	21431	21458	
164	21484	21511	21537	21564	21590	21617	21643	21669	21696	21722	
165	21748	21775	21801	21827	21854	21880	21906	21932	21958	21985	26: 1 3, 2 5, 3 8, 4 10, 5 13, 6 16, 7 18, 8 21, 9 23
166	22011	22037	22063	22089	22115	22141	22167	22194	22220	22246	
167	22272	22298	22324	22350	22376	22401	22427	22453	22479	22505	
168	22531	22557	22583	22608	22634	22660	22686	22712	22737	22763	
169	22789	22814	22840	22866	22891	22917	22943	22968	22994	23019	
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274	25: 1 3, 2 5, 3 8, 4 10, 5 13, 6 15, 7 18, 8 20, 9 23
171	23300	23325	23350	23376	23401	23426	23452	23477	23502	23528	
172	23553	23578	23603	23629	23654	23679	23704	23729	23754	23779	
173	23805	23830	23855	23880	23905	23930	23955	23980	24005	24030	
174	24055	24080	24105	24130	24155	24180	24204	24229	24254	24279	
175	24304	24329	24353	24378	24403	24428	24452	24477	24502	24527	
176	24551	24576	24601	24625	24650	24674	24699	24724	24748	24773	
177	24797	24822	24846	24871	24895	24920	24944	24969	24993	25018	
178	25042	25066	25091	25115	25139	25164	25188	25212	25237	25261	
179	25285	25310	25334	25358	25382	25406	25431	25455	25479	25503	
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744	24: 1 2, 2 5, 3 7, 4 10, 5 12, 6 14, 7 17, 8 19, 9 22
181	25768	25792	25816	25840	25864	25888	25912	25935	25959	25983	
182	26007	26031	26055	26079	26102	26126	26150	26174	26198	26221	
183	26245	26269	26293	26316	26340	26364	26387	26411	26435	26458	
184	26482	26505	26529	26553	26576	26600	26623	26647	26670	26694	
185	26717	26741	26764	26788	26811	26834	26858	26881	26905	26928	23: 1 2, 2 5, 3 7, 4 9, 5 12, 6 14, 7 16, 8 18, 9 21
186	26951	26975	26998	27021	27045	27068	27091	27114	27138	27161	
187	27184	27207	27231	27254	27277	27300	27323	27346	27370	27393	
188	27416	27439	27462	27485	27508	27531	27554	27577	27600	27623	
189	27646	27669	27692	27715	27738	27761	27784	27807	27830	27852	
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081	
191	28103	28126	28149	28171	28194	28217	28240	28262	28285	28307	
192	28330	28353	28375	28398	28421	28443	28466	28488	28511	28533	
193	28556	28578	28601	28623	28646	28668	28691	28713	28735	28758	
194	28780	28803	28825	28847	28870	28892	28914	28937	28959	28981	
195	29003	29026	29048	29070	29092	29115	29137	29159	29181	29203	22: 1 2, 2 4, 3 7, 4 9, 5 11, 6 13, 7 15, 8 18, 9 20
196	29226	29248	29270	29292	29314	29336	29358	29380	29403	29425	
197	29447	29469	29491	29513	29535	29557	29579	29601	29623	29645	
198	29667	29688	29710	29732	29754	29776	29798	29820	29842	29863	
199	29885	29907	29929	29951	29973	29994	30016	30038	30060	30081	
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298	

No.	0	1	2	3	4	5	6	7	8	9	Df.
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298	21
201	30320	30341	30363	30384	30406	30428	30449	30471	30492	30514	1 2
202	30535	30557	30578	30600	30621	30643	30664	30685	30707	30728	2 4
203	30750	30771	30792	30814	30835	30856	30878	30899	30920	30942	3 6
204	30963	30984	31006	31027	31048	31069	31091	31112	31133	31154	4 8
205	31175	31197	31218	31239	31260	31281	31302	31323	31345	31366	5 11
206	31387	31408	31429	31450	31471	31492	31513	31534	31555	31576	6 13
207	31597	31618	31639	31660	31681	31702	31723	31744	31765	31785	7 15
208	31806	31827	31848	31869	31890	31911	31931	31952	31973	31994	8 17
209	32015	32035	32056	32077	32098	32118	32139	32160	32181	32201	9 19
210	32222	32243	32263	32284	32305	32325	32346	32366	32387	32408	20
211	32428	32449	32469	32490	32510	32531	32552	32572	32593	32613	1 2
212	32634	32654	32675	32695	32715	32736	32756	32777	32797	32818	2 4
213	32838	32858	32879	32899	32919	32940	32960	32980	33001	33021	3 6
214	33041	33062	33082	33102	33122	33143	33163	33183	33203	33224	4 8
215	33244	33264	33284	33304	33325	33345	33365	33385	33405	33425	5 10
216	33445	33465	33486	33506	33526	33546	33566	33586	33606	33626	6 12
217	33646	33666	33686	33706	33726	33746	33766	33786	33806	33826	7 14
218	33846	33866	33885	33905	33925	33945	33965	33985	34005	34025	8 16
219	34044	34064	34084	34104	34124	34143	34163	34183	34203	34223	9 18
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420	19
221	34439	34459	34479	34498	34518	34537	34557	34577	34596	34616	1 2
222	34635	34655	34674	34694	34713	34733	34753	34772	34792	34811	2 4
223	34830	34850	34869	34889	34908	34928	34947	34967	34986	35005	3 6
224	35025	35044	35064	35083	35102	35122	35141	35160	35180	35199	4 8
225	35218	35238	35257	35276	35295	35315	35334	35353	35372	35392	5 10
226	35411	35430	35449	35468	35488	35507	35526	35545	35564	35583	6 11
227	35603	35622	35641	35660	35679	35698	35717	35736	35755	35774	7 13
228	35793	35813	35832	35851	35870	35889	35908	35927	35946	35965	8 15
229	35984	36003	36021	36040	36059	36078	36097	36116	36135	36154	9 17
230	36173	36192	36211	36229	36248	36267	36286	36305	36324	36342	
231	36361	36380	36399	36418	36436	36455	36474	36493	36511	36530	
232	36549	36568	36586	36605	36624	36642	36661	36680	36698	36717	
233	36736	36754	36773	36791	36810	36829	36847	36866	36884	36903	
234	36922	36940	36959	36977	36996	37014	37033	37051	37070	37088	
235	37107	37125	37144	37162	37181	37199	37218	37236	37254	37273	
236	37291	37310	37328	37346	37365	37383	37401	37420	37438	37457	
237	37475	37493	37511	37530	37548	37566	37585	37603	37621	37639	
238	37658	37676	37694	37712	37731	37749	37767	37785	37803	37822	
239	37840	37858	37876	37894	37912	37931	37949	37967	37985	38003	
240	38021	38039	38057	38075	38093	38112	38130	38148	38166	38184	18
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487	68753	68762	68771	68780	68789	68797	68806	68815	68824	68833	
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550	74036	74044	74052	74060	74068	74076	74084	74092	74099	74107	

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600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880	
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607	78319	78326	78333	78340	78347	78355	78362	78369	78376	78383	
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649	81224	81231	81238	81245	81251	81258	81265	81271	81278	81285	
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662	82086	82092	82099	82105	82112	82119	82125	82132	82138	82145	
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667	82413	82419	82426	82432	82439	82445	82452	82458	82465	82471	
668	82478	82484	82491	82497	82504	82510	82517	82523	82530	82536	
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671	82672	82679	82685	82692	82698	82705	82711	82718	82724	82730	
672	82737	82743	82750	82756	82763	82769	82776	82782	82789	82795	
673	82802	82808	82814	82821	82827	82834	82840	82847	82853	82860	
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676	82995	83001	83008	83014	83020	83027	83033	83040	83046	83052	
677	83059	83065	83072	83078	83085	83091	83097	83104	83110	83117	
678	83123	83129	83136	83142	83149	83155	83161	83168	83174	83181	
679	83187	83193	83200	83206	83213	83219	83225	83232	83238	83245	
680	83251	83257	83264	83270	83276	83283	83289	83296	83302	83308	6
681	83315	83321	83327	83334	83340	83347	83353	83359	83366	83372	1 1
682	83378	83385	83391	83398	83404	83410	83417	83423	83429	83436	2 1
683	83442	83448	83455	83461	83467	83474	83480	83487	83493	83499	3 2
684	83506	83512	83518	83525	83531	83537	83544	83550	83556	83563	4 2
685	83569	83575	83582	83588	83594	83601	83607	83613	83620	83626	5 3
686	83632	83639	83645	83651	83658	83664	83670	83677	83683	83689	6 4
687	83696	83702	83708	83715	83721	83727	83734	83740	83746	83753	7 4
688	83759	83765	83771	83778	83784	83790	83797	83803	83809	83816	8 5
689	83822	83828	83835	83841	83847	83853	83860	83866	83872	83879	9 5
690	83885	83891	83897	83904	83910	83916	83923	83929	83935	83942	
691	83948	83954	83960	83967	83973	83979	83985	83992	83998	84004	
692	84011	84017	84023	84029	84036	84042	84048	84055	84061	84067	
693	84073	84080	84086	84092	84098	84105	84111	84117	84123	84130	
694	84136	84142	84148	84155	84161	84167	84173	84180	84186	84192	
695	84198	84205	84211	84217	84223	84230	84236	84242	84248	84255	
696	84261	84267	84273	84280	84286	84292	84298	84305	84311	84317	
697	84323	84330	84336	84342	84348	84354	84361	84367	84373	84379	
698	84386	84392	84398	84404	84410	84417	84423	84429	84435	84442	
699	84448	84454	84460	84466	84473	84479	84485	84491	84497	84504	
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566	

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700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566	
701	84572	84578	84584	84590	84597	84603	84609	84615	84621	84628	
702	84634	84640	84646	84652	84658	84665	84671	84677	84683	84689	
703	84696	84702	84708	84714	84720	84726	84733	84739	84745	84751	
704	84757	84763	84770	84776	84782	84788	84794	84800	84807	84813	
705	84819	84825	84831	84837	84844	84850	84856	84862	84868	84874	
706	84880	84887	84893	84899	84905	84911	84917	84924	84930	84936	
707	84942	84948	84954	84960	84967	84973	84979	84985	84991	84997	
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709	85065	85071	85077	85083	85089	85095	85101	85107	85114	85120	
710	85126	85132	85138	85144	85150	85156	85163	85169	85175	85181	
711	85187	85193	85199	85205	85211	85217	85224	85230	85236	85242	
712	85248	85254	85260	85266	85272	85278	85285	85291	85297	85303	
713	85309	85315	85321	85327	85333	85339	85345	85352	85358	85364	
714	85370	85376	85382	85388	85394	85400	85406	85412	85418	85425	
715	85431	85437	85443	85449	85455	85461	85467	85473	85479	85485	
717	85491	85497	85503	85509	85516	85522	85528	85534	85540	85546	
716	85552	85558	85564	85570	85576	85582	85588	85594	85600	85606	
718	85612	85618	85625	85631	85637	85643	85649	85655	85661	85667	
719	85673	85679	85685	85691	85697	85703	85709	85715	85721	85727	
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721	85794	85800	85806	85812	85818	85824	85830	85836	85842	85848	1 1
722	85854	85860	85866	85872	85878	85884	85890	85896	85902	85908	2 1
723	85914	85920	85926	85932	85938	85944	85950	85956	85962	85968	3 2
724	85974	85980	85986	85992	85998	86004	86010	86016	86022	86028	4 2
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726	86094	86100	86106	86112	86118	86124	86130	86136	86141	86147	6 4
727	86153	86159	86165	86171	86177	86183	86189	86195	86201	86207	7 4
728	86213	86219	86225	86231	86237	86243	86249	86255	86261	86267	8 5
729	86273	86279	86285	86291	86297	86303	86308	86314	86320	86326	9 5
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731	86392	86398	86404	86410	86415	86421	86427	86433	86439	86445	
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734	86570	86576	86581	86587	86593	86599	86605	86611	86617	86623	
735	86629	86635	86641	86646	86652	86658	86664	86670	86676	86682	
736	86688	86694	86700	86705	86711	86717	86723	86729	86735	86741	
737	86747	86753	86759	86764	86770	86776	86782	86788	86794	86800	
738	86806	86812	86817	86823	86829	86835	86841	86847	86853	86859	
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740	86923	86929	86935	86941	86947	86953	86958	86964	86970	86976	
741	86982	86988	86994	86999	87005	87011	87017	87023	87029	87035	
742	87040	87046	87052	87058	87064	87070	87075	87081	87087	87093	
743	87099	87105	87111	87116	87122	87128	87134	87140	87146	87151	
744	87157	87163	87169	87175	87181	87186	87192	87198	87204	87210	
745	87216	87221	87227	87233	87239	87245	87251	87256	87262	87268	
746	87274	87280	87286	87291	87297	87303	87309	87315	87320	87326	
747	87332	87338	87344	87349	87355	87361	87367	87373	87379	87384	
748	87390	87396	87402	87408	87413	87419	87425	87431	87437	87442	
749	87448	87454	87460	87466	87471	87477	87483	87489	87495	87500	
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558	

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750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558	
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753	87679	87685	87691	87697	87703	87708	87714	87720	87726	87731	
754	87737	87743	87749	87754	87760	87766	87772	87777	87783	87789	
755	87795	87800	87806	87812	87818	87823	87829	87835	87841	87846	
756	87852	87858	87864	87869	87875	87881	87887	87892	87898	87904	
757	87910	87915	87921	87927	87933	87938	87944	87950	87955	87961	
758	87967	87973	87978	87984	87990	87996	88001	88007	88013	88018	
759	88024	88030	88036	88041	88047	88053	88058	88064	88070	88076	
760	88081	88087	88093	88098	88104	88110	88116	88121	88127	88133	
761	88138	88144	88150	88156	88161	88167	88173	88178	88184	88190	
762	88195	88201	88207	88213	88218	88224	88230	88235	88241	88247	
763	88252	88258	88264	88270	88275	88281	88287	88292	88298	88304	
764	88309	88315	88321	88326	88332	88338	88343	88349	88355	88360	
765	88366	88372	88377	88383	88389	88395	88400	88406	88412	88417	
766	88423	88429	88434	88440	88446	88451	88457	88463	88468	88474	
767	88480	88485	88491	88497	88502	88508	88513	88519	88525	88530	
768	88536	88542	88547	88553	88559	88564	88570	88576	88581	88587	
769	88593	88598	88604	88610	88615	88621	88627	88632	88638	88643	
770	88649	88655	88660	88666	88672	88677	88683	88689	88694	88700	6
771	88705	88711	88717	88722	88728	88734	88739	88745	88750	88756	1 1
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773	88818	88824	88829	88835	88840	88846	88852	88857	88863	88868	3 2
774	88874	88880	88885	88891	88897	88902	88908	88913	88919	88925	4 2
775	88930	88936	88941	88947	88953	88958	88964	88969	88975	88981	5 3
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777	89042	89048	89053	89059	89064	89070	89076	89081	89087	89092	7 4
778	89098	89104	89109	89115	89120	89126	89131	89137	89143	89148	8 5
779	89154	89159	89165	89170	89176	89182	89187	89193	89198	89204	9 5
780	89209	89215	89221	89226	89232	89237	89243	89248	89254	89260	
781	89265	89271	89276	89282	89287	89293	89298	89304	89310	89315	
782	89321	89326	89332	89337	89343	89348	89354	89360	89365	89371	
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785	89487	89492	89498	89504	89509	89515	89520	89526	89531	89537	
786	89542	89548	89553	89559	89564	89570	89575	89581	89586	89592	
787	89597	89603	89609	89614	89620	89625	89631	89636	89642	89647	
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790	89763	89768	89774	89779	89785	89790	89796	89801	89807	89812	
791	89818	89823	89829	89834	89840	89845	89851	89856	89862	89867	
792	89873	89878	89883	89889	89894	89900	89905	89911	89916	89922	
793	89927	89933	89938	89944	89949	89955	89960	89966	89971	89977	
794	89982	89988	89993	89998	90004	90009	90015	90020	90026	90031	
795	90037	90042	90048	90053	90059	90064	90069	90075	90080	90086	
796	90091	90097	90102	90108	90113	90119	90124	90129	90135	90140	
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798	90200	90206	90211	90217	90222	90227	90233	90238	90244	90249	
799	90255	90260	90266	90271	90276	90282	90287	90293	90298	90304	
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358	

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800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358	
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803	90472	90477	90482	90488	90493	90499	90504	90509	90515	90520	
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805	90580	90585	90590	90596	90601	90607	90612	90617	90623	90628	
806	90634	90639	90644	90650	90655	90660	90666	90671	90677	90682	
807	90687	90693	90698	90703	90709	90714	90720	90725	90730	90736	
808	90741	90747	90752	90757	90763	90768	90773	90779	90784	90789	
809	90795	90800	90806	90811	90816	90822	90827	90832	90838	90843	
810	90849	90854	90859	90865	90870	90875	90881	90886	90891	90897	
811	90902	90907	90913	90918	90924	90929	90934	90940	90945	90950	
812	90956	90961	90966	90972	90977	90982	90988	90993	90998	91004	
813	91009	91014	91020	91025	91030	91036	91041	91046	91052	91057	
814	91062	91068	91073	91078	91084	91089	91094	91100	91105	91110	
815	91116	91121	91126	91132	91137	91142	91148	91153	91158	91164	
816	91169	91174	91180	91185	91190	91196	91201	91206	91212	91217	
817	91222	91228	91233	91238	91243	91249	91254	91259	91265	91270	
818	91275	91281	91286	91291	91297	91302	91307	91312	91318	91323	
819	91328	91334	91339	91344	91350	91355	91360	91365	91371	91376	
820	91381	91387	91392	91397	91403	91408	91413	91418	91424	91429	5
821	91434	91440	91445	91450	91455	91461	91466	91471	91477	91482	1 1
822	91487	91492	91498	91503	91508	91514	91519	91524	91529	91535	2 1
823	91540	91545	91551	91556	91561	91566	91572	91577	91582	91587	3 2
824	91593	91598	91603	91609	91614	91619	91624	91630	91635	91640	4 2
825	91645	91651	91656	91661	91666	91672	91677	91682	91687	91693	5 3
826	91698	91703	91709	91714	91719	91724	91730	91735	91740	91745	6 3
827	91751	91756	91761	91766	91772	91777	91782	91787	91793	91798	7 4
828	91803	91808	91814	91819	91824	91829	91834	91840	91845	91850	8 4
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830	91908	91913	91918	91924	91929	91934	91939	91944	91950	91955	
831	91960	91965	91971	91976	91981	91986	91991	91997	92002	92007	
832	92012	92018	92023	92028	92033	92038	92044	92049	92054	92059	
833	92065	92070	92075	92080	92085	92091	92096	92101	92106	92111	
834	92117	92122	92127	92132	92137	92143	92148	92153	92158	92163	
835	92169	92174	92179	92184	92189	92195	92200	92205	92210	92215	
836	92221	92226	92231	92236	92241	92247	92252	92257	92262	92267	
837	92273	92278	92283	92288	92293	92298	92304	92309	92314	92319	
838	92324	92330	92335	92340	92345	92350	92355	92361	92366	92371	
839	92376	92381	92387	92392	92397	92402	92407	92412	92418	92423	
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841	92480	92485	92490	92495	92500	92505	92511	92516	92521	92526	
842	92531	92536	92542	92547	92552	92557	92562	92567	92572	92578	
843	92583	92588	92593	92598	92603	92609	92614	92619	92624	92629	
844	92634	92639	92645	92650	92655	92660	92665	92670	92675	92681	
845	92686	92691	92696	92701	92706	92711	92716	92722	92727	92732	
846	92737	92742	92747	92752	92758	92763	92768	92773	92778	92783	
847	92788	92793	92799	92804	92809	92814	92819	92824	92829	92834	
848	92840	92845	92850	92855	92860	92865	92870	92875	92881	92886	
849	92891	92896	92901	92906	92911	92916	92921	92927	92932	92937	
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988	

No.	0	1	2	3	4	5	6	7	8	9	Dif.
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988	
851	92993	92998	93003	93008	93013	93018	93024	93029	93034	93039	
852	93044	93049	93054	93059	93064	93069	93075	93080	93085	93090	
853	93095	93100	93105	93110	93115	93120	93125	93131	93136	93141	
854	93146	93151	93156	93161	93166	93171	93176	93181	93186	93192	
855	93197	93202	93207	93212	93217	93222	93227	93232	93237	93242	
856	93247	93252	93258	93263	93268	93273	93278	93283	93288	93293	
857	93298	93303	93308	93313	93318	93323	93328	93334	93339	93344	
858	93349	93354	93359	93364	93369	93374	93379	93384	93389	93394	
859	93399	93404	93409	93414	93420	93425	93430	93435	93440	93445	
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861	93500	93505	93510	93515	93520	93526	93531	93536	93541	93546	
862	93551	93556	93561	93566	93571	93576	93581	93586	93591	93596	
863	93601	93606	93611	93616	93621	93626	93631	93636	93641	93646	
864	93651	93656	93661	93666	93671	93676	93682	93687	93692	93697	
865	93702	93707	93712	93717	93722	93727	93732	93737	93742	93747	
866	93752	93757	93762	93767	93772	93777	93782	93787	93792	93797	
867	93802	93807	93812	93817	93822	93827	93832	93837	93842	93847	
868	93852	93857	93862	93867	93872	93877	93882	93887	93892	93897	
869	93902	93907	93912	93917	93922	93927	93932	93937	93942	93947	
870	93952	93957	93962	93967	93972	93977	93982	93987	93992	93997	5
871	94002	94007	94012	94017	94022	94027	94032	94037	94042	94047	1 1
872	94052	94057	94062	94067	94072	94077	94082	94086	94091	94096	2 1
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874	94151	94156	94161	94166	94171	94176	94181	94186	94191	94196	4 2
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876	94250	94255	94260	94265	94270	94275	94280	94285	94290	94295	6 3
877	94300	94305	94310	94315	94320	94325	94330	94335	94340	94345	7 4
878	94349	94354	94359	94364	94369	94374	94379	94384	94389	94394	8 4
879	94399	94404	94409	94414	94419	94424	94429	94433	94438	94443	9 5
880	94448	94453	94458	94463	94468	94473	94478	94483	94488	94493	
881	94498	94503	94507	94512	94517	94522	94527	94532	94537	94542	
882	94547	94552	94557	94562	94567	94571	94576	94581	94586	94591	
883	94596	94601	94606	94611	94616	94621	94626	94630	94635	94640	
884	94645	94650	94655	94660	94665	94670	94675	94680	94685	94689	
885	94694	94699	94704	94709	94714	94719	94724	94729	94734	94738	
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896	95231	95236	95240	95245	95250	95255	95260	95265	95270	95274	
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1099	04100	04104	04108	04112	04116	04120	04123	04127	04131	04135	
1100	04139	04143	04147	04151	04155	04159	04163	04167	04171	04175	

Error greater than	Probability	Difference	Error greater than	Probability	Difference	Error greater than	Probability
0.0	1.00000	5378	2.5	0.09175	1226	5.2	4.53×10^{-4}
0.1	0.94622	5353	2.6	.07949	1090	5.4	2.70×10^{-4}
0.2	.89269	5304	2.7	.06859	964	5.6	1.59×10^{-4}
0.3	.83965	5233	2.8	.05895	849	5.8	9.15×10^{-5}
0.4	.78732	5139	2.9	.05046	744	6.0	5.19×10^{-5}
0.5	0.73593	5023	3.0	0.04302	648	6.2	2.9×10^{-5}
0.6	.68570	4887	3.1	.03654	564	6.4	1.6×10^{-5}
0.7	.63683	4735	3.2	.03090	487	6.6	8.5×10^{-6}
0.8	.58948	4566	3.3	.02603	420	6.8	4.5×10^{-6}
0.9	.54382	4382	3.4	.02183	359	7.0	2.3×10^{-6}
*1.0	0.50000	4188	3.5	0.01824	306	7.2	1.2×10^{-6}
1.1	.45812	3983	3.6	.01518	261	7.4	6.0×10^{-7}
1.2	.41829	3771	3.7	.01257	219	7.6	3.0×10^{-7}
1.3	.38058	3556	3.8	.01038	185	7.8	1.4×10^{-7}
1.4	.34502	3335	3.9	.00853	155	8.0	6.8×10^{-8}
1.5	0.31167	3116	4.0	0.00698	129	9.0	1.3×10^{-9}
1.6	.28051	2898	4.1	.00569	108	10	1.5×10^{-11}
1.7	.25153	2681	4.2	.00461	88	20	$2. \times 10^{-41}$
1.8	.22472	2471	4.3	.00373	73	30	5×10^{-91}
1.9	.20001	2267	4.4	.00300	60	40	$3. \times 10^{-160}$
2.0	0.17734	2069	4.5	0.00240	48	50	$3. \times 10^{-249}$
2.1	.15665	1881	4.6	.00192	40	60	$4. \times 10^{-358}$
2.2	.13784	1702	4.7	.00152	31	70	$1. \times 10^{-486}$
2.3	.12082	1532	4.8	.00121	26	80	$1. \times 10^{-634}$
2.4	.10550	1375	4.9	.00095	20	90	$1. \times 10^{-802}$
2.5	0.09175		5.0	0.00075		100	$1. \times 10^{-990}$

* *1.0 = "Probable Error".*

Name	Symbol	Atomic Weight	Density at 0° and 76cm	Hardness	Breaking Strength	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical, 0° 100°, 76cm	Melting Point	Boiling Point	Specific Heat, 0° 100°, 76cm	Latent Heat Melting	Heat Conductivity	Electrical Conductivity	Thermo-Electric Heights	Electro-Chemical Equiv.
Multiply . . .	by	....	....	....	10^9	10^{12}	10^{12}	.....	....	....	....	....	...	.001	100	.001
Aluminum . .	Al	27.1	2.6	3 —	2.	0.7	0.5?	.000070	700	..	.212	..	.35	.33	— 0.8	...
Antimony . .	Sb	120.	6.7	3 +	..	..	..	.000034	440	1200	.050	..	.04	.63	+24.	0.938
Arsenic . . .	As	74.9	5.7	3 +	..	..	..	.000018	S	450	.083 c.	..	..	.03	+14.	...
Barium . . .	Ba	136.8	3.8	..	..	..	..	...	1300?	..	..	..	..	..	..	...
Beryllium* .	Be	9.1	2.1	..	..	..	..	...	900?	..	.42	..	..	..	..	...
Bismuth . . .	Bi	208.	9.8	2 +	..	..	..	.000040	270	1200	.030	13	.02	.008	—55 c.	...
Boron . . .	B	10.9	2.5?	..	..	..	..	...	a.	..	.25	..	..	..	..	...
Bromine . .	Br	79.77	3.1 l	..	..	..	..	.00104 l	—7	61	.084	16	..	..	..	...
Cadmium . .	Cd	111.8	8.6	2 +	..	..	..	.000094	318	800	.055	14	.22	.14 +	+ 3	8.280
Caesium . . .	Cs	132.7	1.9	..	..	..	..	...	27	..	..	..	..	..	..	...
Calcium . . .	Ca	40.0	1.6	1 +	..	..	..	...	700?	..	.18	..	..	.13	+15	...
Carbon . . .	C	11.97	2. G.	1 — G.	..	..	..	.00002 G.	I.	V.	.2 G.	..	.01 G.	.000	..	...
Cerium . . .	Ce	141.	6.6	..	..	..	..	...	700	..	.045	..	..	..	..	...
Chlorine . .	Cl	35.37	1.31	..	..	..	..	...	..	—33.6	.121 g.	..	..	..	..	3.671
Chromium . .	Cr	52.3	6.7	8 ?	..	..	..	...	2000	..	.1	..	..	..	..	...
Cobalt . . .	Co	58.7	8.5	6 ?	..	..	..	.000037	1500	..	.107	..	..	.10	—22	...
Columbium† .	Cb	94.	7.2	..	..	..	..	...	..	..	..	..	..	..	..	...
Copper . . .	Cu	63.2	8.9	3 —	4.	1.2	1.6	.000051	1100	..	.093	30?	.9 +	.60	+ 4	3.280 C.
Didymium . .	D	145.	6.5	..	..	..	..	...	900?	..	.046	..	..	..	..	...
Erbium . . .	E	167.	..	..	..	..	..	...	..	..	..	..	..	..	..	...
Fluorine . . .	F	19.0	..	..	..	..	..	...	..	..	..	..	..	..	..	...
Gallium . . .	Ga	69.4	5.9	..	..	..	..	...	30	..	.08	19	..	..	..	...

Abbreviations: a, melts in the voltaic arc; c, crystallized; C, cupric; g, gaseous; G, graphite; I, infusible; l, liquid; s, solid; S, sublimes without melting; V, vaporized in the voltaic arc. * Same as Glucinum. † Same as Niobium.

Name	Symbol	Antomic Weight	Density at 0° and 76 cm	Hardness	Breaking Strength	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical, 0° 100°, 76 cm	Melting Point	Boiling Point, 76 cm	Specific Heat, 0° 100°, 76 cm	Latent Heat Melting	Heat Conductivity	Electrical Conductivity	Thermo-Electric Heights	Electro-Chemical Equiv.
Multiply . .	by	....	....	....	10^9	10^{12}	10^{12}	......	....	....	....	....	....	.001	100	.001
Glucinum* .	G	9.1	2.1	..	..	..	..	....	900 ?	..	.42	..	..	..	..	...
Gold . . .	Au	196.2	19.3	3 —	3.—	0.8	0.6	.000044	1100	..	.032	..	.6	.47	+ 2	6.789
Hydrogen .	H	1.0000	0.03 l	..	..	..	..	.00367 g.	b.	b.	3.41 g.	..	.000 g.	..	..	.1038
Indium . .	In	113.4	7.3	3 ?	..	..	..	.00014	176	700 ?	.057	..	..	..	..	...
Iodine . .	I	126.55	4.95	..	..	..	..	....	110	200	.054	12	..	..	..	13.13
Iridium . .	Ir	193.	22.	..	..	..	..	.000021	2200	..	.032	..	..	..	+ 2.5	...
Iron . . .	Fe	55.9	7.8	5 —	6.	1.9	1.5	.000036	1600	..	.113	35 ?	.16	.10	+ 17	1.934 F.
Lanthanum .	La	139.	6.1	..	..	..	..	....	700 ?	..	.045	..	..	..	..	...
Lead . . .	Pb	206.4	11.3 +	2 —	.1—.3	0.1+	..	.000088	330	1500	.032	5.6	.08	.052	0	10.71
Lithium . .	Li	7.01	0.59	..	..	..	..	....	180	..	.94	..	..	.11	+ 14 ?	...
Magnesium .	Mg	24.0	1.7	2 +	..	..	..	.000083	650 ?	1100 ?	.25	..	.4	.3	+ 2	...
Manganese .	Mn	54.6	7 — 8	9 ?	..	..	..	....	1800 ?	..	.12	..	..	..	..	...
Mercury . .	Hg	199.8	14.19 s.	..	..	..	0.5	.000182	—39	357	.032 s.	2.8	.018	.0106	— 0.4	1.037 M.
Molybdenum	Mo	95.7	8.6	1 +	..	..	..	....	1600 ?	..	.067	..	..	..	..	...
Nickel . . .	Ni	58.3	8.9	4 +	..	..	..	.000038	1500	..	.109	..	..	.08	— 20.	3.03
Niobium† .	Nb	94.	7.	..	..	..	..	....	—	..	—	..	..	..	..	...
Nitrogen . .	N	14.01	0.41	..	..	..	..	.00367 g.	b.	b.	.244 g.	..	.000 g.	..	..	.485
Osmium . .	Os	198.	22.	..	..	..	..	.000020	2200 ?	..	.031	..	..	..	..	...
Oxygen . .	O	15.96	0.71	..	..	..	..	.00367 g.	b.	b.	.218 g.	..	.000 g.	..	..	.828
Palladium .	Pd	106.0	11—12	3 +	..	1.0	.7 ?	.000035	1700	..	.059	36	.07	.073	— 7	...
Phosphorus .	P	30.96	1.8	..	..	..	..	0 to .00004	44.3	289	.20.	5	..	.000	+ 30 R.	...
Platinum . .	Pt	195.0	21.5	4 +	3 +	1.6	1.1 ?	.000027	1900	..	.032	27	.09	.10	0 ±	...

Abbreviations: b, below — 100°; F, Ferric; g, gas; l, liquid under 300 atmospheres; M, mercuric; R, red; s, solid.
* Same as Beryllium. † Same as Columbium.

Name	Symbol	Atomic Weight	Density at 0° and 76 cm	Hardness	Breaking Strength	Young's Modulus	Resilience of Volume	Coefficient, Expansion Cubical, 0° 100°, 76 cm	Melting Point	Boiling Point, 76 cm	Specific Heat, 0° 100°, 76 cm	Latent Heat Melting	Heat Conductivity	Electrical Conductivity	Thermo-Electric Heights	Electro-Chemical Equiv.
Multiply . .	by	....	....	..	10^9	10^{12}	10^{12}	....	....	....	....	....	....	.001	100	.001
Potassium .	K	39.03	0.87	0+	..	..	..	.00025	60	725	.17	..	..	.12	—13.?	4.051
Rhodium .	Rh	104.1	11.—12.	..	..	..	..	.000026	2000	..	.058	..	..	..	..	..
Rubidium .	Rb	85.2	1.5	..	..	..	..	...	38	..	—	..	..	..	..	..
Ruthenium .	Ru	104.	12.	..	..	..	..	.000030	2000?	..	.061	..	..	..	..	..
Selenium .	Se	79.	4.8 c.	3—	..	..	..	.0002—.001	217	680	.084 c.	..	..	.000	+800	..
Silicon . .	Si	28.1	2.3 c.	..	..	..	..	.00002	1200?	..	.18 c.	..	..	..	..	..
Silver . . .	Ag	107.7	10.5	2+	3.	0.7	0.5?	.000058	1000	..	.056+	21.	1.1	.64	+2.+	11.18
Sodium . .	Na	23.00	0.98	..	..	..	..	.00021	95	900	.30	..	0.4	.2	—6.?	2.387
Strontium .	Sr	87.4	2.5	..	..	..	..	...	700?	..	—	..	..	.04	+9.?	..
Sulphur . .	S	31.98	2.0	2+	..	..	..	.0003	114	448	.17	9.4	..	..	..	..
Tantalum .	Ta	182.	10.+	..	..	..	..	...	—	..	—	..	..	..	..	..
Tellurium .	Te	128.	6.4 c.	2+	..	..	..	.00005-.0001	450	..	.048 c.	..	..	.000	+500	..
Thallium .	Tl	204.	11.8	2—	..	..	..	.000094	290	700?	.034	..	..	.05+	..	..
Thorium. .	Th	232.	11.?	..	..	..	..	...	..	..	—	..	..	..	..	..
Tin. . . .	Sn	118.	7.2+	2	.2—.4	0.4	0.3?	.000069	230	1500	.055	14.	.15	.09	0.±	3.06S.
Titanium .	Ti	50.	—	8?	..	..	..	...	..	..	—	..	..	..	..	..
Tungsten* .	W	183.7	17.—19.	3+	..	..	..	...	1800?	..	.034	..	..	..	..	..
Uranium. .	U	240.	18.5	..	..	..	..	...	800?	..	.062	..	..	..	..	..
Vanadium .	V	51.2	5.+	..	..	..	..	...	—	..	—	..	..	..	..	..
Wolfram* .	W	183.7	17.—19.	..	..	..	..	...	1800?	..	.034	..	..	..	..	..
Yttrium . .	Y	90.	—	..	..	..	..	...	—	..	—	..	..	..	..	..
Zinc . . .	Zn	64.9	7.0+	3+	.2—.5	0.9	0.5?	.000088	420	1000	.093+	28.	.29	.18	+3.	3.37
Zirconium .	Zr	90.	4.	..	..	..	..	...	1400	..	.066	..	..	..	..	..

c, crystallized. S, Stannic. * Tungsten and Wolfram the same.

Name (Commercial Materials)	Density	Hardness	Breaking Strength	Resistance to Crushing	Resistance to Shearing	Simple Rigidity	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical 0°—100°	Melting Point	Boiling Point 76 cm	Specific Heat 0°—100°	Latent Heat Melting	Heat Con-ductivity	Electrical Con-ductivity	Thermo-Electric Heights
Multiply by	tr	..	10^9	10^9	10^9	10^{12}	10^{12}	10^{12}	......	....	...	....	....	....	.001	100
Aluminum	2.7	3—	2.0	..	..	.25	.7	.5?	.00007	700?	..	.21	..	.3+	.3+	—1.?
Antimony	6.7	3+	..	..	..	..	..	..	...	440	1200	.05	..	.04	.03+	+6.
Bamboo	0.4	..	.044	..	..	..	..	..	...	..	..	..	..	..	..	..
Bismuth pressed . .	9.8	2+	..	..	..	..	..	..	.00004	270	1200	.03	13.	.02—	.008	—97.
Brass* (cast) . . .	8.3+	3+	1.2	.7	..	.24	.6+	.4?	.00005+	900?	..	.094—	..	.2—.3	.1—.2	..
" (hard drawn)	8.5	3+	4.—9.	..	..	.37	1.0	1.0	.000057	..	..	.09+	..	.2—.3	.1—.2	..
Bricks & cement .	1.7	..	.02	.04—.1	..	..	..	..	...	..	..	.2+	..	.003	..	..
Bronze†	8.8	3+	2.5	..	..	.26	.6+	.4?	.000054	900?	..	..	..	..	..	..
Copper (cast) . . .	8.8	3	1.3	4.?	..	.4	1.1	1.6?	.00005	1100?	..	..	30	.8	.6	0±?
" (hard drawn)	8.8+	..	3.—5.	..	..	.45	1.2	1.7	.000051	..	..	..	30?	.8?	.6+	0±?
Cork	0.24	..	..	..	..	..	..	..	...	..	..	..	..	.0005	..	..
Cotton	..	..	..	..	..	..	..	..	...	..	..	..	..	.0001	..	..
Felt	..	..	..	..	..	..	..	..	...	..	..	..	..	.0002	..	..
German silver . .	8.5	3+	..	..	..	..	..	..	.000055	..	..	..	..	.09	.03—.06	—12
Glass (crown) . . .	2.5+	..	.3—.6	..	..	.2	.6+	.4?	.000025	400	..	.19	..	.001±	.0000	..
" (flint) . . .	3.5—	..	..	..	..	.24	.6	.4	.000025	..	..	.19?	..	.001+	.0000	..
Gold (75%)	15?	..	1.—3.	..	..	..	..	..	.000046	1050	..	..	..	..	..	..
Granite	2.7+	..	..	.4 —.8	..	..	..	..	.000026	..	..	..	..	.004	..	..
Hair	..	..	..	..	..	..	..	..	...	..	..	..	..	.0001	..	..
Hemp	..	..	.5	..	..	..	..	..	...	..	..	..	..	.0001?	..	..
India Rubber . . .	0.95	..	..	..	..	..	..	..	...	..	..	..	..	.0004	..	..

* 72% Copper, 28% Zinc.

† 86% Copper, 10% Tin, 4% Zinc.

Name	Density	Hardness	Breaking Strength	Resistance to Crushing	Resistance to Shearing	Simple Rigidity	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical, 0°, 100°, 76 cm	Melting Point	Boiling Point, 76 cm	Specific Heat, 0°, 100°, 76 cm	Latent Heat Melting	Heat Conductivity	Electrical Conductivity	Thermo-Electric Heights
Multiply . . .	by	..	10^9	10^9	10^9	10^{12}	10^{12}	10^{12}	.....	...	..	...	....	....	.001	100.
Iron (grey cast)	7.5+	6?	1.?	8.	2.	.4—.5	1.1—1.3	1.0	.000033	1200	..	.13?	25?	..	..	...
" (white cast)	7+	8?	1.?	8.	2.	.4—.5	1.1—1.3	1.0	.000036	1100	..	.13	34?	..	..	...
" (wrought) .	7.8	4+	4.—7.	3.	3.+	0.75	1.9+	1.5	.000037	1600	..	.11	..	.16	.07—.1	+18.
Lead (pressed)	11.3	2—	0.2	.5?	..	0.02+	.07—.13	..	.000088	300+	1500	..	6.	.08	.05	0?
Leather . . .	..	..	0.3	..	..	..	..	..	...	..	..	..	..	..	..	...
Marble. . . .	2.7	3	..	.4	..	..	..	..	.00002+?	..	..	.21	..	.002	..	...
Paper	..	..	..	..	..	..	..	..	...	..	..	..	..	.0001	..	...
Platinum. . .	20?	..	3.+	..	..	0.6?	1.6+	1.1?	.000027	1800?	..	.03+	27?	.09	.07—.1	—1.+3.
Sand (with air spaces) . . .	1.45	7	..	..	..	..	..	..	...	..	..	.19	..	.0004	..	...
Silk.	..	..	50?	..	..	..	..	..	...	..	..	..	..	..	..	...
Silver (Sterling)	..	..	2.—3.?	..	..	0.3?	0.7?	0.5?	.00006	900	..	..	21?	..	..	...
Slate	2.8	..	0.7+	..	..	..	..	..	...	..	..	..	..	.004?	..	...
Steel (cast) . .	7.8+	..	8.—10.	..	..	0.8+	2.2	1.3?	.000034	1400	..	.12	..	.14	..	+11.
" (tempered)	7.8+	9?	7.+	..	..	1.0?	2.7?	1.8?	.000037	..	..	.12?	..	..	..	...
" (wire) . .	7.9?	..	10.—20.	..	..	..	2.8?	1.9?	.000037	..	..	.12?	..	..	..	...
Tin (pressed) .	7.3	2?	.2—.4	..	..	0.15	0.4	0.3	.000068	..	..	.05+	14?	..	.08	...
Vulcanite . .	1.0+	..	..	..	..	..	..	..	.0002?	..	..	..	..	.0003	..	...
Wood (hard) .	.6—1.	..	1.0+	.6+	.8±	0.010?	0.10	..	.00002—	..	..	.6+	..	.0006	..	...
" (soft) .	.4—.7?	..	0.8—	.4+	.05—.12	0.007?	0.09±	..	.00001+	..	..	.6—	..	.0004	..	...
Wool	..	..	..	..	..	..	..	..	...	..	..	..	..	.0001—	..	...
Zinc (rolled) .	7.0	3+?	.5—1.5	..	..	0.4	0.9	0.6	.000089	400	1000	..	28?	.3—?	.16	+3.

Note: Annealing generally increases electrical conductivity, but greatly diminishes breaking strength (10—30%). Powdering reduces heat conductivity of most substances to about .0002.

Name	Symbol	Density	Hardness	Coefficient Expansion cubic. 0—100°	Melting Point	Boiling Point, 76 cm	Specific Heat	Latent Heat Melting	Solubility at 20° in %	Solubility at 100° in %
Acid										
„ Acetic . . .	$HC_2H_3O_2$ . . .	1.1	.	..	16	117	.46	44	100	100
„ Oxalic . . .	$H_2C_2O_4.2H_2O$.	1.6	.	..	.	.	.	.	12	90?
„ Phosphoric .	H_3PO_4	1.9	.	..	40	.	.	26	.	100
„ Phosphorous	H_3PO_3	1.7	.	..	72	.	.	37	.	100
„ „ Hypo-	H_3PO_2	1.5	.	..	17	.	.	36	.	100
„ Sulphuric (hyd.)	$H_2SO_4.H_2O$ . .	1.8	.	..	10	.	.32	.	100	100
„ Tartaric . .	$H_2C_4H_4O_6$ . .	1.8	.	..	135	.	.29	.	58	77
Arsenate of										
„ Lead. . . .	$Pb_3As_2O_8$ . . .	.	.	..	.	.	.073	.	0	0
„ Potassium .	KH_2AsO_4 . . .	2.9	.	..	.	.	.175	.	.	.
„ „ (anhyd.)	$KAsO_3$	.	.	..	.	.	.156	.	70	.
Borate of										
„ Lead . . .	PbB_2O_4	.	.	..	.	.	.090	.	0	.
„ „ Bi-. .	PbB_4O_7	.	.	..	.	.	.114	.	0+	.
„ Potassium .	KBO_2	.	.	..	.	.	.205	.	sol	.
„ „ Bi-. .	$K_2B_4O_7$	.	.	..	.	.	.220	.	sol	sol
„ Sodium . .	$NaBO_2$	.	.	..	.	.	.257	.	47	.
„ „ Bi-. .	$Na_2B_4O_7$ . . .	2.4	.	..	600	.	.233	.	4	36
„ (Borax) . .	$Na_2B_4O_7.10H_2O$	1.7	.	..	.	.	.385	.	7	67
Bromide of										
„ Lead . . .	$PbBr_2$.	6.6	.	..	500	.	.053	.	0?	0+
„ Potassium .	KBr	2.7	.	.000126	700	.	.113	.	39	51
„ Silver . . .	AgBr	6.3	.	.000104	430	.	.074	.	0	0
Carbonate of										
„ Barium . .	$BaCO_3$	4.3	.	..	.	.	.110	.	0	0
„ Calcium . .	$CaCO_3$	2.7	1+	..	.	.	.210	.	0	0
„ Iron . . .	$FeCO_3$	3.8	4	..	.	.	.193	.	0	0
„ Lead . . .	$PbCO_3$	6.5	.	..	.	.	.079	.	0	0
„ Potassium .	K_2CO_3	2.3	.	..	850	.	.211	.	51	62
„ Sodium . .	Na_2CO_3	2.5	.	..	850	.	.26	.	20	33
„ „ (acid).	$NaHCO_3$ . . .	2.2	.	..	.	.	.	.	9	.
„ Strontium .	$SrCO_3$	3.6	.	..	.	.	.148	.	0	0
Chloral	$C_2H_3Cl_3O_2$ . .	1.8	.	..	50	97	.	33	sol	.
Chlorate of										
„ Barium . .	$BaCl_2O_6.H_2O$.	3.2	.	..	400	.	.157	.	29	59
„ Potassium .	$KClO_3$	2.3	.	..	350	.	.20	.	7	38
„ „ Per- .	$KClO_4$	2.5	.	..	600	.	.19	.	.	.
„ Sodium . .	$NaClO_3$	2.3	.	..	300	.	.	.	50	70
Chloride of										
„ Ammonium.	H_4NCl	1.5	2—	.000188	sub	400	.38	.	27	42
„ Barium . .	$BaCl_2$	3.8	.	..	.	.	.090	.	26	37
„ „ (crystals)	$BaCl_2.2H_2O$. .	3.0	.	..	.	.	.171	.	31	44
„ Calcium . .	$CaCl_2$	2.2	.	..	720	.	.164	.	42	60
„ „ (crystals)	$CaCl_2.6H_2O$. .	1.6	.	.0006	29	.	.4+	40	83	100
„ Carbon . .	C_2Cl_6	2.0	.	..	187	187	.2	.	0+	.
„ Copper . .	Cu_2Cl_2	3.5	.	..	434	1000	.138	.	0	.
„ Iron . . .	$FeCl_3$.	2.5	.	..	300	.	.	.	47	.

Name	Symbol	Density	Hardness	Coefficient Expansion Cubic 0—100°	Melting Point	Boiling Point, 76 cm	Specific Heat 0—100°	Latent Heat Melting	Solubility at 20° in %	Solubility at 100° in %
Chloride of										
,, Lead . . .	$PbCl_2$	5.8	.	. .	500	900	.067	.	1	5
,, Lithium . .	$LiCl$	2.0	.	. .	600	.	.282	.	45	57
,, Magnesium .	$MgCl_2$	2.2	.	. .	700	.	.194	.	70?	.
,, Mercury . .	$HgCl_2$	5.4	.	. .	290	300	.067	.	7	35
,, ,, (calomel)	Hg_2Cl_2 . . .	7.1	.	. .	.	.	.052	.	0	.
,, Potassium .	KCl	2.0	.	.000114	730	.	.172	.	26	36
,, & platinum .	K_2PtCl_6 . . .	3.5	.	. .	.	.	.113	.	1	5
,, Rubidium .	$RbCl$	2.2	.	. .	.	.	.112	.	.	.
,, Silver . . .	$AgCl$	5.6	.	.00010	450	.	.091	.	0	0
,, Sodium . .	$NaCl$	2.1	2	.000121	775	.	.214	.	27	28
,, Strontium .	$SrCl_2$	3.0	.	. .	850	.	.120	.	35	50
,, Tin	$SnCl_2$	.	.	. .	250	625	.102	.	67?	.
,, ,, (crystals)	$SnCl_2.2H_2O$.	2.7	.	. .	.	.	.	.	80?	.
,, Zinc . . .	$ZnCl_2$	2.8	.	. .	260	700	.136	.	80	.
Chromate of										
,, Lead . . .	$PbCrO_4$ . . .	5.9	.	. .	.	.	.090	.	0	.
,, Potassium .	K_2CrO_4 . . .	2.7	.	. .	.	.	.187	.	39	45
,, ,, Bi- . . .	$K_2Cr_2O_7$ . . .	2.7	.	. .	.	.	.188	.	11	50
,, Sodium . .	Na_2CrO_4 . . .	2.7	.	. .	.	.	.	.	.	.
,, ,, Bi- . . .	$Na_2Cr_2O_7$ . .	.	.	. .	.	.	.	.	60?	.
Cyanide of										
,, Mercury . .	HgC_2N_2 . . .	4.0	.	. .	.	.	.100	.	12	35
,, Potassium .	KCN	1.5	.	. .	.	.	.	.	.	55
,, ,, Ferri- . .	$K_6Fe_2C_{12}N_{12}$.	1.8	.	. .	.	.	.233	.	30	44
,, ,, Ferro . .	$K_4FeC_6N_6.3H_2O$	1.9	.	. .	.	.	.280	.	25	50
Fluoride of										
,, Calcium . .	CaF_2	3.2	4	.00004?	900	.	.212	.	0	0
Hyposulphite of							anh.			
,, Barium . .	$BaS_2O_3.H_2O$.	3.4	. .	. .	.	.	.163	.	0+	.
,, Lead . . .	PbS_2O_3 . . .	.	.	. .	.	.	.092	.	0+	.
,, Potassium .	$K_2S_2O_3$ . . .	.	.	. .	.	.	.197	.	sol	.
,, Sodium . .	$Na_2S_2O_3$ . . .	.	.	. .	.	.	.221	.	41	100
,, ,, (crystals)	$Na_2S_2O_3.5H_2O$	1.7	.	.00013	30?	.	.445	.38	64	.
Iodide of										
,, Copper . .	Cu_2I_2	4.4	.	. .	600	760	.069	.	.	.
,, Lead . . .	PbI_2	6.2	.	.000101	380	900	.043	.	0.1	0.5
,, Mercury . .	HgI_2	6.1	.	.000072	250	350	.042	.	5?	.
,, ,, (mercurous)	Hg_2I_2	9.7	.	. .	290	310	.039	.	0	.
,, Potassium .	KI	3.1	.	.000128	640	.	.082	.	59	67?
,, Silver . . .	AgI	5.7	.	—.000004	530	.	.062	.	0	0
,, Sodium . .	NaI	3.6	.	. .	630	.	.088	.	64	76
Naphthalene	$C_{10}H_8$	1.2	.	. .	80	215	.	36	0	0+
Nitrate of										
,, Ammonium .	H_4NNO_3 . . .	1.7	.	. .	150	.	.455	.	67?	.
,, Barium . .	BaN_2O_6 . . .	3.2	.	. .	600	.	.150	.	8	26
,, Lead . . .	PbN_2O_6 . . .	4.4	.	. .	.	.	.114	.	36	58
,, Potassium .	KNO_3	2.1	2	. .	340	.	.235	49	24	71

Name	Symbol	Density	Hardness	Coefficient Expansion Cubic 0—100°	Melting Point	Boiling Point, 76 cm	Specific Heat 0—100°	Latent Heat Melting	Solubility at 20° in %	Solubility at 100° in %
Nitrate of										
„ Silver . . .	$AgNO_3$	4.3	.	..	210	.	.144	.	70	90
„ Sodium . .	$NaNo_3$	2.2	2—	..	310	.	.270	65	47	64
„ Strontium .	SrN_2O_6	2.9	.	..	650	.	.181	.	42	50
Oxalate of										
„ Potassium .	$K_2C_2O_4.H_2O$.	.	.	..	.	.	.236	.	25	40
„ „ Tetr- . .	$KH_3C_4O_8.2H_2O$.	.	.	..	.	.	.283	.	5	.
Oxide of										
„ Aluminum .	Al_2O_3	3.9	9	..	.	.	.198	.	0	0
„ Antimony .	Sb_2O_3	5.5	.	..	.	.	.093	.	0+	0+
„ Arsenic . .	As_2O_3	3.7	.	..	.	.	.128	.	2	8?
„ Bismuth . .	Bi_2O_3	8.1	.	..	.	.	.061	.	0	.
„ Boron . . .	B_2O_3	1.8	.	..	580	.	.237	.	2	.
„ Calcium . .	CaO	3.1	.	..	.	.	.	.	.1+	.1—
„ „ (hydrate)	CaO_2H_2	2.1	.	..	.	.	.	.	.1+	0.1
„ Chromium	Cr_2O_3	5.0	.	..	.		.177	.	0	.
„ Copper . .	CuO	6.4	.	..	.	.	.135	.	0	0
„ „ (cuprous)	Cu_2O	6.0	4—	..	.	.	.111	.	0	0
„ Iron . . .	Fe_2O_3	5.2	5+	.00004	.	.	.16	.	0	0
„ Lead . . .	PbO	9.3	.	..	.	.	.051	.	0+	.
„ Magnesium .	MgO	3.3	.	..	.	.	.244	.	0	0
„ „ (hydrate)	MgO_2H_2	.	.	..	.	.	.312	.	0+	.
„ Manganese .	MnO	5.1	.	..	.	.	.157	.	0	.
„ „ Per- . .	MnO_2	5.0	2+	..	.	.	.159	.	0	.
„ Mercury . .	HgO	11.	.	..	.	.	.052	.	0	0+
„ Molybdenum	MoO_3	4.4	.	..	.	.	.154	.	0.2	0.1
„ Nitrogen . .	N_2O_5	.	.	..	30	.	.	77	80?	.
„ Potassium .	K_2O	2.6	.	..	.	47	.	.	50	.
„ „ (hydrate)	KOH	2.0	.	..	.	.	.	.	67	.
„ Silicon . .	SiO_2	2.2	.	.00004	.	.	.19	.	0	0
„ Sodium . .	Na_2O	.	.	..	.	.	.	.	40	70
„ „ (hydrate)	$NaOH$.	2.1	.	..	.	.	.	.	60	.
„ Tin	SnO_2	6.9	6+	.00002	.	.	.091	.	0	.
„ Titanium . .	TiO_2	4.	.	.00003	.	.	.172	.	0	.
„ Tungsten. .	WO_3	6.8	.	..	.	.	.085	.	0	.
„ Zinc . . .	ZnO	5.7	.	..	.	.	.125	.	0	0
Phosphate of						.			(?)	
„ Calcium . .	CaP_2O_6	.	.	..	.		.199	.	0+	.
„ Lead . . .	$Pb_2P_2O_7$	.	.	..	.	.	.082	.	0	.
„ Potassium .	$K_4P_2O_7$	.	.	..	.	.	.191	.	sol	.
„ „ (acid) .	KH_2PO_4	2.3	.	..	.	.	.208	.	sol	.
„ Sodium . .	$Na_4P_2O_7$	2.4	.	..	900	.	.228	.	5	25
„ „ (acid). .	$Na_2HPO_4.12H_2O$	1.5			36	.	.454	.	20	
Silicate of										
„ Al etc. (clay)	$Al_2Si_2O_7.2H_2O$,etc.	2.	.	.00002?	.	.	.2+	.	0	0
„ Calcium . .	$CaSiO_3$	4.5	.	..	.	.	.178	.	0	0
„ Zirconium .	$ZrSiO_4$	.	7+	..	.	.	.132	.	0	0

Name	Symbol	Density	Hardness	Coefficient Expansion cubic. 0–100°	Melting Point	Boiling Point, 76 cm.	Specific Heat 0–100°	Latent Heat Melting	Solubility at 20° in %	Solubility at 100° in %
Sulphate of										
„ Ammonium	$(H_4N)_2SO_4$	1.8	.	. .	140	.	.350	.	43	50
„ Barium	$BaSO_4$	4.4	.	.00006	.	.	.110	.	0	0
„ Calcium	$CaSO_4$	3.0	3	. .	.	.	.19	.	0.2	0.2
„ „ (hydrat.)	$CaSO_4.2H_2O$	2.3	2—	. .	.	.	.26	.	0.2	0.2
„ Cobalt	$CoSO_4.7H_2O$	1.9	.	. .	.	.	.343	.	48	.
„ Copper	$CuSO_4$	3.6	.	. .	.	.	.184	.	19	43
„ „ (crystals)	$CuSO_4.5H_2O$	2.3	2+	. .	.	.	.30	.	30	67
„ Iron	$FeSO_4.7H_2O$	1.9	2	. .	.	.	.350	.	50	80
„ Lead	$PbSO_4$	6.3	.	. .	.	.	.083	.	0	0
„ Magnesium	$MgSO_4$	2.7	.	. .	.	.	.225	.	26	40
„ „ (hydrat.)	$MgSO_4.7H_2O$	1.7	.	. .	.	.	.38	.	55	87
„ Manganese	$MnSO_4$	3.0	.	. .	.	.	.18	.	31	.
„ „ (hydrat.)	$MnSO_4.5H_2O$	2.1	.	. .	.	.	.33	.	50	.
„ Nickel	$NiSO_4$	.	.	. .	.	.	.216	.	28	.
„ „ (hydrat.)	$NiSO_4.7H_2O$	2.0	.	. .	.	.	.341	.	52	.
„ Potassium	K_2SO_4	2.6	.	. .	.	.	.193	.	10	21
„ „ (acid)	$KHSO_4$	2.3	.	. .	205	.	.244	.	32	53
„ „ & Al (alum)	$K_2Al_2S_4O_{16}.24H_2O$	1.7	2+	. .	.	.	.371	.	13	78
„ „ & Cr.	$K_2Cr_2S_4O_{16}.24H_2O$	1.8	.	. .	.	.	.324	.	14	—
„ Sodium	Na_2SO_4	2.7	.	. .	900	.	.230	.	30?	25?
„ „ (crystals)	$Na_2SO_4.10H_2O$	1.5	.	. .	.	.	.	.	60?	.
„ Strontium	$SrSO_4$	3.7	.	.00006	.	.	.140	.	0	0
„ Zinc	$ZnSO_4$	3.5	.	. .	.	.	.174	.	35	51
„ „ (hydrat.)	$ZnSO_4.7H_2O$	2.0	2+	. .	.	.	.34	.	62	87
Sulphide of										
„ Antimony	SbS_3	4.5	.	. .	.	.	.084	.	0	.
„ Bismuth	Bi_2S_3	7.4	.	. .	.	.	.060	.	0	.
„ Copper	CuS	4.0	.	. .	.	.	.	.	0	.
„ „ (cuprous)	Cu_2S	5.6	3—	. .	.	.	.121	.	0	.
„ „ & iron	$CuFeS_2$	4.2	4—	. .	.	.	.131	.	0	.
„ Iron	FeS	4.8	.	. .	.	.	.136	.	0	.
„ „ Bi-	FeS_2	5.0	6+	.00003	.	.	.128	.	0	.
„ Lead	PbS	7.5	2+	.00007	.	.	.050	.	0	.
„ Mercury	HgS	7.9	2+	. .	.	.	.051	.	0	.
„ Nickel	NiS	4.6	.	. .	.	.	.128	.	0	.
„ Potassium	K_2S	2.1	.	. .	.	.	.	.	50?	.
„ Silver	Ag_2S	7.2	2+	. .	.	.	.075	.	0	.
„ Tin	SnS	5.0	.	. .	.	.	.084	.	0	.
„ „ Bi-	SnS_2	4.5	.	. .	.	.	.119	.	0	.
„ Zinc	ZnS	4.1	4—	.000036	.	.	.122	.	0	.
Talc	$3MgO,4SiO_2.H_2O$	2.7	1	. .	.	.	.	.	0	.
Tartrate of										
„ Potass. (acid)	$KHC_4H_4O_6$	.	.	. .	.	.	.	.	0.6	6
„ „ & sodium	$KNaC_4H_4O_6.4H_2O$	.	.	. .	50	.	.33	.	50?	.

Name	Symbol	Density	Hartness	Coefficient Expansion cubical, 0°—100°	Melting Point	Boiling Point	Specific Heat	Latent Heat of Melting	Heat Conductivity	Specific Inductive Capacity	Index of Refraction (D) Minimum	Index Refract. Medium (D) or Ordinary	Index of Refraction (D) Maximum	Index of Dispersion A—H
Agate	$Si O_2$	2.6	7	..	..	..	.19?	..	..	..	..	1.540	..	..
Albumen	$C_{44} H_{70} N_{11} O_{14}$ (?) etc.	1.+	..	..	..	..	..	..	..	..	..	1.36	..	..
Alum	$Al_2 K_2 S_4 O_{16}.24 H_2 O$	1.7	2.+	..	..	..	.371	..	..	..	..	1.456	..	.018
„ chrome	$Cr_2 K_2 S_4 O_{16}.24 H_2 O$	..	..	..	..	..	.324	..	..	..	..	1.481	..	..
„ iron	$Fe_2 K_2 S_4 O_{16}.24 H_2 O$	..	..	..	..	..	..	..	..	..	..	1.482	..	..
„ „ & ammonium	$Fe_2 (H_4 N)_2 S_4 O_{16}.24 H_2 O$	..	..	..	..	..	..	..	..	..	..	1.485	..	..
„ selenium (?)*	$Al_2 K_2 Se_4 O_{16}.24 H_2 O$?	1.97	..	..	..	..	..	..	..	..	..	1.480	..	..
„ thallium	$Tl_2 K_2 S_4 O_{16}.24 H_2 O$	..	..	..	..	..	..	..	..	..	..	1.489	..	..
Amber	$C_{10} H_{16} O$ (?)	1.+	2.+	..	..	..	..	..	..	..	..	1.532	..	..
Amethyst	$Si O_2$	2.7	7	..	..	..	.19?	..	..	..	..	1.544	1.553	..
Anatase	$Ti O_2$	3.8	..	..	..	..	..	..	..	..	2.496	2.535	..	..
Anglesite	$Pb S O_4$	6.3	..	..	..	..	..	..	..	..	1.877	1.882	1.894	.065?
Apatite	$Ca_5 P_3 O_{12} F + Ca_5 P_3 O_{12} Cl$	3.2	5	..	..	..	..	..	..	..	1.638	1.642	..	..
Aragonite	$Ca C O_3$	2.94	4.+	.000065	..	..	.205	..	..	..	1.530	1.682	1.686	.033
Blende (zinc)	$Zn S$	4.0	4.—	..	..	..	..	..	..	..	..	2.369	..	..
Borax	$Na_2 B_4 O_7.10 H_2 O$	1.7	..	..	..	..	.385	..	..	..	1.446	1.468	1.471	..
Bromide of Potassium	$K Br$	2.7	..	.00013?	700	..	.113	..	..	..	..	1.559	..	..
„ „ Silver	$Ag Br$	6.5	..	00010	430	..	.074	..	..	..	..	2.253	..	.20?
Calc spar †	$Ca C O_3$	2.71	3	.000018	..	..	..	..	..	..	1.486	1.658	..	.033
Camphor	$C_{10} H_{16} O$ (?)	1.—	..	..	175	205	..	..	..	..	..	..	..	..
Canada Balsam	....	1.07	..	..	..	..	..	..	..	..	..	1.53	..	.02?

* See Landolt and Börnstein, Table 95.

† Same as Iceland Spar.

Name	Symbol	Density	Hardness	Coefficient Expansion cubical, 0°—100°	Melting Point	Boiling Point	Specific Heat	Latent Heat of Melting	Heat Con-ductivity	Specific Inductive Capacity	Index of Refraction (D) Minimum	Index Refract. Medium (D) or Ordinary	Index of Refraction (D) Maximum	Index of Dispersion A—H
Caoutchouc	. . . .	0.95	..	...	..	..	..	..	..	2.2	...	...	...	..
Carbonate of Lead	$Pb\,C\,O_3$	6.5	..	...	..	..	.079	..	..	..	1.804	2.076	2.078	.10?
Celestine	$Sr\,S\,O_4$	3.9	3+	.000061	..	..	.140	..	..	..	1.622	1.624	1.631	.021?
Chlorate of Sodium	$Na\,Cl\,O_3$	2.29	..	...	300	..	..	..	..	..	...	1.515	...	..
Chloride of Silver	$Ag\,Cl$	5.6	..	.00010	450	..	.091	..	..	..	...	2.061	...	.12?
Chromate Magnesium	$Mg\,Cr\,O_4$	..	..	...	..	..	..	..	..	..	1.521	1.550	1.568	..
" Potassium	$K_2\,Cr\,O_4$	2.7	..	...	..	..	.189	..	..	..	...	1.725	...	..
Chrysoberyl	$Be\,Al_2\,O_4$	3.7	8+	...	..	..	..	..	..	..	...	...	...	..
Chrysolite	$Mg\,Si\,O_3$ (?)	3.4	7—	...	..	..	..	..	..	..	...	...	...	..
Coal*	C, 75—95 %	1.3	2+	.00006	..	..	.3?	..	.001?	..	...	...	...	..
Cyanide Potas., Ferri-	$K_6\,Fe_2\,C_{12}\,N_{12}$	1.8	..	...	..	..	.23	..	..	..	1.566	1.569	1.583	..
" " Ferro-	$K_4\,Fe\,C_6\,N_6 . 3\,H_2O$	1.9	..	...	..	..	.28	..	..	..	...	1.575	1.581	..
Diamond	C (?)	3.5	10	.000004	..	..	.16	..	..	..	...	2.5	...	.06
Ebonite	. . . .	..	..	...	..	..	..	..	..	2—3	...	...	...	..
Emerald	$Be_3\,Al_2\,Si_6\,O_{18}$	2.7	8	...	..	..	..	..	..	..	...	1.58?	...	..
Feldspar ("Adular")	$Si\,O_2$, K_2O, etc.	2.5	6	...	..	..	..	..	..	..	...	...	...	..
Fluor Spar	$Ca\,F_2$	3.2	4	.000060	900	..	.21	..	.010	..	...	1.434	...	.011
Garnet	SiO_2, Al_2O_3, CaO, etc.	3.8	7±	...	..	..	..	..	..	..	...	...	...	..
	SiO_2 | PbO | K_2O | Na_2O													
Glass (crown) . . . %	59 | 9 | 21 | 3	2.5	..	.000023	..	..	.19	..	..	6+	...	1.53	...	.023
" (flint)	55 | 37 | 6 | 1	3.6	..	.000023	..	..	.19?	..	..	7+	...	1.61	...	.043

* The density of coal varies from 1.2 to 1.5; that of coal with air spaces varies from .8 to 1.1.

Name	Symbol	Density	Hardness	Coefficient Expansion Cubical $0^\circ-100^\circ$	Melting Point	Boiling Point, 76 cm	Specific Heat $0^\circ-100^\circ$	Latent Heat of Melting	Heat Conductivity	Specific Inductive Capacity	Index of Refraction (D) Minimum	Index Refract Medium (D) or Ordinary	Index of Refraction (D) Maximum	Index of Dispersion A—H
Heavy Spar	$Ba\,SO_4$	4.48	3+	.000058	..	..	.110	..	..	..	1.636	1.638	1.648	.021
Ice*	H_2O	.917	1.5	.0001 ?*	0	100	.50	79	.002	..	..	1.310	1.311	.013?
Iodide of Ammonium	H_4NI	2.4	..	...	..	..	..	..	..	..	..	1.703	..	..
" " Potassium	KI	3.07	..	.00013	650	..	.082	..	..	..	..	1.667	..	..
" " Silver	AgI	5.6	..	—.000004	..	..	.062	..	..	..	..	2.182	..	.30?
Ivory		1.9	..	...	..	..	..	..	..	..	..	1.539	1.541	..
Mica	$K_2O, Al_2O_3, Si\,O_2$, etc.	2.8	2+	...	..	..	.2?	..	..	..	1.561	1.594	1.600	..
Nitrate of Barium	$Ba\,N_2O_6$	3.2	..	...	..	..	..	..	..	..	..	1.571	..	.08?
" " Lead	$Pb\,N_2O_6$	4.4	..	...	..	..	.114	..	..	..	..	1.782	..	..
" " Potassium	KNO_3	2.0	2	...	340	..	.235	..	..	..	1.335	1.506	1.506	..
" " Sodium	$Na\,NO_3$	2.2	..	...	320	..	.270	..	..	..	1.337	1.586	..	..
Paraffine	$C_{20}?\,H_{42}?$ + etc.	0.90	..	.001+	55	400	..	..	..	3	..	..	..	..
Phosphorus	P	1.8	..	.0003	44	289	.20	5.0	..	..	..	2.14	..	.20?
Quartz	$Si\,O_2$	2.65	7	.000040	..	..	.186	..	.005	..	..	1.544	1.553	.019
Realgar	As_2S_2	3.5	..	...	..	..	..	..	..	..	..	2.4?	..	..
Resin		1.07	..	.0002	..	..	..	..	.0003	2	..	1.55?	..	..
Rochelle Salt	$KNa\,C_4H_4O_6.4H_2O$	..	..	...	..	..	..	..	..	..	1.491	1.495	1.498	..
Rock Salt	$Na\,Cl$	2.2	2	.00012	800	..	.219	..	.011	..	..	1.544	..	.031
Ruby	Al_2O_3	4.0	9	...	..	..	.22?	..	..	..	..	1.78?	..	..
Sapphire	Al_2O_3	3.8	9	...	..	..	.217	..	..	..	..	1.79?	..	..
Selenite†	$Ca\,SO_4.2H_2O$	2.3	2—	...	..	..	.26	..	..	..	1.521	1.523	1.530	.014

* The coefficient of expansion of ice is quoted by Landolt and Börnstein as **negative** according to Schumacher, **positive** according to four other observers. † Gypsum.

Name	Symbol	Density	Hardness	Coefficient Expansion Cubical 0°—100°	Melting Point	Boiling Point, 76 cm	Specific Heat 0°—100°	Latent Heat of Melting	Heat Conductivity	Specific Inductive Capacity	Index of Refraction (D) Minimum	Index Refract. Medium (D) or Ordinary	Index of Refraction (D) Maximum	Index of Dispersion A—H
Selenium (crystals)	Se	4.6	..	.0002	217	620	.084	..	...	...	...	2.98	...	1.3?
Shellac		1.+	..	...	..	...	...	..	...	3	...	...	...	...
Spermaceti		0.91	..	...	44	...	...	..	...	1.5?	...	1.54	...	...
Spinel	$MgAl_2O_4$	3.8	8	...	..	...	...	..	...	...	...	1.715	...	...
Sugar (crystals)	$C_{12}H_{22}O_{11}$	1.59	..	...	170	...	.30	..	...	...	1.537	1.565	1.570	.017?
Sulphate* of Copper	$CuSO_4.5H_2O$	2.27	2+	...	..	...	.30	..	...	...	1.515	1.538	1.545	.024?
" Magnesium	$MgSO_4.7H_2O$	1.7	..	...	..	...	.38	..	...	...	1.432	1.455	1.461	...
" Nickel	$NiSO_4.7H_2O$	2.0	..	...	..	...	.34	..	...	...	1.467	1.489	1.492	...
" Potassium	K_2SO_4	2.6	..	...	..	...	.193	..	...	...	1.493	1.495	1.498	...
" Zinc	$ZnSO_4.7H_2O$	2.0	..	...	..	...	.33	..	...	...	1.457	1.480	1.484	...
Sulphur	S	2.0	..	.0002	..	...	.17	9.4	...	3+	1.951	2.038	2.241	...
Tallow		0.95	..	...	40	...	...	..	...	...	...	1.49	...	...
Tartar Emetic	$K(SbO)C_4H_4O_6.H_2O$	..	..	...	..	...	...	..	...	...	1.620	1.636	1.638	...
Tartaric Acid	$H_2C_4H_4O_6$	1.75	..	...	135	...	.29	..	...	...	1.495	1.535	1.605	...
Thallium (prisms)	Tl (?)	11.8	..	.0001	290	700?	.034	..	...	...	...	1.752	...	.07?
Topaz	$Al_{12}Si_6O_{25}F_{10}$ (?)	3.5	8	...	..	...	...	..	...	...	1.612	1.614	1.621	.017
Tourmaline	Al_2O_3, SiO_2, etc.	3.0	7+	...	..	...	...	..	...	...	1.626	1.648	...	...
Varnishes		1.1	..	...	..	...	...	..	...	...	...	1.53	...	...
Vulcanite		1.±	..	...	..	...	...	..	...	3	...	...	...	...
Wax		0.96	..	.001	63	...	.5	42	.0001	2?	...	1.54	...	...
Zircon	$ZrSiO_4$	4.4	7+	...	..	...	.132	..	...	...	...	1.92	1.97	...

* See Alums.

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point, 76cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat, 0°–100°	Latent Heat Vaporization	Heat Conductivity	Spec. Induct Capacity	Index of Refraction (D)	Index of Dispersion A–H.
Multiply by	. . . .	..	..	..	10^{12}	...	..	..	..	10^6	10^6	..	..	....	..	..	..
Acetate of Amyl	$C_5H_{11}OC_2H_3O$	0.896	.0071	..	..	.00115	..	150	..	..	..	..	..	.00030	..	1.404	.017
" " Ethyl	$C_2H_5OC_2H_3O$	0.909	.0035	..	..	.00127	..	75	245	43	..	.56	100	.00035	..	1.373	.016
" " Isobutyl	$(CH_3)_2(C_2H_3O)_2$	0.817	..	..	..	.00103	..	108	296	..	..	..	..	...	..	..	..
" " Methyl	$CH_3OC_2H_3O$	0.956	.0032	..	..	.00129	..	58	233	59	..	.51	110	.00038	..	1.361	.015
" " Propyl	$C_3H_7OC_2H_3O$	0.90	.0046	..	..	.00111	..	..	282	..	..	..	..	.00033	..	1.384	.016
Acetone	$(CH_3)_2CO$	0.814	.0031	..	..	.00135	..	56	233	53	.240	.53	126	...	..	1.36	.017
Acid* Acetic	$HOC_2H_3O_2$	1.080	.0101	..	..	.00105	.17	117	222	..	.025	.53	120	.00047	..	1.374	.016
" " (anhyd.)	$(C_2H_3O)_2O$	1.097	..	..	..	.00105	..	138	..	..	..	..	66	...	..	1.390	.017
" Butyric	HOC_4H_7O	0.98	.013	..	..	.00103	..	160	..	..	.010	..	115	.00036	..	1.398	.017
" " Iso-†	$HO(CH_3)_2CHCO$	0.96	.01 ?	..	..	...	..	154 †	..	..	..	..	..	.00034	..	1.393	.017
" Formic	$HOCHO$	1.23	..	..	..	.00099	5	105	..	..	.042	..	115	.00065	..	1.371	.018
" Nitric	HNO_3	1.56	..	..	.031 ?	.00111	—47	..	..	..	..	..	115	...	..	..	..
" Propionic	HOC_3H_5O	1.016	.0094	..	..	.00110	..	142	340	..	.011	..	..	.00039	..	1.387	.017
" Sulphuric	H_2SO_4	1.84	..	..	.032 ?	.00059	10	330 ?	..	..	..	.34	122	.00077	..	1.43 ?	.017 ?
" Valeric	HOC_5H_9O	0.959	..	..	..	.00105	..	184	..	..	..	..	104	.00032	..	..	..
" " Iso-	$HO(CH_3)_2C_2H_3CO$	0.950	..	..	..	...	..	173	..	..	.008	..	..	.00031	..	1.404	.018
Alcohol, Amyl	$C_5H_{11}OH$	0.83	.04 ?	..	..	...	..	136	..	..	..	.65	121	.00033	..	1.41 ?	.018 ?
" " Benzyl	C_7H_7OH	1.063	..	..	..	.00079	..	207	..	..	..	..	..	...	..	1.540	.044
" " Butyl	C_4H_9OH	0.826	.023	..	..	...	..	115	..	..	..	..	..	.00034	..	1.399	.017
" (common)†† Ethyl	C_2H_5OH	0.810	..	25.4	.012	.00106	..	78.2	236	63	.058	.65	206	.00042	..	1.36	.015
" (wood) § Methyl	CH_3OH	0.817	..	..	..	.00114	..	66	..	..	.118	.65	264	.00050	..	1.33	.013
" " Propyl.	C_3H_7OH	0.81	.020	..	..	...	..	97	..	..	..	..	..	.00037	..	1.385	.016

* See Hydrochloric Acid. † Isobutyric acid boils at 205° acc. to Lossen. †† Ordinary (grain) alcohol. § Wood spirit.

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point, 76cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat, 0°–100°	Latent Heat Vaporizatin	Heat Conductivity	Spec. Induct Capacity	Index of Refraction (D)	Index of Dispersion A–H
Multiply	....	..	...	..	10^{12}	....	..	..	..	10^6	10^6	..	....	....	..	..	..
Aldehyde	CH_3CHO	0.805	...	..	..	.00160	..	22	..	..	..	..	136	...	..	1.332	.014
Aniline	$C_6H_5NH_2$	1.03	...	..	..	.00082	—8	183	..	..	..	..	..	.00041	..	1.58	.06+
Benzene	C_6H_6	0.899	.0052	..	.016	.00118	4	80	286	70?	.100	.44	92	.00033	2.2	1.500	.043
Benzoate of Ethyl	$C_2H_5OC_7H_5O$	1.066	...	..	..	.00093	..	213	..	..	..	..	..	...	..	1.506	.042
" " Methyl	$CH_3OC_7H_5O$	1.107	...	..	..	.00089	..	200	..	..	..	..	..	...	..	1.517	.045
Bromide of Amyl	$C_5H_{11}Br$	1.24	...	..	..	.00105	..	129	..	..	..	..	48	.00024	..	1.44?	.023
" " Antimony	$SbBr_3$	3.6	...	..	..	.00058	92	275	..	..	..	..	..	...	..	...	..
" " Ethyl	C_2H_5Br	1.473	.0031	..	..	.00134	..	40	236	..	.516	.22	62	.00025	..	1.424	.024
" " Ethylene	$C_2H_4Br_2$	2.17	...	..	..	.00099	10	131	..	..	.013	.17	44	...	..	1.538	.036
" " Methyl	CH_3Br	1.664	...	..	..	.00142	..	13	..	..	..	..	..	...	..	...	..
" " Phosphorus	PBr_3	2.925	...	..	..	.00084	..	175	..	..	..	..	..	...	..	...	..
" " Propyl	C_3H_7Br	1.38	...	..	..	...	..	71	..	..	..	..	..	.00026	..	1.434	.023
" " Silicon	$SiBr_4$	2.813	...	..	..	.00095	—14	150	..	..	..	..	..	...	..	...	..
Bromine	Br	3.19	...	74?	..	.00104	—7	61	..	..	..	.11	46	...	..	...	..
Bromobenzene	C_6H_5Br	1.52	.0096	..	..	...	..	155	..	..	..	..	..	.00027	..	1.560	.050
Butyrate of Ethyl	$C_2H_5OC_4H_7O$	0.903	.0053	..	..	.00119	..	117	304	..	..	..	..	.00032	..	1.396	.017
" " Methyl	$CH_3OC_4H_7O$	0.91	...	..	..	.00122	..	101	..	..	..	..	87	.00034	..	1.389	.017
" Propyl	$C_3H_7OC_4H_7O$	...	.0070	..	..	.00099	..	..	333	..	..	..	..	...	..	...	..
Carbonate of Ethyl	$(C_2H_5)_2CO_3$	1.000	...	..	..	.00117	..	126	..	..	..	..	..	...	..	1.385	.016
Chloral	CCl_3CHO	1.53	...	..	..	.00095	—75	99	..	..	..	..	54	...	..	1.456	.024
" hydrate	$CCl_3CHO . H_2O$	...	...	..	..	...	50	98	..	..	..	..	132	...	..	...	..

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point, 76cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat, 0°–100°	Latent Heat Vaporization	Heat Conductivity	Spec. Induct Capacity	Index of Refraction (D)	Index of Dispersion A–H.
Multiply by	. . . .	..	..	..	10^{12}	...	..	..	..	10^{6}	10^{6}	..	..	....	..	..	..
Chloride of Acetyl .	C_2H_3OCl . . .	1.130	..	..	..	.00131	..	53	..	..	..	..	..	...	..	1.390	.020
" Allyl	C_3H_5Cl	0.95	.0028	..	..	.00129	..	45	240	..	..	..	..	...	..	1.415	.025
" Amyl	$C_5H_{11}Cl$ . . .	0.891	..	..	..	.00117	..	102	..	..	..	..	56	.00028	..	...	..
" Antimony . .	$SbCl_3$	2.68	..	..	..	.00080	73	224	..	..	..	..	..	...	..	...	..
" Arsenic . . .	$AsCl_3$	2.205	..	..	..	.00098	..	130	..	..	..	.18	40?	...	..	...	..
" Boron	BCl_3	1.35	..	..	..	...	..	18	..	..	..	..	..	...	..	...	..
" Carbon, Tetra-	CCl_4	1.630	.0080	..	..	.00118	—25	77	285	60	.121	..	45	.00025	..	1.461	.024
" " Proto-	C_2Cl_4	1.649	..	..	..	...	..	122	..	..	..	.20	..	...	..	...	..
" Ethyl	C_2H_5Cl	0.921	..	..	..	.0015?	..	11	183	53	.132	.43	90	...	..	...	..
" Ethylene . . .	$(CH_2Cl)_2$ . . .	1.280	.007?	..	..	.00112	..	85	283	..	..	.31	..	...	..	1.444	.021
" Ethylidene . .	CH_3CHCl_2 . .	1.20	.0040	..	..	.0012	..	59	255	..	..	..	67	...	..	1.417	.020
" Isobutyl . . .	$(CH_3)_2C_2H_3Cl$.	0.895	.0038	..	..	...	..	68	..	..	..	..	..	.00028	..	...	..
" Phosphorus .	PCl_3	1.612	..	..	..	.00109	..	77	285	..	.134	.20	51	...	..	...	..
" Propyl	C_3H_7Cl	0.91	.0060	..	..	...	..	50	..	..	..	..	..	.00028	..	1.389	.017
" Silicon . . .	$SiCl_4$	1.524	..	..	..	.00129	..	59	..	..	.260	.19	..	...	..	...	..
" Sulphur . . .	S_2Cl_2	1.706	..	..	..	.00096	..	140	..	..	..	.20	49	...	..	...	..
" Tin, Tetra- .	$SnCl_4$	2.27	..	..	..	.00113	..	115	..	..	..	.14	31	...	..	...	..
" Titanium . .	$TiCl_4$	1.761	..	..	..	.00094	..	136	..	..	..	.18	..	...	..	...	..
Chlorobenzene . . .	C_6H_5Cl	1.12	..	..	..	...	—45	132	..	..	..	..	..	.00030	..	1.525	.044
Chloroform	$CHCl_3$	1.525	.0045	30	..	.0012?	—70	61	260	56	.215	.24	61	.00029	..	1.446	.022
Cyanide of Ethyl . .	C_2H_5CN . . .	0.801	..	..	..	...	..	97	..	..	..	.63	..	...	..	...	..
" " Methyl .	CH_3CN	0.835	..	..	..	.00121	..	75	..	..	..	..	..	...	..	...	..

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point, 76cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat, 0°—100°	Latent Heat Vaporization	Heat Conductivity	Spec. Induct Capacity	Index of Refraction	Index of Dispersion A—H.
Multiply by		..	..	..	10^{12}	...	..	..	..	10^{6}	10^{6}	..	..	...	..	..	..
Cyanide of Phenyl	C_6H_5CN	1.023	..	..	..	.00093	—17	191	..	..	..	..	..	...	..	..	..
Diethylamine	$(C_2H_5)_2NH$	..	..	..	..	.00136	..	57	220	39	..	..	..	...	..	..	..
Ether	$(C_2H_5)_2O$	0.73	.0019	20?	.009	.00148	..	35	193	38	.578	.54	91	.00030	3.3	1.353	.015
Formate of Ethyl	C_2H_5OCHO	0.94	.0032	..	..	.00134	..	54	230	50	..	..	105	.00038	..	1.36	.016
„ „ Isoamyl	$(CH_3)_2C_3H_5OCHO$	..	..	..	..	.00099	..	..	305	..	..	..	..	...	..	..	..
„ „ Methyl	CH_3OCHO	0.998	..	..	..	.00140	..	33	..	..	..	..	114	...	..	..	..
„ „ Propyl	C_3H_7OCHO	0.919	.0042	..	..	.00118	..	82	267	..	..	..	..	.00036	..	..	..
Glycerine	$C_3H_5O_3H_3$	1.270	..	..	.040	.0005	17	290	..	..	..	..	..	.00067	..	1.473	.019
Hydrochloric acid	HCl	0.90	..	..	..	...	low	—80	51	..	..	..	..	...	..	..	..
Iodide of Amyl	$C_5H_{11}I$	1.544	..	..	..	.00096	..	155	..	..	..	..	47	.00020	..	1.49?	.032?
„ „ Ethyl	C_2H_5I	1.975	..	..	..	.00114	..	71	..	..	.147	.17	47	.00022	..	1.513	.041
„ „ Isobutyl	$(CH_3)_2C_2H_3I$	1.64	.0069	..	..	...	..	121	..	..	..	..	..	...	..	1.496	.035
„ „ Methyl	CH_3I	2.20	.0041	..	..	.00120	..	44	..	..	..	..	46	.00021	..	1.530	.047
„ „ Propyl	C_3H_7I	1.78	.0060	..	..	...	..	103	..	..	..	..	..	.00022	..	1.505	.038
Iodine (melted)	I	..	..	..	..	...	110	200	..	..	..	..	24	...	..	..	..
Mercury *†	Hg	13.596	..	540	.5	.00018	—39	350	..	..	†	.034	62	...	..	..	..
Nitrobenzene	$C_6H_5NO_2$	1.21	.015?	..	..	.00083	3	210	..	..	..	.35	..	...	..	1.553	.063
Nitroglycerine	$C_3H_5(NO_3)_3$	1.6+	..	..	..	...	10	185	..	..	..	..	..	...	..	..	..
Oil, †† Bitter almond	C_6H_5CHO	1.064	..	..	..	.00094	..	179	..	..	..	..	..	...	..	1.546	.061
„ Olive		0.92	..	35	.021	.00080	..	..	..	..	..	..	..	...	3+	1.47	..
„ Rape		0.92	..	..	.021	...	..	..	..	..	..	..	..	...	2+	1.47	..
„ Sperm		0.92	..	..	..	...	..	..	..	..	..	..	..	...	3	1.46	..

* See Table 24. † See Table 13, C. †† Oils, see also Petroleum and Turpentine. Oil of Vitriol, see Acid, sulphuric

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point 76 cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat 0°–100°	Latent Heat Vaporization	Heat Con-ductivity	Spec. Induct. Capacity	Index of Refraction (D)	Index of Dispersion A–H
Multiply by	. . . .	. .	. .	.	10^{12}	. . .	.	. .	.	10^6	10^6	. .	.	. . .	.	. .	. .
Oxalate of Ethyl .	$(C_2H_5)_2O_2C_2O_2$	1.102	. .	.	.	.00107	.	185	.	.	. .	. .	73	. . .	.	1.410	.018
„ „ Methyl	$(CH_3)_2O_2C_2O_2$	1.16	. .	.	.	.00108	50	161	.	.	. .	. .	.	. . .	.	. .	. .
Petroleum (light) .	$C_8?H_{18}?$ etc.	0.7	. .	.	.	. .	.	110?	.	.	. .	. .	76?	. . .	2	1.4+	. .
„ (heavy)	$C_{13}?H_{28}?$ etc.	0.8+	. .	32	.016	.00090	.	220?	.	.	. .	. .	.	. . .	2	1.45	. .
Phenol	C_6H_5OH	1.08	. .	.	.	.00067	38	186	.	.	. .	. .	.	. . .	.	1.550	.049
Phosphorus (melted) .	P	1.76?	. .	.	.	. .	44.3	289	.	.	. .	. .	.	. . .	.	2.08	.16?
Propionate of Ethyl .	$C_2H_5OC_3H_5O$	0.923	.0045	.	.	.00129	.	98	280	.	. .	. .	.	. . .	.	. .	. .
„ „ Isobutyl	$(CH_3)_2C_2H_3OC_3H_5O$	0.893	.0068	.	.	.00101	.	137	320	.	. .	. .	.	. . .	.	. .	. .
„ „ Methyl .	$CH_3OC_3H_5O$	0.92	.0040	.	.	.00120	.	98	263	.	. .	. .	.	. . .	.	. .	. .
„ „ Propyl .	$C_3H_7OC_3H_5O$	0.902	.0059	.	.	.00103	.	123	305	.	. .	. .	.	. . .	.	. .	. .
Salicylate of Methyl .	$CH_3OC_7H_5O_2$	1.20	. .	.	.	.00084	.	224	.	.	. .	. .	.	. . .	.	1.537	.060
Succinate of Ethyl .	$(C_2H_5)_2O_2C_4H_4O_2$	1.05	. .	.	.	.00101	.	217	.	.	. .	. .	.	. . .	.	. .	. .
Sulphide Carbon. Bi-	CS_2	1.29	. .	32	.018	.00114	.	47	272	76	.397	.24	90	.00034	2.1	1.63	.091
„ Ethyl . . .	$(C_2H_5)_2S$	0.825	.0034	.	.	.00120	.	91?	.	.	. .	.48	.	.00033	.	1.442	.025?
Sulphur (melted) . .	S	. .	. .	.	.	. .	114	448	.	.	. .	. .	362	. . .	.	. .	. .
Toluene	$C_6H_5CH_3$	0.882	.0047	.	.	. .	.	111	320	.	. .	. .	.	.00031	.	1.496	.041
Turpentine	$C_{10}H_{16}$	0.88	. .	29	.017	.00071	—10	160	.	.	.006	.46	70	.00026	2.1	1.47	.024
Valeraldehyde. Iso- .	$(CH_3)_2C_2H_3CHO$	0.822	. .	.	.	.00119	.	93	.	.	. .	. .	.	. . .	.	1.39	.018?
Valerate of Amyl . .	$C_5H_{11}OC_5H_9O$	0.87	. .	.	.	.00103	.	189	.	.	. .	. .	.	. . .	.	1.412	.018
„ „ Ethyl . .	$C_2H_5OC_5H_9O$	0.88	.0061	.	.	. .	.	133	.	.	. .	. .	.	.00031	.	1.397	.017
„ „ Methyl .	$CH_3OC_5H_9O$	0.90	. .	.	.	.00112	.	116	.	.	. .	. .	.	.00032	.	1.395	.017
Water*†∓§	H_2O	1.000	.0140	80	.021	†	0	100∓	400?	.	.023§	1.005	536	.00170	.	1.333	.014

* See Table 25. † See Table 23. ∓ See Table 14. § See Table 13, C

Note, **Density of blood, 1.060; milk, 1.032; sea water, 1.026. Resilience of sea water, .022 × 10^{12}.**

Name	Symbol	Sp.Gr.ref.to Hydrogen	Density, δ^0, 1,000,000 dynes per sq. cm.	Resilience of Volume, δ^0, $p=10^6$	Coefficient Expansion 0–100°, 76 cm.	Temperature of Solidification	Temp. of Condensation 76 cm.	Critical Temperature	Critical Pressure	Specif. Heat Constant Pressure	Specif. Heat Const. Vol.	Latent Heat Condensation	Heat Conductivity	Specific Inductive Capacity	Index of Refraction δ^0, 76 cm. Line D	Index of Dispersion A—H	Solubility %, in Water, 20°, 76 cm.
Multiply by.	. . . .	..	...	10^6	...	..	..	..	10^6	..	..	..	.001	...	...	10^{-6}	..
Acetate of Ethyl . .	$C_2H_5OC_2H_3O$	44.3	...	..	...	..	75	245	43	.35	.3?	100	..	...	...	..	..
Acetylene	C_2H_2	13.3	.00117	..	...	..	low?	37	..	..	..	..	..	...	...	..	..
Acetone	$(CH_3)_2CO$ . .	29.2	solid	..	...	17	117	233	53	.35	.3?	126	..	...	...	..	..
Air*	77% N, 23% O	14.43	.001276	.999	.00367	..	low	—158	15	.238	.169	..	.055	1.0015	1.000293	7.5	.0022
Alcohol, Ethyl** . .	C_2H_5OH . .	23.3	liquid	..	...	..	78.2	236	63	45	.4?	206	..	...	1.00087?	35?	..
„ Methyl† . .	CH_3OH . . .	16.2	liquid	..	...	..	66	..	..	.45	.4?	264	..	...	...	..	..
Aldehyde	CH_3CHO . .	22.1	liquid	..	...	..	22	..	..	..	..	136	..	...	...	..	..
Ammoniac	H_3N	8.55	.000759	..	...	—75	—38	..	..	.50	.38	295	.050	...	1.000385	..	33
Arsenic	As_4	153	solid	..	...	subl.	450	..	..	..	..	..	..	...	1.0011?	..	..
Arsen d'Hydrogen .	H_3As	38.9	.0035	..	...	..	—58	..	..	..	..	..	..	...	...	..	..
Benzene	C_6H_6	40.0	solid	..	...	+4	80	286	70	.34	.3?	92	..	...	...	..	..
Bromide of Ethyl .	C_2H_5Br . . .	..	...	..	...	..	40	236	..	.16	.1?	62	..	...	...	..	..
„ „ Ethylene	$C_2H_4Br_2$ . . .	..	solid	..	...	10	131	..	..	..	..	44	..	...	...	..	..
Bromine	Br_2	79.5	liquid	..	...	—7	61	..	..	.055	.042	46	..	...	...	..	..
Carbonic Dioxide†† .	CO_2	22.1	.001951	.992	.00371	††	—80	31	78	.20	.15	48	.033	1.0023	1.000454	11	.18
„ Oxide . .	CO	13.9	.001218	.999	.00367	..	low	..	..	.24	.17	..	.054	...	1.000335	12	.0028
Chloride Arsenic . .	$AsCl_3$	90.9	...	..	...	..	130	..	..	.11	.1?	40?	..	...	...	..	..
„ Boron . . .	BCl_3	49.?	...	..	...	..	18	..	..	..	..	..	..	...	...	..	..
„ Carbon, Tetra-§	CCl_4	78.1	liquid	..	...	—25	79	285	60	..	..	45?	..	...	...	..	..
„ Ethyl . . .	C_2H_5Cl . . .	32.0	...	..	...	..	11	183	53	.27	.24?	90	..	...	...	..	..
„ Ethylene §§	$C_2H_4Cl_2$ . . .	49.7	...	..	...	..	85	283	..	.23	.2?	..	..	...	...	..	..
„ Iodine . . .	ICl	..	solid	..	...	27	101	..	..	.051	.039	..	..	...	...	..	..

* Air contains 79% N and 21% O by volume.
†† Carbonic Acid Gas. Solid at —58° acc. to Faraday.
** Ordinary (grain) alcohol.
§ Formerly called Bi-chloride.
† Wood Spirit.
§§ Dutch Liquid.

Name	Symbol	Sp. Gr. ref. to Hydrogen	Density 0° 1.000000 dynes per sq. cm.	Resilience of Volume 0°, p=10⁶	Coefficient Expansion 0°–100°, 76 cm	Temperature of Solidification	Temp. of Condensation 76 cm.	Critical Temperature	Critical Pressure	Specif. Heat Constant Pressure	Specif. Heat Const. Vol.	Latent Heat Condensation	Heat Conductivity	Specific Inductive Capacity	Index of Refraction 0°, 76 cm. Line D	Index of Dispersion A–H	Solubility % in Water 20°, 76 cm.
Multiply by		..	...	10^6	...	...	...	...	10^6	..	..	..	.001	...	...	10^{-6}	..
Chloride Methyl	CH_3Cl	25.0	...	..	...	...	22	...	..	..	..	..	..	...	...	..	..
" Phosphorus	PCl_3	70.3	...	..	...	...	77	285	..	.13	.1?	51	..	...	...	..	..
" Silicon	$SiCl_4$	85.7	...	..	...	...	59	...	..	.13	.1?	..	..	...	...	..	..
" Silver	$AgCl$	..	solid	..	...	453	...	...	..	..	..	..	..	...	...	..	..
" Sulphur	S_2Cl_2	68.0	...	..	...	...	140	...	..	..	..	49	..	...	...	..	..
" Tin, Stannic*	$SnCl_4$	133	...	..	...	...	115	...	..	.094	.08?	31	..	...	...	..	..
" Titanium	$TiCl_4$	98.7	...	..	...	...	136	...	..	.13	.1?	..	..	...	...	..	..
Chlorine	Cl_2	35.6	.003091	..	...	...	—34	...	..	.12	.09	..	..	...	1.000772	..	.7
Chloroform	$CHCl_3$	..	liquid	..	...	—70	61	260	56	.14	.1?	61	..	...	1.00144	53	..
Coal-gas	H_2, CH_4, etc.	6?	.0005?	..	...	...	low?	...	..	..	..	..	.15?	1.0019	...	..	..
Cyanide Ethyl	C_2H_5CN	..	...	..	...	...	97	...	..	.43	..	..	..	...	...	..	..
Cyanogen	$(CN)_2$	26.1	.00230	.987	.00388	—35	—21?	...	..	..	..	..	..	...	1.000822	40	..
Ethane †	C_2H_6	..	gas	..	...	...	low?	...	..	..	..	..	..	...	...	..	.006
Ether	$(C_2H_5)_2O$	37.3	liquid	..	...	...	35	193	38	.43	.4?	91	..	...	1.00152?	53	..
Ethylene ††	C_2H_4	14.1	.001253	..	...	...	—110	+5	44	.39	.32	..	.041	...	1.000678	..	.02
" Oxide	C_2H_4O	20.5	...	..	...	...	13	...	..	..	..	139	..	...	...	..	..
Fluoride Boron	BF_3	34.2	.0030	..	...	...	low?	...	..	..	..	..	..	...	...	..	..
" Silicon	SiF_4	51.9	.0046	..	...	...	low?	...	..	..	..	..	..	...	...	..	..
Hydriodic Acid Gas	HI	64.1	.0057	..	...	—51	—40?	...	..	.055	.039	..	..	...	...	..	..
Hydrobromic " "	HBr	39.1	.0035	..	...	—87	—60?	...	..	.082	.057	..	..	...	...	..	..
Hydrochloric " "	HCl	18.0	.0017	..	...	—110	—80	51	..	.190	.135	..	..	...	...	..	..

* Tetra-formerly called Bi-chloride. † Bicarburetted Hydrogen. †† Olefiant Gas.

Name	Symbol	Sp.Gr.ref.to Hydrogen	Density, 0°, 1,000,000 dynes per sq. cm.	Resilience of Volume, 0°, p=10⁶	Coefficient Expansion 0–100°, 76 cm.	Temperature of Solidification	Temp. of Condensation 76 cm.	Critical Temperature	Critical Pressure	Specif.Heat Constant Pressure	Specif.Heat Const. Vol.	Latent Heat Condensation	Heat Conductivity	Specific Inductive Capacity	Index of Refraction 0°, 76 cm. Line D	Index of Dispersion A–H	Solubility % in Water, 20°, 76 cm.
Multiply by.	. . . .	..	...	10^6	...	..	..	..	10^6	..	..	..	.001	...	...	10^{-6}	..
Hydrocyanic Acid .	HCN	13.7	liquid	..	...	—15	26	...	..	..	..	..	..	...	...	..	..
Hydrofluoric „ .	HF	..	liquid	..	...	—34	20	...	..	..	..	..	..	...	...	..	..
Hydrogen	H_2	1.000	.0000884	1.001	.00366	low.	low.	—174	100	3.40	2.40	..	.39	1.0013	1.000139	2.8	.0002
Iodine	I_2	126	solid	..	...	110	200	...	..	.033	.025	24	..	...	...	..	..
Mercury*.	Hg	100.6	liquid	..	...	—39	350	...	..	..	..	62	..	...	1.00056	..	..
Methane**	CH_4	8.0+	.000717	..	...	..	low.	—76	47	.59	.47?	..	.076	...	...	..	.002
Nitric Anhydride. .	N_2O_5 . . .	..	solid	..	...	30	47	...	..	..	..	45	..	...	...	..	..
Nitric Oxide . . .	NO	15.0	.001325	..	...	..	low	...	..	.22	.16	..	.052	...	1.000298	..	.09
Nitrous Oxide† . .	N_2O	22.0	.001943	.988	.00372	—100	—90	...	..	.21	.16	101	.035	...	1.000516	30	—
Nitrogen	N_2	14.03	.001239	.999	.00367	..	low	—124	43	.24	.17	..	.054	...	1.000298	9.6	.0020
Oxygen	O_2	15.95	.001411	..	.00367	..	low	—105	49	.22	.16	..	.056	...	1.000271	8.3	.0040
Phosphorus	P_4	63.8	solid	..	...	44.3	289	...	..	..	..	..	..	...	1.0014?	..	...
Phosph'd Hydrogen .	H_3P	17.5	.0015+	..	...	..	low?	...	..	..	..	..	..	...	...	..	...
Sulphide Carbon, Bi-	CS_2	38.1	liquid	..	...	..	47	272	76	.16?	.13?	90	..	...	1.0015?	150?	...
„ Ethyl. . .	$(C_2H_5)_2S$ . .	..	...	..	...	..	91	...	..	.40	.38?	..	..	...	...	..	...
Sulphur††	S_6 (at 450°)	95.5	solid	..	...	114	448	...	..	..	..	362	..	...	1.0016?	..	...
Sulph'd Hydrogen .	H_2S	17.2	.0015	..	...	—86	—62?	...	..	.24	.18?	..	..	...	1.00064?	..	.44
Sulphuric Anhydride	SO_3	39.9	solid	..	...	16	46+	...	..	..	..	147	..	...	...	..	..
Sulphurous „ §.	SO_2	32.	.002699	.984	.00390	—78	—9	155	80	.15	.11?	92	..	1.0052	1.00070	..	..
Turpentine	$C_{10}H_{16}$ . . .	69.	liquid	..	...	—10	160	...	..	.5	..	70	..	...	...	..	9.
Water§§	H_2O	9.00	freezes	..	...	0	100	400?	..	.48	.37?	536	..	...	...	..	..

* See Table 13, C.
†† See Table 13, C.
** Marsh Gas.
§ Sulphurous Acid Gas.
† Laughing Gas.
§§ Aqueous Vapor, or Steam. See Tables 13, C–D.

18, A. Maximum Pressure of Vapors at Different Temperatures (−70° to +120°) in Megadynes per sq. c.

Name	Symbol	−70°	−60°	−50°	−40°	−30°	−20°	−10°	0°	+10°	+20°	+30°	+40°	+50°	+60°	+70°	+80°	+90°	+100	+110	+120
Acetylene	C_2H_2	..	..	..	..	..	12?	20?	30?	45?	60?	80?	..	..	..	..	..	..	..	..	..
Ammonia	H_3N	..	..	..	..	1.2	1.9	2.8	4.2	6.0	8.5	12	15	20	26	33	41	51	62	..	..
Arsen'd Hydrogen	H_3As	..	0.9	1.5	2	3	5	7	9	12	..	..	..	..	..	..	..	..	..	..	..
Carbonic Dioxide	CO_2	2	4	7	10	15	20	27	36	47	60	75	92	..	..	..	..	..	..	..	..
Chloride Boron ..	BCl_3	..	..	..	..	..	.23	.34	.51	.75	1.1	1.5	2.1	2.7	3.6	4.5	5.7	..	..	..	..
„ Phosphorus	PCl_3	..	..	..	..	..	..	..	.05	.08	.13	.21	.31	.46	.65	.90	..	..	..	..	..
„ Silicon	$SiCl_4$	..	..	..	..	..	.03	.06	.11	.17	.26	.39	.57	.81	1.1	..	..	..	..	..	..
Chlorine	Cl_2	..	..	..	..	1?	2?	2?	3?	4?	4?	..	..	..	..	..	..	..	..	..	..
Cyanogen	C_2N_2	..	..	..	..	..	1.2	1.7	2.4	3.3	4.4	6	8	..	..	..	..	..	..	..	..
Ethane	C_2H_6	..	..	..	..	..	..	..	46	(at	4°)	..	..	..	..	..	..	..	..	..	..
Ethylene	C_2H_4	5	7	10	14	19	25	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Fluoride Boron ..	BF_3	5	8	13	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Hydriodic Acid ..	HI	..	..	..	..	..	2.8	3.3	4.0	5	7	..	..	..	..	..	..	..	..	..	..
Hydrochloric „ ..	HCl	2	3	5	7+	10	14	29	27	36	45	55	68	85	..	..	..	..	..	..	..
Methylether	$CH_3OC_2H_5$	..	..	..	..	.77	1.2	1.7	2.5	3.4	4.8	6.4	..	..	..	..	..	..	..	..	..
Nitrous Oxide ...	N_2O	3	5	8	11	16	22	29	36	45	56	69	84	..	..	..	..	..	..	..	..
Sulph'd Hydrogen	H_2S	1.1	1.5	2	3	4	6	8	11	14	19	24	30	37	45	54	..	..	..	..	..
Sulphurous Anh.	SO_2	..	..	..	..	0.4	0.6	1.0	1.5	2.3	3.3	4.6	6.2	8.3	11	14	18	22	28	..	42

13 B. Maximum Pressure of Vapors at Different Temperatures (0°–190°) in Megadynes per sq. cm.

Name	Symbol	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100	110	120	130	140	150	160	170	180	190
Acetone	$(CH_3)_2CO$	..	..	.24	.38	.56	.81	1.2	1.6	2.2	2.9	3.7	4.8	6.1	7.6	9.3	..	..	..	..	..
Acid Acetic . .	HOC_2H_3O	.012	.016	.025	.039	.059	.088	.13	.19	.27	.39	.54	.76	1.04	1.42	1.91	..	..	..	..	..
" Butyric . .	HOC_4H_7O	..	.007	.010	.014	.019	.027	.038	.053	.074	.10	.14	.20	.27	.38	.52	.70	96	1.31	1.8	..
" Formic . .	HOCHO	..	.025	.042	.069	.11	.17	.26	.37	.53	.74	1.02	..	..	..	..	..	..	..	..	..
" Propionic .	HOC_3H_5O	..	.007	.011	.017	.026	.037	.057	.084	.12	.18	.26	.36	.52	.73	1.02	1.42	..	..	..	..
" Valeric, Iso-	HOC_5H_9O	..	.006	.008	.012	.016	.022	.030	.041	.056	.077	.11	.14	.20	.26	.36	.49	.66	.88	1.19	1.6
Alcohol Ethyl. .	C_2H_5OH	.017	.032	.059	.10	.18	.29	.47	.72	1.08	1.6	2.3	3.2	4.3	5.8	7.6	9.8	..	..	..	..
" Methyl .	CH_3OH	.036	.067	.12	.20	.33	.51	.77	1.14	1.7	2.3	3.2	4.4	5.8	7.6	9.8	13	..	..	..	..
Benzene	C_6H_6	.034	.060	.10	.16	.25	.36	.52	.73	1.00	1.4	1.8	2.3	3.0	3.8	4.7	5.8	7.0	8.5	..	..
Bromide Ethyl .	C_2H_5Br	.022	.035	.052	.075	.11	.14	.20	.27	.35	.45	.58	.72	.89	1.08	1.3	..	..	..	..	..
" Ethylene	$C_2H_4Br_2$	.005	.009	.014	.023	.037	.057	.088	.13	.19	.28	.39	.54	.73	.97	1.28	1.6	2.1	2.6	3.3	4.0
Chloride Carbon.	CCl_4	.044	.075	.12	.19	.29	.42	.60	.83	1.12	1.5	2.0	2.5	3.2	4.0	5.0	6.1	7.3	8.9	11	13
" Ethyl .	C_2H_5Cl	.62	.92	1.3	1.9	2.6	3.4	4.5	5.9	7.5	9.4	12	15	18	21	26	31	36	43	..	..
" Methyl .	CH_3Cl	2.5	3.5	4.9	6.6	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Chloroform . .	$CHCl_3$	..	..	.21	.33	.48	.72	1.01	1.4	1.9	2.5	3.2	4.2	5.2	6.5	8.0	9.7	12	..	..	..
Ether	$(C_2H_5)_2O$	.25	.38	.58	.85	1.2	1.7	2.3	3.1	4.0	5.2	6.6	8.3	10	..	..	..	..	..	..	..
Iodide Ethyl . .	C_2H_5I	.056	.093	.15	.23	.34	.49	.68	..	..	..	..	..	..	..	..	..	..	..	..	..
Sulphide Carbon, Bi-	CS_2	.17	.27	.40	.58	.82	1.14	1.6	2.1	2.7	3.5	4.4	5.5	6.9	8.4	10	12	..	..	..	..
Turpentine . . .	$C_{10}H_{16}$	.003	.004	.006	.009	.014	.023	.035	.054	.082	.12	.18	.25	.34	.47	.62	.81	1.03	1.3	1.6	2.0

13C. Pressure of the Vapors of Mercury, Sulphur and Water in Megadynes per sq. cm (0°—600°).

Temperature	0°	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190°
Mercury Hg	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0004	.0006	.0010	.0015	.0022	.0033	.0045	.007	.010	.013	.018
Temperature	200°	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	360	370	380	390°
Mercury Hg	.025	.034	.045	.060	.078	.101	.129	.165	.208	.260	.323	.400	.492	.602	.732	.885	1.06	1.27	1.52	1.80
Temperature	400°	410	420	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	590°
Mercury Hg	2.12	2.48	2.90	3.38	3.91	4.51	5.19	5.93	6.77	7.68	8.97	9.81	11.1							
Sulphur S.	.439	.529	.630	.748	.884	1.04	1.22	1.42	1.64	1.90	2.18	2.50	2.84	3.23	3.65	4.12	4.63	5.17		
Temperature	0°	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190°
Aqueous Vapor from Sulphuric Acid. 85 % $H_2SO_4.H_2O$		.0001	.0002	.0003																
73 % $H_2SO_4.2H_2O$		.0007	.0011	.0020																
52 % $H_2SO_4.5H_2O$		.0040	.0077	.0142																
33 % $H_2SO_4.11H_2O$		.0085	.0161	.0295																
Water H_2O	.006	.012	.023	.042	.073	.123	.198	.310	.472	.701	1.014	1.44	2.0	2.7	3.6	4.8	6.2	8.0	10.1	12.6

13D. Density of Steam saturated at Different Temperatures.

Temperature	0°	10	20	30	40	50	60	70	80	90	100°
Density	.000005	.000009	.000017	.000030	.000051	.000083	.000131	.000199	.000296	.000428	.000606
Temperature	100°	110	120	130	140	150	160	170	180	190	200°
Density	.000606	.000840	.00114	.00153	.00201	.00260	.00333	.00421	.00526	.00650	.00796

14. Boiling Points of Water at Different Pressures (g = 980.61).

Barometric Pressure, cm.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
68	96.92	96.96	97.00	97.05	97.09	97.13	97.17	97.21	97.25	97.29
69	97.33	97.36	97.40	97.44	97.48	97.52	97.56	97.60	97.64	97.68
70	97.72	97.76	97.80	97.84	97.88	97.92	97.96	98.00	98.03	98.07
71	98.11	98.15	98.19	98.23	98.27	98.31	98.34	98.38	98.42	98.46
72	98.50	98.54	98.58	98.61	98.65	98.69	98.73	98.77	98.80	98.84
73	98.88	98.92	98.96	98.99	99.03	99.07	99.11	99.14	99.18	99.22
74	99.26	99.30	99.33	99.37	99.41	99.44	99.48	99.52	99.56	99.59
75	99.63	99.67	99.71	99.74	99.78	99.82	99.85	99.89	99.93	99.96
76	100.00	100.04	100.07	100.11	100.15	100.18	100.22	100.26	100.29	100.33
77	100.36	100.40	100.44	100.47	100.51	100.55	100.58	100.62	100.65	100.69

14A. Dew Points corresponding to Different Degrees of Temperature and Relative Humidity.

Temperature of the Air.	Relative Humidity of the Air. 10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
0°	..	..	−16	−12	−9	−7	−5	−3	−1	0°
1	..	..	15	11	8	6	4	2	0	+1
2	..	−19	14	10	7	5	3	−1	+1	2
3	..	18	13	9	6	4	2	0	2	3
4°	..	−17	−12	−8	−6	−3	−1	+1	+3	+4
5°	..	−16	−11	−7	−5	−2	0	+2	+3	+5°
6	..	15	10	7	4	−1	+1	3	4	6
7	..	15	9	6	3	0	2	4	5	7
8	..	14	−9	5	2	+1	3	5	6	8
9°	..	−13	−8	−4	−1	+2	+4	+6	+7	+9°
10°	..	−12	−7	−3	0	+3	+5	+7	+8	+10°
11	..	11	6	2	+1	3	6	8	9	11
12	−19	10	5	−1	2	4	7	9	10	12
13	18	10	4	0	3	5	8	10	11	13
14°	−17	−9	−3	+1	+4	+6	+9	+11	+12	+14°
15°	−17	−8	−3	+2	+5	+7	+10	+12	+13	+15°
16	16	7	2	2	6	8	11	13	14	16
17	15	6	−1	3	6	9	11	14	15	17
18	14	5	0	4	7	10	12	14	16	18
19°	−13	−5	+1	+5	+8	+11	+13	+15	+17	+19°
20°	−13	−4	+2	+6	+9	+12	+14	+16	+18	+20°
21	12	3	3	7	10	13	15	17	19	21
22	11	2	4	8	11	14	16	18	20	22
23	10	−1	4	9	12	15	17	19	21	23
24°	−10	0	+5	+10	+13	+16	+18	+20	+22	+24°
25°	−9	0	+6	+10	+14	+17	+19	+21	+23	+25°
26	8	+1	7	11	15	18	20	22	24	26
27	7	2	8	12	16	19	21	23	25	27
28	7	3	9	13	17	20	22	24	26	28
29°	−6	+4	+10	+14	+18	+20	+23	+25	+27	+29°
30°	−5	+5	+11	+15	+18	+21	+24	+26	+28	+30°
31	4	5	11	16	19	22	25	27	29	31
32	3	6	12	17	20	23	26	28	30	32
33	3	7	13	18	21	24	27	29	31	33
34	2	8	14	18	22	25	28	30	32	34
35°	−1	+9	+15	+19	+23	+26	+29	+31	+33	+35°

15. Hygrometric Table, showing at a given temperature (T), the maximum pressure (P) of aqueous vapor in mercurial centimetres, the maximum density (D) of aqueous vapor, and the factor (F) by which the difference between a wet and a dry bulb thermometer must be multiplied to find the difference between the dew-point and the temperature (T) of the air.

T	P	D	F	T	P	D	F
—10°	0.22	.0000023	8.8	+10°	0.91	.0000093	2.1
—9	.23	25	8.5	11	0.98	.0000100	2.0
—8	.25	27	8.2	12	1.04	106	2.0
—7	.27	29	7.9	13	1.11	112	2.0
—6	.29	32	7.6	14	1.19	120	1.9
—5°	0.32	.0000034	7.3	+15°	1.27	.0000128	1.9
—4	.34	37	6.8	16	1.35	135	1.9
—3	.37	40	6.0	17	1.44	144	1.9
—2	.39	42	5.0	18	1.53	152	1.8
—1	.42	45	4.1	19	1.63	162	1.8
0°	0.46	.0000049	3.3	+20°	1.74	.0000172	1.8
+1	.49	52	2.9	21	1.85	182	1.8
+2	.53	56	2.6	22	1.96	193	1.7
+3	.57	60	2.5	23	2.09	204	1.7
+4	.61	64	2.4	24	2.22	216	1.7
+5°	0.65	.0000068	2.3	+25°	2.35	.0000229	1.7
+6	.70	73	2.2	26	2.50	242	1.7
+7	.75	77	2.2	27	2.65	256	1.7
+8	.80	82	2.1	28	2.81	270	1.7
+9	.85	87	2.1	29	2.97	285	1.7
10°	.91	.0000093	2.1	+30°	3.15	.0000301	1.6

15 A. Specific Heat of Moist Air under Constant Pressure (76 cm.)

Dew-Point	Specific Heat	Dew-Point	Specific Heat	Dew-Point.	Specific Heat
—∞°	.2383	—11°	.2387	+12°	.2404
—33	.2383	—10	.2387	13	.2405
—32	.2384	— 9	.2388	14	.2407
—31	.2384	— 8	.2388	15	.2408
—30	.2384	— 7	.2388	16	.2410
—29	.2384	— 6	.2389	17	.2412
—28	.2384	— 5	.2389	18	.2414
—27	.2384	— 4	.2390	19	.2416
—26	.2384	— 3	.2390	20	.2418
—25	.2384	— 2	.2391	21	.2420
—24	.2384	— 1	.2392	22	.2423
—23	.2384	0	.2392	23	.2425
—22	.2385	+ 1	.2393	24	.2428
—21	.2385	2	.2394	25	.2430
—20	.2385	3	.2394	26	.2433
—19	.2285	4	.2395	27	.2436
—18	.2385	5	.2396	28	.2440
—17	.2385	6	.2397	29	.2443
—16	.2386	7	.2398	30	.2447
—15	.2386	8	.2399	31	.2451
—14	.2386	9	.2400	32	.2455
—13	.2386	10	.2401	33	.2459
—12°	.2387	11°	.2403	100°	.4805

15, B. Velocity of Sound in centimetres per second through Atmospheric Air at Different Temperatures and under Different Conditions of Relative Humidity.

Temperature of the Air. / Relative Humidity	0 %	20 %	40 %	60 %	80 %	100 %
0°	33,220	33,225	33,231	33,236	33,242	33,247
1°	33,281	33,286	33,292	33,298	33,304	33,310
2°	33,341	33,347	33,353	33,360	33,367	33,373
3°	33,402	33,408	33,415	33,422	33,429	33,436
4°	33,462	33,469	33,476	33,484	33,491	33,499
5°	33,523	33,530	33,538	33,546	33,554	33,562
6°	33,583	33,591	33,600	33,608	33,617	33,625
7°	33,643	33,652	33,661	33,670	33,679	33,689
8°	33,703	33,713	33,722	33,732	33,742	33,752
9°	33,763	33,773	33,784	33,794	33,805	33,815
10°	33,823	33,834	33,845	33,856	33,867	33,879
11°	33,882	33,894	33,906	33,918	33,930	33,942
12°	33,942	33,955	33,967	33,980	33,993	34,006
13°	34,001	34,015	34,029	34,043	34,056	34,070
14°	34,060	34,075	34,090	34,105	34,119	34,134
15°	34,120	34,136	34,151	34,167	34,183	34,198
16°	34,179	34,196	34,213	34,229	34,246	34,263
17°	34,238	34,256	34,274	34,292	34,310	34,328
18°	34,297	34,316	34,335	34,354	34,374	34,393
19°	34,356	34,376	34,397	34,417	34,438	34,458
20°	34,415	34,436	34,458	34,480	34,502	34,524
21°	34,474	34,496	34,520	34,543	34,566	34,589
22°	34,532	34,557	34,581	34,606	34,630	34,655
23°	34,590	34,617	34,643	34,669	34,695	34,722
24°	34,649	34,677	34,705	34,732	34,761	34,789
25°	34,707	34,737	34,766	34,796	34,826	34,856
26°	34,765	34,797	34,828	34,860	34,892	34,924
27°	34,823	34,857	34,890	34,924	34,958	34,992
28°	34,881	34,917	34,953	34,988	35,025	35,061
29°	34,939	34,977	35,015	35,053	35,092	35,130
30°	34,997	35,037	35,077	35,118	35,158	35,199
31°	35,055	35,097	35,139	35,182	35,225	35,269
32°	35,113	35,157	35,202	35,247	35,293	35,340
33°	35,170	35,218	35,265	35,313	35,362	35,412

15, C. Coefficients of Interdiffusion of Gases. (C. G. S.)*

	Air	Carbonic Oxide CO	Hydrogen H_2	Methane CH_4	Nitrous Oxide N_2O	Oxygen O_2	Sulphurous Anhydride SO_2
Carbonic Dioxide CO_2	.1423		.5614	.1586	.0982	.1409	
Hydrogen H_2		.6422				.7214	.4800
Oxygen O_2 . .		.1802	.7214				

* See Maxwell's Theory of Heat, 4th Ed. page 332. (Everett Art. 131.)

Reduction of Inches to Centimetres.

	Inches.	0	1	2	3	4	5	6	7	8	9
	28.0	71.119	.145	.170	.196	.221	.246	.272	.297	.323	.348
	28.1	71.373	.399	.424	.450	.475	.500	.526	.551	.577	.602
	28.2	71.627	.653	.678	.704	.729	.754	.780	.805	.831	.856
	28.3	71.881	.907	.932	.958	.983	*008	*034	*059	*085	*110
	28.4	72.135	.161	.186	.212	.237	.262	.288	.313	.339	.364
	28.5	72.389	.415	.440	.466	.491	.516	.542	.567	.593	.618
	28.6	72.643	.669	.694	.720	.745	.770	.796	.821	.847	.872
	28.7	72.897	.923	.948	.974	.999	*024	*050	*075	*101	*126
	28.8	73.151	.177	.202	.228	.253	.278	.304	.329	.355	.380
	28.9	73.405	.431	.456	.482	.507	.532	.558	.583	.609	.634
	29.0	73.659	.685	.710	.736	.761	.786	.812	.837	.863	.888
	29.1	73.913	.939	.964	.990	*015	*040	*066	*091	*117	*142
	29.2	74.167	.193	.218	.244	.269	.294	.320	.345	.371	.396
	29.3	74.421	.447	.472	.498	.523	.548	.574	.599	.625	.650
	29.4	74.675	.701	.726	.752	.777	.802	.828	.853	.879	.904
	29.5	74.929	.955	.980	*006	*031	*056	*082	*107	*133	*158
	29.6	75.183	.209	.234	.260	.285	.310	.336	.361	.387	.412
	29.7	75.437	.463	.488	.514	.539	.564	.590	.615	.641	.666
	29.8	75.691	.717	.742	.768	.793	.818	.844	.869	.895	.920
	29.9	75.945	.971	.996	*022	*047	*072	*098	*123	*149	*174
	30.0	76.199	.225	.250	.276	.301	.326	.352	.377	.403	.428
	30.1	76.453	.479	.504	.530	.555	.580	.606	.631	.657	.682
	30.2	76.707	.733	.758	.784	.809	.834	.860	.885	.911	.936
	30.3	76.961	.987	*012	*038	*063	*088	*114	*139	*165	*190
	30.4	77.215	.241	.266	.292	.317	.342	.368	.393	.419	.444
	30.5	77.469	.495	.520	.546	.571	.596	.622	.647	.673	.698
	30.6	77.723	.749	.774	.800	.825	.850	.876	.901	.927	.952
	30.7	77.977	*003	*028	*053	*079	*104	*130	*155	*180	*206
	30.8	78.231	.257	.282	.307	.333	.358	.384	.409	.434	.460
	30.9	78.485	.511	.536	.561	.587	.612	.638	.663	.688	.714
	31.0	78.739	.765	.790	.815	.841	.866	.892	.917	.942	.968
	31.1	78.993	*019	*044	*069	*095	*120	*146	*171	*196	*222
	31.2	79.247	.273	.298	.323	.349	.374	.400	.425	.450	.476
	31.3	79.501	.527	.552	.577	.603	.628	.654	.679	.704	.730
	31.4	79.755	.781	.806	.831	.857	.882	.908	.933	.958	.984
	31.5	80.009	.035	.060	.085	.111	.136	.162	.187	.212	.238
Dif.	In.	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010
	Cm.	.003	.005	.008	.010	.013	.015	.018	.020	.023	.025

* The star indicates that the number of whole centimetres is to be read from the line underneath it.

16 A. Reduction of Mercurial Centimetres to Megadynes per sq. cm. g=980.

cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
70	0.9327	0.9340	0.9354	0.9367	0.9380	0.9393	0.9407	0.9420	0.9433	0.9447	13.3
71	.9460	.9473	.9487	.9500	.9513	.9527	.9540	.9553	.9567	.9580	1 1
72	.9593	.9607	.9620	.9633	.9647	.9660	.9673	.9687	.9700	.9713	2 3, 3 4
73	.9727	.9740	.9753	.9767	.9780	.9793	.9807	.9820	.9833	.9847	4 5
74	.9860	.9873	.9886	.9900	.9913	.9926	.9940	.9953	.9966	.9980	5 7
75	.9993	1.0006	1.0020	1.0033	1.0046	1.0060	1.0073	1.0086	1.0100	1.0113	6 8
76	1.0126	1.0140	1.0153	1.0166	1.0180	1.0193	1.0206	1.0220	1.0233	1.0246	7 9, 8 11
77	1.0260	1.0273	1.0286	1.0300	1.0313	1.0326	1.0339	1.0353	1.0366	1.0379	9 12

16 B. Reduction of Mercurial Centimetres to Megadynes per sq. cm. g=981.

cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
70	0.9336	0.9350	0.9363	0.9376	0.9390	0.9403	0.9416	0.9430	0.9443	0.9456	13.3
71	.9470	.9483	.9496	.9510	.9523	.9536	.9550	.9563	.9576	.9590	1 1
72	.9603	.9616	.9630	.9643	.9656	.9670	.9683	.9696	.9710	.9723	2 3, 3 4
73	.9737	.9750	.9763	.9777	.9790	.9803	.9817	.9830	.9843	.9857	4 5
74	.9870	.9883	.9897	.9910	.9923	.9937	.9950	.9963	.9977	.9990	5 7
75	1.0003	1.0017	1.0030	1.0043	1.0057	1.0070	1.0083	1.0097	1.0110	1.0123	6 8
76	1.0137	1.0150	1.0163	1.0177	1.0190	1.0203	1.0217	1.0230	1.0243	1.0257	7 9, 8 11
77	1.0270	1.0283	1.0297	1.0310	1.0323	1.0337	1.0350	1.0363	1.0377	1.0390	9 12

17. Elevation in Metres above the Sea Level corresponding to Different Barometric Pressures at 10° Centigrade (g=980.6).

cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
60	1959	1945	1931	1918	1904	1890	1876	1863	1849	1836
61	1822	1808	1795	1782	1768	1754	1741	1727	1714	1701
62	1687	1674	1660	1647	1634	1621	1607	1594	1581	1568
63	1555	1541	1528	1515	1502	1489	1476	1463	1450	1437
64	1424	1411	1398	1385	1372	1360	1347	1334	1321	1308
65	1295	1283	1270	1257	1245	1232	1219	1207	1194	1182
66	1169	1157	1144	1131	1119	1107	1094	1082	1069	1057
67	1044	1032	1020	1007	995	983	971	958	946	934
68	922	910	897	885	873	861	849	837	825	813
69	801	789	777	765	753	741	729	717	705	693
70	681	670	658	646	634	623	611	599	587	576
71	564	552	541	529	517	506	494	483	471	460
72	448	437	425	414	402	391	379	368	356	345
73	334	322	311	300	288	277	266	255	243	232
74	221	210	199	187	176	165	154	143	132	121
75	110	99	88	77	66	55	44	33	22	11
76	0	—11	—22	—33	—43	—54	—65	—76	—87	—98
77	—108	—119	—130	—141	—151	—162	—173	—183	—194	—205
78	—215	—226	—236	—247	—258	—268	—279	—289	—300	—310

17 A. Correction for Temperature in 17.

Mean Temp.	Subtr. %	Mean Temp.	Add %	Mean Temp.	Add %
0°	3.5	10	0.0	20	3.5
1	3.2	11	0.4	21	3.9
2	2.8	12	0.7	22	4.2
3	2.5	13	1.1	23	4.6
4	2.1	14	1.4	24	5.0
5	1.8	15	1.8	25	5.3
6	1.4	16	2.1	26	5.7
7	1.1	17	2.5	27	6.0
8	0.7	18	2.8	28	6.4
9°	0.4	19	3.2	29	6.7

17 B. Correction for Humidity in 17.

Dew-Point	Add %	Dew-Point	Add %	Dew-Point	Add %
—∞	0.0	+10	0.5	+20	0.9
—20	0.0	11	0.5	21	0.9
—15	0.1	12	0.5	22	1.0
—10	0.1	13	0.6	23	1.1
—5	0.2	14	0.6	24	1.1
0	0.2	15	0.6	25	1.2
+2	0.3	16	0.7	26	1.3
+4	0.3	17	0.7	27	1.3
+6	0.3	18	0.8	28	1.4
+8	0.4	19	0.8	29	1.5

18a. Reduction of Mercurial Columns to 0°. Corrections for Expansion to be subtracted.

Temperature	Length in centimetres of the Mercurial Column measured by a Brass Scale. 70	71	72	73	74	75	76	77	78	Correction for glass scale
	cm	cm	cm	cm	cm	cm	cm	cm	cm	
0°	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	011	011	012	012	012	012	012	012	013	001
2	023	023	023	024	024	024	024	025	025	002
3	034	034	035	035	036	036	037	037	038	002
4	045	046	046	047	048	048	049	050	050	003
5	0.056	0.057	0.058	0.059	0.060	0.060	0.061	0.062	0.063	0.004
6	068	069	069	071	072	072	073	074	075	005
7	079	080	081	082	083	085	086	087	088	006
8	090	092	093	094	095	097	098	099	101	006
9	102	103	104	106	107	109	110	112	113	007
10	0.113	0.114	0.116	0.118	0.119	0.121	0.122	0.124	0.126	0.008
11	124	126	128	129	131	133	135	137	138	009
12	135	137	139	141	143	145	147	149	151	009
13	147	149	151	153	155	157	159	161	164	010
14	158	160	163	165	167	169	172	174	176	011
15	0.169	0.172	0.174	0.177	0.179	0.181	0.184	0.186	0.189	0.012
16	181	183	186	188	191	194	196	199	201	013
17	192	195	197	200	203	206	208	211	214	013
18	203	206	209	212	215	218	221	224	227	014
19	215	218	221	224	227	230	233	236	239	015
20	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.248	0.252	0.016
21	237	241	244	247	251	254	258	261	264	017
22	249	252	256	259	263	266	270	273	277	017
23	260	264	267	271	275	278	282	286	290	018
24	271	275	279	283	287	291	294	298	302	019
25	0.283	0.287	0.291	0.295	0.299	0.303	0.307	0.311	0.315	0.020
26	294	298	302	306	311	315	319	323	327	021
27	305	310	314	318	323	327	331	336	340	021
28	317	321	326	330	335	339	344	348	353	022
29	328	333	337	342	347	351	356	361	365	023
30	0.339	0.344	0.349	0.354	0.359	0.363	0.368	0.373	378	024

18b. Correction for the Capillarity of Mercurial Columns to be added.

Internal Diameter of Tube	Height of Meniscus unknown	Height of Meniscus in Centimetres 0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18
0.1 cm	.9?								
0.2	.46								
0.3	.29								
0.4	.26	0.083	0.122	0.154	0.198	0.237	...	...	...
0.5	.15	.047	.065	.086	.119	.145	0.180	...	...
0.6	.11	.027	.041	.056	.078	.098	.121	0.143	...
0.7	.09	.018	.028	.040	.053	.067	.082	.097	0.113
0.8	.07		.020	.029	.038	.046	.056	.065	.077
0.9	.05		.015	.021	.028	.033	.040	.046	.052
1.0	.04			.015	.020	.025	.029	.033	.037
1.1	.03			.010	.014	.018	.021	.024	.027
1.2	.03			.007	.010	.013	.015	.018	.019
1.3	.02			.004	.007	.010	.012	.013	.014

18c. Correction for the Pressure of Mercurial Vapor to be added.

Temperature	0°	5°	10°	15°	20°	25°	30°	35°	40°
Add cm.	0.001?	.001?	.002?	.002?	.002?	.002?	.003?	.003?	.004?

18, d. Factors for the Reduction of the Density of a Gas to 76 cm.

Pressure cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
70	1.0857	1.0842	1.0826	1.0811	1.0795	1.0780	1.0765	1.0750	1.0734	1.0719	
71	1.0704	1.0689	1.0674	1.0659	1.0644	1.0629	1.0615	1.0600	1.0585	1.0570	15
72	1.0556	1.0541	1.0526	1.0512	1.0497	1.0483	1.0468	1.0454	1.0440	1.0425	
73	1.0411	1.0397	1.0383	1.0368	1.0354	1.0340	1.0326	1.0312	1.0298	1.0284	14
74	1.0270	1.0256	1.0243	1.0229	1.0215	1.0201	1.0188	1.0174	1.0160	1.0147	
75	1.0133	1.0120	1.0106	1.0093	1.0080	1.0066	1.0053	1.0040	1.0026	1.0013	
76	1.0000	.9987	.9973	.9961	.9948	.9935	.9922	.9909	.9896	.9883	13
77	.9870	.9857	.9845	.9832	.9819	.9806	.9794	.9781	.9769	.9756	

18, e. Factors for the Reduction of the Density of a Gas to 0° Centigrade.

Temperature	+0°	+1°	+2°	+3°	+4°	+5°	+6°	+7°	+8°	+9°	Dif.
0°	1.0000	1.0037	1.0073	1.0110	1.0147	1.0184	1.0220	1.0257	1.0294	1.0330	36.7
10	1.0367	1.0404	1.0440	1.0477	1.0514	1.0551	1.0587	1.0624	1.0661	1.0697	1 4
20	1.0734	1.0771	1.0807	1.0844	1.0881	1.0918	1.0954	1.0991	1.1028	1.1064	2 7
30	1.1101	1.1138	1.1174	1.1211	1.1248	1.1285	1.1321	1.1358	1.1395	1.1431	3 11
40	1.1468	1.1505	1.1541	1.1578	1.1615	1.1652	1.1688	1.1725	1.1762	1.1798	4 15
50	1.1835	1.1872	1.1908	1.1945	1.1982	1.2019	1.2055	1.2092	1.2129	1.2165	5 18
60	1.2202	1.2239	1.2275	1.2312	1.2349	1.2386	1.2422	1.2459	1.2496	1.2532	6 22
70	1.2569	1.2606	1.2642	1.2679	1.2716	1.2753	1.2789	1.2826	1.2863	1.2899	7 26
80	1.2936	1.2973	1.3009	1.3046	1.3083	1.3120	1.3156	1.3193	1.3230	1.3266	8 29
90	1.3303	1.3340	1.3376	1.3413	1.3450	1.3487	1.3523	1.3560	1.3597	1.3633	9 33
100°	1.3670	1.3707	1.3743	1.3780	1.3817	1.3854	1.3890	1.3927	1.3964	1.4000	

18, f. Factors for the Reduction of the Volume of a Gas to 76 cm.

Pressure cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
70	0.9211	0.9224	0.9237	0.9250	0.9263	0.9276	0.9289	0.9303	0.9316	0.9329	13.13
71	.9342	.9355	.9368	.9382	.9395	.9408	.9421	.9434	.9447	.9461	1 1 1
72	.9474	.9487	.9500	.9513	.9526	.9539	.9553	.9566	.9579	.9592	2 3
73	.9605	.9618	.9632	.9645	.9658	.9671	.9684	.9697	.9711	.9724	3 4
74	.9737	.9750	.9763	.9776	.9789	.9803	.9816	.9829	.9842	.9855	4 5 5 7
75	.9868	.9882	.9895	.9908	.9921	.9934	.9947	.9961	.9974	.9987	6 8
76	1.0000	1.0013	1.0026	1.0039	1.0053	1.0066	1.0079	1.0092	1.0105	1.0118	7 9 8 11
77	1.0132	1.0145	1.0158	1.0171	1.0184	1.0197	1.0211	1.0224	1.0237	1.0250	9 12

18, g. Factors for the Reduction of the Volume of a Gas to 0° Centigrade.

Temperature													
0°	1.0000	5°	0.9820	10°	0.9646	15	0.9478	20°	0.9316	25°	0.9160	30°	0.9008
1	0.9963	6	.9785	11	.9612	16	.9445	21	.9285	26	.9129	31	.8978
2	.9927	7	.9750	12	.9578	17	.9413	22	.9253	27	.9098	32	.8949
3	.9891	8	.9715	13	.9545	18	.9380	23	.9222	28	.9068	33	.8920
4	.9855	9	.9680	14	.9511	19	.9348	24	.9190	29	.9038	34	.8891
Dif.	36		35		34		33		32		31		29

19. Weight in grams of 1 cubic centimetre of dry air.

Temperature of the Air.	Barometric pressure ($g = 980.6$)						
	72 cm	73 cm	74 cm	75 cm	76 cm	77 cm	Diff. per cm.
0°	.001225	.001242	.001259	.001276	.001293	.001310	17
1	1220	1237	1254	1271	1288	1305	.1 2
2	1216	1233	1249	1267	1283	1300	.2 3
3	1212	1228	1245	1262	1279	1296	.3 5
4	1207	1224	1241	1257	1274	1290	.4 7
5°	.001203	.001219	.001236	.001253	.001270	.001286	.5 8
6	1198	1215	1232	1248	1265	1282	.6 10
7	1194	1211	1227	1244	1260	1277	.7 12
8	1190	1206	1223	1239	1256	1272	.8 14
9	1186	1202	1219	1235	1251	1268	.9 15
10°	.001181	.001198	.001214	.001231	.001247	.001263	16
11	1177	1194	1210	1226	1243	1259	.1 2
12	1173	1189	1206	1222	1238	1255	.2 3
13	1169	1185	1202	1218	1234	1250	.3 5
14	1165	1181	1197	1214	1230	1246	.4 6
15°	.001161	.001177	.001193	.001209	.001225	.001242	.5 8
16	1157	1173	1189	1205	1221	1237	.6 10
17	1153	1169	1185	1201	1217	1233	.7 11
18	1149	1165	1181	1197	1213	1229	.8 13
19	1145	1161	1177	1193	1209	1224	.9 14
20°	.001141	.001157	.001173	.001189	.001204	.001220	15
21	1137	1153	1169	1185	1200	1216	.1 2
22	1133	1149	1165	1181	1196	1212	.2 3
23	1130	1145	1161	1177	1192	1208	.3 4
24	1126	1141	1157	1173	1188	1204	.4 6
25°	.001122	.001138	.001153	.001169	.001184	.001200	.5 7
26	1118	1134	1149	1165	1180	1196	.6 9
27	1114	1130	1145	1161	1176	1192	.7 10
28	1110	1126	1142	1157	1172	1188	.8 12
29	1107	1122	1138	1153	1169	1184	.9 13
30°	.001103	.001119	.001134	.001149	.001165	.001180	

20. Correction for Moisture in Table 19.

Dew-Point	Subtract	Dew-Point	Subtract	Dew-Point	Subtract	Dew-Point	Subtract
−10°	.000,001	0°	.000,003	+10°	.000,006	+20°	.000,010
−8	.000,002	+2	.000,003	+12	.000,006	+22	.000,012
−6	.000,002	+4	.000,004	+14	.000,007	+24	.000,013
−4	.000,002	+6	.000,004	+16	.000,008	+26	.000,015
−2	.000,003	+8	.000,005	+18	.000,009	+28	.000,016

20A. Weight in grams of air displaced by 1 gram of brass of density 8.4.

Density of Air	.00110	.00112	.00114	.00116	.00118	.00120
Weight Displaced	.000131	.000133	.000136	.000138	.000140	.000143
Density of Air	.00120	.00122	.00124	.00126	.00128	.00130
Weight Displaced	.000143	.000145	.000148	.000150	.000152	.000155

21. Factors for the Reduction of Apparent Weighings in Air with Brass Weights to Vacuo.

Density of the Substance Weighed.	Density of the Air. .00115	.00120	.00125	Density of the Substance Weighed.	Density of the Air. .00115	.00120	.00125
0.70	1.00151	1.00157	1.00164	2.0	1.00044	1.00046	1.00048
0.75	„ 140	„ 146	„ 152	2.5	„ 32	„ 34	„ 35
0.80	„ 130	„ 136	„ 141	3.0	„ 25	„ 26	„ 27
0.85	„ 122	„ 127	„ 132	3.5	„ 19	„ 20	„ 21
0.90	„ 114	„ 119	„ 124	4.0	„ 15	„ 16	„ 16
0.95	„ 107	„ 112	„ 117	4.5	„ 12	„ 12	„ 13
1.0	1.00101	1.00106	1.00110	5.0	1.00009	1.00010	1.00010
1.1	1.00091	1.00095	1.00099	6.0	1.00005	1.00006	1.00006
1.2	„ 82	„ 86	„ 89	7.0	„ 3	„ 3	„ 3
1.3	„ 75	„ 78	„ 81	8.0	„ 1	„ 1	„ 1
1.4	„ 68	„ 71	„ 74	9.0	0.99999	0.99999	0.99999
1.5	1.00063	1.00066	1.00068	10	0.99998	0.99998	0.99998
1.6	„ 58	„ 61	„ 63	12	„ 6	„ 6	„ 5
1.7	„ 54	„ 56	„ 59	14	„ 5	„ 4	„ 4
1.8	„ 50	„ 52	„ 55	16	„ 3	„ 3	„ 3
1.9	„ 47	„ 49	„ 51	18	„ 3	„ 2	„ 2
2.0	1.00044	1.00046	1.00048	20	0.99992	0.99992	0.99991

Apparent Specific Volume of Water.

22. Space in cubic centimetres occupied by a quantity of Water weighing apparently 1 gram when counterpoised in Air with Brass Weights of the Density 8.4.

Temperature of the Water.	Density of the Air .00110	.00115	.00120	.00125	.00130
0°	1.00109	1.00113	1.00117	1.00122	1.00126
1	„ 103	„ 107	„ 112	„ 116	„ 121
2	1.00099	„ 103	„ 108	„ 112	„ 116
3	„ 97	„ 101	„ 106	„ 110	„ 114
4	„ 96	„ 100	„ 105	„ 109	„ 114
5°	1.00097	1.00101	1.00106	1.00110	1.00114
6	„ 99	„ 103	„ 108	„ 112	„ 117
7	1.00103	„ 107	„ 111	„ 116	„ 120
8	„ 108	„ 112	„ 116	„ 121	„ 125
9	„ 114	„ 118	„ 123	„ 127	„ 131
10°	1.00122	1.00126	1.00131	1.00135	1.00139
11	„ 131	„ 135	„ 140	„ 144	„ 148
12	„ 141	„ 146	„ 150	„ 155	„ 159
13	„ 153	„ 158	„ 162	„ 166	„ 171
14	„ 166	„ 171	„ 175	„ 179	„ 184
15°	1.00180	1.00185	1.00189	1.00194	1.00198
16	„ 196	„ 200	„ 205	„ 209	„ 214
17	„ 212	„ 217	„ 221	„ 225	„ 230
18	„ 231	„ 235	„ 239	„ 244	„ 248
19	„ 250	„ 254	„ 258	„ 263	„ 267
20°	1.00270	1.00275	1.00279	1.00283	1.00288
21	„ 291	„ 295	„ 300	„ 304	„ 309
22	„ 313	„ 318	„ 322	„ 326	„ 331
23	„ 336	„ 340	„ 344	„ 349	„ 353
24	„ 360	„ 364	„ 368	„ 373	„ 377
25°	1.00384	1.00389	1.00393	1.00398	1.00402
26	„ 410	„ 414	„ 419	„ 423	„ 428
27	„ 437	„ 441	„ 445	„ 450	„ 454
28	„ 464	„ 468	„ 473	„ 477	„ 482
29	„ 492	„ 497	„ 501	„ 505	„ 510
30°	1.00521	1.00525	1.00530	1.00534	1.00538

23. Space in cu. cm. occupied by a quantity of Water weighing 1 gram in Vacuo.

0°	1.00012	Dif.	25°	1.00287	Dif.	50°	1.01194	Dif.	75°	1.02565	Dif
1	1.00006	−4	26	1.00313	26	51	1.01242	48	76	1.02629	64
2	1.00002	−4	27	1.00339	26	52	1.01291	49	77	1.02693	64
3	1.00000	−2	28	1.00367	28	53	1.01340	49	78	1.02757	64
4	0.99999	−1	29	1.00395	28	54	1.01389	49	79	1.02821	64
5°	1.00000	+1	30°	1.00424	29	55°	1.01438	49	80°	1.02886	65
6	1.00002	2	31	1.00454	30	56	1.01487	49	81	1.02951	65
7	1.00006	4	32	1.00485	31	57	1.01536	49	82	1.03017	66
8	1.00011	5	33	1.00517	32	58	1.01586	50	83	1.03084	67
9	1.00017	6	34	1.00550	33	59	1.01637	51	84	1.03152	68
10°	1.00025	8	35°	1.00585	35	60°	1.01690	53	85°	1.03220	68
11	1.00034	9	36	1.00620	35	61	1.01743	53	86	1.03288	68
12	1.00044	10	37	1.00656	36	62	1.01797	54	87	1.03357	69
13	1.00056	12	38	1.00693	37	63	1.01851	54	88	1.03426	69
14	1.00069	13	39	1.00731	38	64	1.01907	56	89	1.03496	70
15°	1.00083	14	40°	1.00769	38	65°	1.01963	56	90°	1.03566	70
16	1.00099	16	41	1.00808	39	66	1.02020	57	91	1.03637	71
17	1.00115	16	42	1.00848	40	67	1.02077	57	92	1.03709	72
18	1.00133	18	43	1.00888	40	68	1.02136	59	93	1.03781	72
19	1.00152	19	44	1.00928	40	69	1.02195	59	94	1.03855	74
20°	1.00173	21	45°	1.00970	42	70°	1.02255	60	95°	1.03930	75
21	1.00194	21	46	1.01013	43	71	1.02315	60	96	1.04005	75
22	1.00216	22	47	1.01056	43	72	1.02377	62	97	1.04081	76
23	1.00238	22	48	1.01101	45	73	1.02439	62	98	1.04157	76
24	1.00262	24	49	1.01147	46	74	1.02502	63	99	1.04234	77
25°	1.00287	25	50°	1.01194	47	75°	1.02565	63	100°	1.04311	77

23, A. Space in cu. cm. occupied by 1 gram of Mercury.

0°	0.073,551	10°	0.073,684	20°	0.073,816	Dif.		
1	.073,564	11	.073,697	21	.073,830		13	14
2	.073,578	12	.073,710	22	.073,843	.1	1	1
3	.073,591	13	.073,723	23	.073,856	.2	3	3
4	.073,604	14	.073,737	24	.073,870	.3	4	4
5°	0.073,617	15°	0.073,750	25°	0.073,883	.4	5	6
6	.073,631	16	.073,763	26	.073,896	.5	7	7
7	.073,644	17	.073,776	27	.073,910	.6	8	8
8	.073,657	18	.073,790	28	.073,923	.7	9	10
9	.073,670	19	.073,803	29	.073,936	.8	10	11
10°	0.073,684	20°	0.073,816	30°	0.073,950	.9	12	13

23, B. Space in cu. cm. occupied by a quantity of Mercury weighing apparently 1 gram when balanced by Brass Weights of Density 8.4 in Air of Density .0012.

0°	0.073,547	10°	0.073,680	20°	0.073,812	Dif.		
1	.073,560	11	.073,693	21	.073,826		13	14
2	.073,574	12	.073,706	22	.073,839	.1	1	1
3	.073,587	13	.073,719	23	.073,852	.2	3	3
4	.073,600	14	.073,733	24	.073,866	.3	4	4
5°	0.073,613	15°	0.073,746	25°	0.073,879	.4	5	6
6	.073,627	16	.073,759	26	.073,892	.5	7	7
7	.073,640	17	.073,772	27	.073,906	.6	8	8
8	.073,653	18	.073,786	28	.073,919	.7	9	10
9	.073,666	19	.073,799	29	.073,932	.8	10	11
10°	0.073,680	20°	0.073,812	30°	0.073,946	.9	12	13

Density of Water, Mercury and Glycerine.

24. Density of Mercury at different temperatures.

0°	13.596	90°	13.377	180°	13.162	270°	12.948
10°	13.572	100°	13.353	190°	13.138	280°	12.924
20°	13.547	110°	13.329	200°	13.114	290°	12 900
30°	13.523	120°	13.305	210°	13.091	300°	12.876
40°	13.498	130°	13.281	220°	13.067	310°	12.853
50°	13.474	140°	13.257	230°	13.043	320°	12.829
60°	13.450	150°	13.233	240°	13.019	330°	12.805
70°	13.426	160°	13.210	250°	12.995	340°	12.781
80°	13.401	170°	13.186	260°	12.972	350°	12.757

25. Density of Water at different temperatures.

0°	0.99988	25°	0.99714	50°	0.98819	75°	0.97497
1	94	26	.99687	51	772	76	437
2	98	27	61	52	725	77	376
3	1.00000	28	34	53	677	78	315
4	01	29	06	54	629	79	254
5°	1.00000	30°	0.99578	55°	0.98582	80°	0.97193
6	0.99998	31	548	56	534	81	131
7	94	32	518	57	486	82	069
8	89	33	486	58	437	83	006
9	83	34	453	59	388	84	.96942
10°	0.99975	35°	0.99419	60°	0.98338	85°	0.96878
11	66	36	384	61	286	86	814
12	56	37	348	62	234	87	750
13	44	38	311	63	181	88	686
14	31	39	274	64	127	89	621
15°	0.99916	40°	0.99236	65°	0.98073	90°	0.96554
16	01	41	198	66	018	91	488
17	.99885	42	158	67	.97963	92	421
18	67	43	118	68	907	93	354
19	48	44	078	69	850	94	286
20°	0.99828	45°	0.99037	70°	0.97793	95°	0.96216
21	07	46	.98996	71	735	96	146
22	.99785	47	954	72	676	97	076
23	62	48	910	73	617	98	005
24	39	49	865	74	557	99	.95934
25°	0.99714	50°	0.98819	75°	0.97497	100°	0.95863

26. Density of Commercial Glycerine (0°—30°).

0°	1.269	5°	1.266	10°	1.263	15°	1.260	20°	1.257	25°	1.253
1°	1.268	6°	1.265	11°	1.262	16°	1.259	21°	1.256	26°	1.253
2°	1.268	7°	1.265	12°	1.262	17°	1.258	22°	1.255	27°	1.252
3°	1.267	8°	1.264	13°	1.261	18°	1.258	23°	1.255	28°	1.252
4°	1.267	9°	1.263	14°	1.260	19°	1.257	24°	1.254	29°	1.251
5°	1.266	10°	1.263	15°	1.260	20°	1.257	25°	1.253	30°	1.251

w't	VOL. 15°	15°	16°	17°	18°	19°	20°	21°	22°
0	0.00	·9992	·9990	·9988	·9987	·9985	·9983	·9981	·9979
1	1.26	·9971	·9969	·9967	·9966	·9964	·9962	·9960	·9958
2	2.51	·9953	·9951	·9949	·9947	·9945	·9943	·9942	·9940
3	3.75	·9936	·9934	·9932	·9930	·9928	·9926	·9924	·9922
4	5.00	·9920	·9918	·9916	·9914	·9912	·9909	·9907	·9905
5	6.24	·9903	·9901	·9899	·9897	·9895	·9892	·9890	·9888
6	7.47	·9887	·9885	·9883	·9880	·9878	·9876	·9874	·9872
7	8.70	·9871	·9869	·9866	·9864	·9861	·9859	·9857	·9855
8	9.93	·9856	·9854	·9851	·9849	·9846	·9844	·9842	·9839
9	11.16	·9842	·9839	·9837	·9834	·9832	·9829	·9827	·9824
10	12.38	·9828	·9825	·9823	·9820	·9817	·9815	·9813	·9810
11	13.59	·9814	·9811	·9809	·9806	·9803	·9800	·9798	·9795
12	14.81	·9801	·9798	·9795	·9793	·9790	·9787	·9784	·9781
13	16.03	·9789	·9786	·9783	·9780	·9777	·9774	·9772	·9769
14	17.24	·9777	·9774	·9771	·9768	·9765	·9762	·9759	·9756
15	18.45	·9765	·9762	·9759	·9755	·9752	·9749	·9746	·9743
16	19.65	·9753	·9750	·9746	·9743	·9740	·9736	·9733	·9730
17	20.85	·9741	·9738	·9734	·9731	·9727	·9724	·9721	·9717
18	22.05	·9729	·9725	·9722	·9718	·9715	·9711	·9708	·9704
19	23.25	·9718	·9714	·9711	·9707	·9703	·9699	·9696	·9692
20	24.45	·9707	·9703	·9699	·9695	·9691	·9687	·9683	·9679
21	25.64	·9695	·9691	·9687	·9683	·9679	·9674	·9670	·9666
22	26.83	·9683	·9679	·9674	·9670	·9666	·9661	·9657	·9653
23	28.01	·9671	·9666	·9662	·9657	·9653	·9648	·9644	·9639
24	29.19	·9659	·9654	·9650	·9645	·9640	·9635	·9631	·9626
25	30.37	·9647	·9642	·9637	·9632	·9627	·9621	·9617	·9612
26	31.54	·9633	·9628	·9623	·9618	·9613	·9607	·9602	·9597
27	32.71	·9619	·9614	·9608	·9603	·9598	·9592	·9587	·9582
28	33.86	·9604	·9599	·9593	·9588	·9583	·9577	·9571	·9566
29	35.02	·9589	·9583	·9578	·9572	·9567	·9561	·9555	·9549
30	36.17	·9573	·9567	·9561	·9556	·9550	·9544	·9538	·9532
31	37.30	·9556	·9550	·9544	·9538	·9532	·9526	·9520	·9514
32	38.44	·9539	·9533	·9527	·9521	·9515	·9508	·9502	·9496
33	39.57	·9522	·9516	·9509	·9503	·9497	·9490	·9484	·9478
34	40.69	·9504	·9498	·9491	·9485	·9479	·9472	·9466	·9459
35	41.81	·9486	·9479	·9473	·9466	·9460	·9453	·9447	·9440
36	42.92	·9467	·9460	·9454	·9447	·9440	·9433	·9427	·9420
37	44.02	·9448	·9441	·9434	·9428	·9421	·9414	·9407	·9400
38	45.12	·9429	·9422	·9415	·9408	·9401	·9394	·9388	·9381
39	46.21	·9410	·9403	·9396	·9389	·9382	·9375	·9368	·9361
40	47.30	·9390	·9383	·9376	·9369	·9362	·9354	·9347	·9340
41	48.38	·9370	·9363	·9356	·9348	·9341	·9334	·9327	·9320
42	49.45	·9349	·9342	·9334	·9327	·9320	·9312	·9305	·9298
43	50.51	·9328	·9321	·9313	·9306	·9298	·9291	·9284	·9276
44	51.57	·9307	·9299	·9292	·9284	·9277	·9269	·9262	·9254
45	52.62	·9286	·9278	·9271	·9263	·9256	·9248	·9240	·9233
46	53.67	·9265	·9257	·9250	·9242	·9234	·9226	·9219	·9211
47	54.71	·9244	·9236	·9229	·9221	·9213	·9205	·9198	·9190
48	55.75	·9223	·9215	·9207	·9200	·9192	·9184	·9176	·9168
49	56.78	·9201	·9193	·9185	·9178	·9170	·9162	·9154	·9146
50	57.80	·9179	·9171	·9163	·9155	·9147	·9139	·9132	·9124

W't	VOL. 15°	15°	16°	17°	18°	19°	20°	21°	22°
50	57.80	.9179	.9171	.9163	.9155	.9147	.9139	.9132	.9124
51	58.81	.9157	.9149	.9141	.9133	.9125	.9117	.9110	.9102
52	59.82	.9135	.9127	.9119	.9111	.9103	.9095	.9087	.9079
53	60.82	.9113	.9105	.9097	.9089	.9081	.9073	.9065	.9057
54	61.82	.9091	.9083	.9075	.9067	.9059	.9050	.9042	.9034
55	62.81	.9069	.9061	.9053	.9045	.9037	.9028	.9020	.9012
56	63.79	.9046	.9038	.9030	.9022	.9013	.9005	.8997	.8989
57	64.77	.9023	.9015	.9007	.8998	.8990	.8982	.8974	.8966
58	65.74	.9000	.8992	.8984	.8975	.8967	.8959	.8951	.8943
59	66.70	.8977	.8969	.8961	.8952	.8944	.8936	.8928	.8920
60	67.65	.8954	.8946	.8938	.8929	.8921	.8913	.8905	.8897
61	68.60	.8931	.8923	.8914	.8906	.8898	.8890	.8882	.8873
62	69.55	.8908	.8900	.8891	.8883	.8875	.8867	.8859	.8850
63	70.49	.8885	.8877	.8868	.8860	.8852	.8844	.8836	.8827
64	71.42	.8862	.8854	.8845	.8837	.8829	.8821	.8813	.8804
65	72.34	.8838	.8830	.8821	.8813	.8805	.8797	.8789	.8780
66	73.26	.8815	.8807	.8798	.8790	.8782	.8773	.8765	.8756
67	74.18	.8792	.8784	.8775	.8767	.8759	.8750	.8742	.8733
68	75.08	.8768	.8760	.8751	.8743	.8735	.8726	.8718	.8709
69	75.98	.8744	.8736	.8727	.8719	.8711	.8702	.8694	.8685
70	76.88	.8721	.8713	.8704	.8696	.8688	.8679	.8671	.8662
71	77.77	.8698	.8689	.8681	.8672	.8664	.8655	.8647	.8638
72	78.65	.8674	.8665	.8657	.8648	.8640	.8631	.8623	.8614
73	79.51	.8649	.8640	.8632	.8623	.8615	.8606	.8598	.8589
74	80.37	.8625	.8616	.8608	.8599	.8591	.8582	.8574	.8565
75	81.23	.8601	.8592	.8584	.8575	.8567	.8558	.8550	.8541
76	82.08	.8576	.8567	.8559	.8550	.8542	.8533	.8525	.8516
77	82.92	.8552	.8543	.8535	.8526	.8518	.8509	.8501	.8492
78	83.76	.8528	.8519	.8511	.8502	.8494	.8485	.8476	.8468
79	84.59	.8503	.8494	.8486	.8477	.8469	.8460	.8451	.8443
80	85.41	.8478	.8469	.8461	.8452	.8444	.8435	.8426	.8418
81	86.22	.8453	.8444	.8436	.8427	.8419	.8410	.8401	.8393
82	87.03	.8428	.8419	.8411	.8402	.8394	.8385	.8376	.8368
83	87.84	.8404	.8395	.8387	.8378	.8370	.8361	.8352	.8344
84	88.63	.8379	.8370	.8362	.8353	.8345	.8336	.8327	.8319
85	89.42	.8354	.8345	.8337	.8328	.8320	.8311	.8302	.8294
86	90.20	.8329	.8320	.8312	.8303	.8295	.8286	.8277	.8269
87	90.97	.8303	.8294	.8286	.8277	.8269	.8260	.8251	.8243
88	91.72	.8277	.8268	.8260	.8251	.8243	.8234	.8225	.8217
89	92.47	.8251	.8242	.8234	.8225	.8217	.8208	.8199	.8191
90	93.22	.8225	.8216	.8208	.8199	.8190	.8181	.8173	.8164
91	93.96	.8199	.8190	.8182	.8173	.8164	.8155	.8147	.8138
92	94.68	.8172	.8163	.8155	.8146	.8137	.8128	.8120	.8111
93	95.39	.8145	.8136	.8128	.8119	.8110	.8101	.8093	.8084
94	96.09	.8118	.8109	.8101	.8092	.8083	.8074	.8066	.8057
95	96.78	.8090	.8081	.8073	.8064	.8055	.8046	.8038	.8029
96	97.45	.8061	.8052	.8044	.8035	.8026	.8017	.8009	.8000
97	98.11	.8032	.8023	.8015	.8006	.7997	.7988	.7980	.7971
98	98.75	.8002	.7993	.7985	.7976	.7967	.7958	.7950	.7941
99	99.38	.7972	.7963	.7955	.7946	.7937	.7928	.7920	.7911
100	100.00	.7941	.7932	.7924	.7915	.7906	.7897	.7889	.7880

Per Cent %	Acetic Acid $C_2H_4O_2$	Nitric Acid. HNO_3	Phosphoric Acid. H_3PO_4	Sulphuric Acid. H_2SO_4	Tart. Acid. $C_4H_6O_6$	Alcohol sol. in Ether.	Methyl Alcohol. CH_4O	Hydrate of Sodium NaOH.	Hydrate of Potassium KOH	Glycerine $C_3H_8O_3$	Sugar (Cane) $C_{12}H_{22}O_{11}$
0	0.999	0.999	0.999	0.999	0.999	0.719	0.999	0.999	0.999	0.999	0.999
2	1.002	1.010	1.010	1.010	1.008	.721	.993	1.02	1.02	1.004	1.007
4	1.005	1.022	1.021	1.024	1.017	.723	.989	1.04	1.03	1.009	1.015
6	1.008	1.035	1.032	1.039	1.026	.724	.985	1.06	1.05	1.014	1.023
8	1.011	1.047	1.044	1.053	1.036	.726	.982	1.09	1.07	1.019	1.031
10	1.014	1.059	1.056	1.068	1.045	0.728	0.980	1.11	1.09	1.024	1.039
12	1.017	1.071	1.068	1.084	1.055	.729	.978	1.13	1.11	1.030	1.047
14	1.020	1.083	1.080	1.099	1.065	.731	.976	1.16	1.12	1.035	1.056
16	1.023	1.096	1.093	1.114	1.075	.733	.974	1.18	1.14	1.040	1.065
18	1.026	1.108	1.106	1.129	1.085	.734	.972	1.20	1.16	1.045	1.073
20	1.028	1.121	1.119	1.144	1.095	0.736	0.970	1.22	1.18	1.050	1.082
22	1.031	1.134	1.132	1.160	1.106	.738	.968	1.24	1.20	1.055	1.091
24	1.034	1.147	1.146	1.175	1.116	.739	.965	1.27	1.22	1.061	1.100
26	1.036	1.160	1.159	1.191	1.127	.741	.963	1.29	1.24	1.066	1.110
28	1.039	1.173	1.174	1.207	1.138	.743	.961	1.31	1.26	1.071	1.119
30	1.041	1.186	1.188	1.224	1.149	0.745	0.959	1.33	1.29	1.076	1.129
32	1.044	1.199	1.204	1.240	1.160	.746	.957	1.35	1.31	1.081	1.138
34	1.046	1.213	1.218	1.257	1.171	.748	.955	1.37	1.33	1.086	1.148
36	1.048	1.226	1.233	1.274	1.182	.750	.953	1.39	1.36	1.092	1.158
38	1.050	1.239	1.248	1.290	1.193	.751	.950	1.41	1.39	1.097	1.168
40	1.052	1.252	1.264	1.306	1.205	0.753	0.947	1.44	1.41	1.102	1.178
42	1.054	1.265	1.280	1.323	1.217	.754	.945	1.46	1.43	1.107	1.189
44	1.056	1.278	1.297	1.340	1.229	.756	.943	1.48	1.45	1.112	1.199
46	1.058	1.292	1.313	1.361	1.240	.757	.940	1.50	1.48	1.117	1.210
48	1.060	1.305	1.330	1.380	1.253	.759	.938	1.52	1.51	1.122	1.221
50	1.062	1.318	1.348	1.399	1.26	0.760	0.935	1.54	1.53	1.127	1.232
52	1.063	1.330	1.365	1.418	1.28	.761	.932	1.56	1.56	1.132	1.243
54	1.065	1.342	1.383	1.438	1.29	.763	.929	1.58	1.58	1.137	1.254
56	1.066	1.353	1.401	1.459	1.30	.764	.926	1.60	1.61	1.143	1.266
58	1.067	1.364	1.420	1.480	1.32	.765	.923	1.62	1.64	1.148	1.277
60	1.069	1.375	1.439	1.502		0.766	0.919	1.64	1.66	1.153	1.289
62	1.070	1.386		1.525		.767	.915	1.66	1.69	1.158	1.301
64	1.071	1.396		1.540		.769	.911	1.68	1.72	1.163	1.313
66	1.072	1.405		1.568		.770	.905	1.70	1.75	1.168	1.325
68	1.073	1.414		1.591		.771	.900	1.73	1.77	1.173	1.337
70	1.073	1.423		1.615		0.773	0.896	1.75	1.79	1.178	1.350
72	1.074	1.431		1.638		.774	.890			1.183	1.363
74	1.074	1.438		1.662		.775	.885			1.188	1.375
76	1.075	1.445		1.686		.777	.880			1.193	
78	1.075	1.453		1.710		.778	.873			1.198	
80	1.075	1.460		1.734		0.779	0.868	1.8?	2.0?	1.203	
82	1.075	1.467		1.758		.781	.862			1.209	
84	1.074	1.474		1.774		.782	.857			1.214	
86	1.074	1.481		1.791		.784	.851			1.220	
88	1.073	1.488		1.807		.785	.846			1.225	
90	1.071	1.495		1.819		0.786	0.840	1.9?	2.1?	1.231	
92	1.070	1.502		1.829		.788	.835			1.237	
94	1.069	1.509		1.836		.789	.829			1.242	
96	1.064	1.516		1.840		.791	.823			1.248	
98	1.060	1.523		1.841		.793	.817			1.254	
100	1.055	1.530		1.839		0.794	0.810	2.0?	2.2?	1.260	

Per Cent. %	Hydrochloric Acid. HCl	Ammonia Gas H_3N	Carbonate of Potas. K_2CO_3	Chloride of Calcium $CaCl_2$	Chloride of Zinc $ZnCl_2$	Hypo Sodium $Na_2S_2O_3 .5H_2O$	Nitrate of Copper CuN_2O_6	Nitrate of Sodium $NaNO_3$	Sulph. Iron $FeSO_4 .7H_2O$	Sulph. Magnes. $MgSO_4 .7H_2O$	Sulph. Zinc $ZnSO_4 .7H_2O$
0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2	1.009	.990	1.017	1.016	1.019	1.010	1.011	1.012	1.010	1.009	1.012
4	1.019	.982	1.036	1.033	1.036	1.020	1.024	1.025	1.020	1.018	1.023
6	1.029	.974	1.054	1.051	1.052	1.031	1.038	1.039	1.031	1.028	1.034
8	1.039	.966	1.073	1.068	1.071	1.041	1.052	1.053	1.042	1.038	1.046
10	1.049	0.958	1.092	1.086	1.090	1.052	1.065	1.067	1.053	1.048	1.058
12	1.059	.951	1.111	1.105	1.109	1.063	1.077	1.082	1.064	1.058	1.072
14	1.069	.944	1.131	1.123	1.127	1.074	1.091	1.096	1.076	1.068	1.084
16	1.079	.937	1.151	1.142	1.145	1.085	1.105	1.110	1.087	1.078	1.096
18	1.089	.930	1.171	1.162	1.164	1.097	1.119	1.125	1.099	1.088	1.109
20	1.100	0.924	1.192	1.181	1.185	1.108	1.134	1.141	1.111	1.099	1.123
22	1.110	.918	1.213	1.202	1.206	1.120	1.151	1.157	1.124	1.109	1.136
24	1.120	.912	1.234	1.222	1.227	1.131	1.171	1.173	1.136	1.120	1.149
26	1.130	.907	1.256	1.244	1.248	1.143	1.191	1.189	1.148	1.131	1.163
28	1.140	.902	1.278	1.265	1.269	1.155	1.211	1.206	1.160	1.142	1.178
30	1.150	0.897	1.300	1.287	1.290	1.167	1.231	1.223	1.173	1.153	1.193
32	1.160	.892	1.323	1.310	1.315	1.179	1.250	1.240	1.186	1.164	1.208
34	1.170	.887	1.346	1.332	1.339	1.191	1.270	1.258	1.199	1.175	1.223
36	1.180	.883	1.370	1.355	1.365	1.204	1.290	1.276	1.212	1.186	1.239
38	1.190		1.394	1.379	1.391	1.216	1.310	1.295	1.225	1.198	1.254
40	1.200		1.418	1.402	1.419	1.229	1.331	1.315	1.238	1.210	1.270
42			1.442		1.445	1.242	1.352	1.335		1.222	1.287
44			1.467		1.472	1.255	1.374	1.355		1.234	1.303
46			1.492		1.499	1.268	1.397	1.375		1.246	1.319
48			1.518		1.532	1.281	1.420	1.396		1.259	1.336
50			1.543		1.565	1.294	1.443	1.417		1.271	1.352
52			1.570		1.599		1.468			1.284	1.369
54					1.633		1.493			1.297	1.387
56					1.668		1.519				1.405
58					1.703						1.424
60					1.739						1.444

Per Cent. %	Acet. Lead $PbC_4H_6O_4 .3H_2O$	Carbonate of Sodium Na_2CO_3	Chloride of Ammonium H_4NCl	Chloride of Magnesium $MgCl_2$	Chloride of Sodium NaCl	Chloride of Potassium KCl	Bichromate of Potas. $K_2Cr_2O_7$	Nitrate of Potassium KNO_3	Sulph. Copper $CuSO_4 .5H_2O$	Sulph. Sodium $Na_2SO_4 .10H_2O$	Sulphurous Anhydride SO_2
0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1·999
2	1.013	1.021	1.005	1.016	1.014	1.012	1.01	1.011	1.012	1.007	1·004
4	1.027	1.042	1.012	1.033	1.028	1.025	1.03	1.023	1.024	1.015	1·009
6	1.042	1.063	1.018	1.050	1.043	1.038	1.04	1.035	1.037	1.023	1·015
8	1.057	1.084	1.024	1.067	1.058	1.052	1.05	1.048	1.051	1.031	1·021
10	1.072	1.106	1.030	1.085	1.072	1.065	1.07	1.061	1.064	1.039	1.027
12	1.088	1.128	1.036	1.103	1.088	1.079	1.08	1.074	1.078	1.047	1.033
14	1.105	1.150	1.042	1.121	1.103	1.093	1.10	1.088	1.091	1.055	1.040
16	1.121	1.175	1.047	1.140	1.118	1.107		1.102	1.105	1.063	1.047
18	1.132	1.200	1.053	1.158	1.134	1.121		1.116	1.120	1.072	1.054
20	1.155		1.058	1.177	1.150	1.135		1.131	1.134	1.080	1.062
22	1.173		1.064	1.197	1.167	1.150		1.146	1.149	1.088	
24	1.192		1.069	1.217	1.183	1.165		1.161	1.165	1.096	
26			1.075	1.237	1.200				1.181	1.105	
28				1.258					1.197	1.113	
30				1.278					1.214	1.122	

Per Cent. %	Hydrochloric Acid. HCl	Nitric Acid HNO_3	Sulphuric Acid H_2SO_4	Alcohol C_2H_6O	Ammonia Gas. H_3N	Carbonate Potassium K_2CO_3	Carbonate Sodium Na_2CO_3	Chloride Calcium $CaCl_2$	Chloride Sodium NaCl	Hydrate Potassium KOH	Hydrate Sodium NaOH
0	100	100	100	100	100	100	100	100	100	100	100
2	102	..	101	98	92	100	100	100	100	100	100
4	103	..	101	96	86	100	100	101	101	100	100
6	105	..	102	94	79	101	100	101	101	101	101
8	107	..	102	93	71	101	101	101	101	101	101
10	109	..	103	91	65	101	101	101	102	101	101
12	111	..	103	90	59	101	101	102	102	101	102
14	109	..	104	89	53	101	101	102	103	101	102
16	106	..	104	88	47	101	101	103	103	102	103
18	102	..	105	87	41	102	102	103	104	103	103
20	88	104	105	86	36	102	102	104	105	103	104
22	73?	104	106	86	30	102	102	105	105	104	105
24	59?	105	106	85	25	103	103	106	106	105	106
26	48?	106	107	85	20	103	103	107	107	106	107
28	..	107	108	84	15	104	104	108	108	107	108
30	..	108	109	84	10	104	104	109	..	109	110
32	..	109	111	84	5	105	105	111	..	110	112
34	..	110	112	83	0	106	105	113	..	112	114
36	..	110	114	83	—5?	107	..	114	..	114	116
38	..	111	116	83	..	108	..	116	..	116	118
40	..	112	118	83	..	109	..	118	..	119	120
42	..	113	120	82	..	110	..	120	..	122	123
44	..	114	122	82	..	111	..	122	..	125	125
46	..	115	124	82	..	112	..	124	..	128	128
48	15?	116	126	82	..	113	..	126	..	131	131
50	..	117	128	82	..	114	..	128	..	134	134
52	..	118	130	81	..	115	..	131	..	137	137
54	..	119	133	81	..	116	..	134	..	140	140
56	..	119	137	81	..	..	..	138	..	143	143
58	..	119	139	81	..	..	..	141	..	146	146
60	..	120	142	81	..	..	..	144	..	149	149
62	..	120	145	81	..	..	..	148	..	..	..
64	..	120	150	81	..	..	..	152	..	..	..
66	..	119	156	80	..	..	..	156	..	..	..
68	..	116	163	80	..	..	..	160	..	..	..
70	..	..	170	80	..	..	..	164	..	..	..
72	..	..	176	80	..	..	..	169	..	..	..
74	..	..	183	80	..	..	..	173	..	..	..
76	..	..	190	80	..	..	..	178	..	..	..
78	..	..	198	80	..	..	..	..	..	..	..
80	..	100?	206	79	..	..	..	..	..	..	..
82	..	..	214	79	..	..	..	..	..	..	316?
84	..	..	225	79	..	..	..	..	..	..	..
86	..	..	236	79	..	..	..	..	..	316?	..
88	..	..	248	79	..	..	..	..	..	..	..
90	..	..	260	79	..	..	..	..	..	..	..
92	..	..	274	79	..	..	..	..	..	..	..
94	..	..	288	78	..	..	..	..	..	..	..
96	..	40?	303	78	..	..	..	..	..	..	..
98	..	..	318	78	..	..	..	..	..	red	red
100	..	..	333	78	..	..	..	..	..	heat	heat

Name	Symbol	Per Cent by Weight 0	1	2	3	4	5	6	8	10	12	14	16	18	20	25	30	35	40	45	50	60	70	80	90	100
Acid Acetic	$HC_2H_3O_2$	**1.00**	.99	.99	.99	.98	.98	.98	.97	.96	.95	.95	.94	.93	.92	.90	.88	.86	.83	.81	**.78**	.73	.68	**.62**	.56	**.50**
" Hydrochloric...	HCl	**1.00**	.98	**.96**	.95	.94	.93	.91	.88	.85	.82	.79	**.76**	.	.	.	.	.	.	.	.	.	.	.	.	.
" Nitric..........	HNO_3	**1.00**	1.00	.99	**.98**	.97	.95	**.93**	.91	.90	.88	.87	.85	.84	.82	**.78**	.	.	.	.	.	.	.	.	.	.
" Sulphuric......	H_2SO_4	**1.00**	.99	**.98**	.97	**.97**	.95	**.95**	.93	.92	.90	.89	.87	.86	.84	.80	.76	.72	.68	.64	.60	**.53**	.47	.42	.38	**.33**
" Tartaric.........	$H_2C_4H_4O_6$	**1.00**	1.00	.99	.99	**.98**	.98	.97	.95	.93	.92	**.91**	.90	.89	.88	.85	.82	.80	.77	**.74**	.	.	.	.	.	.
Alcohol. Ethyl.......	C_2H_5OH	**1.00**	1.00	1.01	1.01	1.01	1.02	1.02	1.03	**1.03**	1.03	1.04	1.04	1.05	**1.05**	1.04	**1.03**	1.00	**.98**	.	.	.	.	.	.	**.65**
" Methyl	CH_3OH	**1.00**	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06	**1.07**	1.07	1.07	1.07	**1.07**	1.03	**.98**	.	.	.	.	.	.	.	.	**.65**
Ammonia	H_3N	**1.00**	1.00	**.999**	.999	**.999**	.998	**.997**	.995	.99	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Carbonate Sodium ..	Na_2CO_3	**1.00**	.99	.97	**.95**	.94	.93	.92	.91	**.89**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Chloride Ammonium.	H_4NCl	**1.00**	.99	**.98**	.96	.95	**.94**	.94	.93	.92	.90	.88	.86	.85	.83	.79	**.75**	.	.	.	.	.	.	.	.	.
" Calcium....	$CaCl_2$	**1.00**	.99	.98	.97	**.96**	.94	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Potassium ..	KCl	**1.00**	.99	**.97**	.96	.94	.93	.92	**.90**	.88	.86	.84	.82	.80	**.78**	.73	.	.	.	.	.	.	.	.	.	.
" Sodium	$NaCl$	**1.00**	**.98**	.97	.96	.95	.94	**.93**	.91	.89	.87	.86	.84	.82	.81	**.79**	.	.	.	.	.	.	.	.	.	.
Hydrate Potassium...	KOH	**1.00**	.98	**.97**	**.95**	.94	.93	.92	.90	**.87**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Sodium.....	$NaOH$	**1.00**	**.98**	.97	.95	**.94**	.94	.93	.92	.91	.90	.89	.88	.87	.86	**.83**	.	.	.	.	.	.	.	.	.	.
Nitrate Ammonium..	H_4NNO_3	**1.00**	.99	.98	.97	**.96**	.95	.94	**.92**	.91	.89	.88	.87	.86	.85	.82	.79	.76	.73	**.70**	.	.	.	.	.	.
" Potassium ...	KNO_3	**1.00**	.99	.97	**.96**	.95	.94	.93	.92	**.90**	.88	.87	.85	**.83**	.	.	.	.	.	.	.	.	.	.	.	.
" Sodium......	$NaNO_3$	**1.00**	.99	**.98**	.97	.96	.95	.94	.93	**.91**	.89	.87	.85	.83	.82	.80	**.78**	.76	.	.	.	.	.	.	.	.
Sugar...............	$C_{12}H_{22}O_{11}$	**1.00**	.99	.99	.98	**.98**	.97	.96	.95	.94	.93	.92	**.91**	.90	.89	.86	.83	.81	.78	**.75**	.	.	.	.	.	.
Sulphate Ammonium	$(H_4N)_2SO_4$	**1.00**	.99	.98	**.97**	.95	.94	**.93**	.92	.90	.88	.86	.85	.83	**.82**	.	.	.	.	.	.	.	.	.	.	.
" Copper	$CuSO_4$	**1.00**	.99	.98	.96	**.95**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Iron	$FeSO_4$	**1.00**	.99	.98	.96	**.95**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Magnesium	$MgSO_4$	**1.00**	.99	.97	**.95**	.94	.93	**.92**	.90	.88	.86	.84	.82	.80	.78	**.74**	.	.	.	.	.	.	.	.	.	.
" Sodium....	Na_2SO_4	**1.00**	.99	.98	.97	**.96**	.94	.93	**.92**	**.90**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Zinc	$ZnSO_4$	**1.00**	.99	.98	.96	**.95**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Most of the numbers in this table were obtained by interpolation. Those nearest observed values are printed in heavy type.

Name	Symbol	Per Cent by Weight																								
		0	1	2	3	4	5	6	8	10	12	14	16	18	20	25	30	35	40	45	50	60	70	80	90	100
Acid, Acetic . . .	$HC_2H_3O_2$	0	.05	.07	.08	.10	.11	.12	.14	.15	.15	.15	.16	.16	.16	.16	.15	.13	.11	.09	.07	.05	.04	.02	.01	0.0
„ Hydrochloric	HCl	0	8	16	24	32	39	44	53	62	69	73	74	75	75	72	66	59	51							
„ Nitric . . .	HNO_3	0	6	11	16	21	26	31	39	46	53	59	64	69	73	76	77	76	73	68	63	51	40	27		
„ Oxalic . . .	$H_2C_2O_4$	0	1	3	4	5	6	6	8																	
„ Phosphoric .	H_3PO_4	0	1	2	2	3	3	4	5	6	7	8	9	10	11	13	15	17	18	19	20	18	14	10	6	
„ Sulphuric .	H_2SO_4	0	4	8	12	16	20	25	32	39	45	51	57	61	65	71	73	72	68	62	54	37	21	11	10	8
„ Tartaric . .	$H_2C_4H_4O_6$	0	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.8	0.9	0.9	0.9	1.0	1.0	1.0	1.0	0.9	0.8	0.6	0.5					
Bromide Potassium	KBr	0	1	2	3	4	5	6	8	10	12	14	16	18	19	24	29	34								
Carbonate „ .	K_2CO_3	0	1	2	3	4	6	7	9	12	14	16	18	20	21	22	22	21	19	17	15					
„ Sodium	Na_2CO_3	0	1	2	3	4	4	5	6	7	7	8														
Chloride Ammon. .	H_4NCl	0	2	4	6	8	9	11	14	18	21	24	27	30	33	40										
„ Potassium	KCl	0	1	3	4	6	7	9	11	14	16	19	20	22	24	28										
„ Sodium .	$NaCl$	0	1	3	4	5	7	8	10	11	13	14	16	17	19	21										
„ Zinc . .	$ZnCl_2$	0	1	2	3	4	5	6	6	7	8	8	8	9	9	9	9	8	7	6	5	4				
Hydrate Potas. .	KOH	0	3	6	9	12	16	19	25	31	37	42	46	49	51	54	54	51	45							
Iodide „ .	KI	0	1	2	2	3	3	4	6	7	9	10	11	13	15	19	23	27	31	34	38	46				
Nitrate Ammon. .	H_4NNO_3	0	1	2	3	4	6	7	9	11	13	15	17	19	21	25	28	30	32	34	36					
„ Copper .	CuN_2O_6	0	1	2	3	3	4	4	5	6	6	7	8	8	9	11	11	10								
„ Potassium.	KNO_3	0	1	2	3	4	4	5	6	8	9	11	13	14	15											
„ Silver . .	$AgNO_3$	0	1	1	2	2	3	3	4	5	6	7	8	9	9	10	12	14	16	18	20	21				
„ Sodium .	$NaNO_3$	0	1	2	3	4	4	5	6	7	8	10	11	12	13	15	16									
Sulphate Ammon. .	$(H_4N)_2SO_4$	0	1	2	3	4	5	6	8	9	11	13	15	16	18	20	22									
„ Copper . .	$CuSO_4$	0	0	1	2	2	2	2	3	3	3	4	4	4												
„ Magnesium	$MgSO_4$	0	1	1	2	2	2	3	4	4	4	5	5	5	5	4										
„ Potassium .	K_2SO_4	0	1	1	2	3	4	5	7	8																
„ Sodium .	Na_2SO_4	0	1	1	2	3	4	5	6	7	8	8	9													
„ Zinc . . .	$ZnSO_4$	0	0	1	1	2	2	2	3	3	3	4	4	4	4.8	4.4	4.1	3								

The numbers in this table must be multiplied by 0.000,000,000,01 (10^{-11}) to reduce them to the C.G.S. System. The conductivity of the solutions named above diminishes as the temperature rises at the rate of about 2 % per degree, with the exception of sulphuric and phosphoric acids, in which the rate varies from 1 to 4% according to the strength of the solution. The conductivities were determined at about 18°.

81. B. Refractive and Dispersive Indices of Solutions at about 18°.

Name and Symbol	Per Cent.	Index of Refract.	Index of Dispers.	Name and Symbol	Per Cent.	Index of Refract.	Index of Dispers.
Acid, acetic $HC_2H_3O_2$	0	1.333	.014	Chloride Amm. H_4NCl	10	1.351	.016
" " "	20	1.348	..	" " "	20	1.370	.018
" " "	40	1.362	..	" Calcium $CaCl_2$	20	1.384	.019
" " "	60	1.371	..	" " "	40	1.441	.023
" " "	80	1.378	..	" Sodium $NaCl$	10	1.350	.016
" " "	100	1.374	.017	" " "	20	1.368	.018
" Hydrochloric, HCl	35	1.413	.023	" Zinc $ZnCl_2$	20	1.370	.018
" Nitric, HNO_3	50	1.401	.024	" " "	40	1.410	.021
" Sulphuric, H_2SO_4	0	1.333	.014	Hydrate Potas. KOH	40	1.403	.018
" " "	20	1.358	..	" Sodium, $NaOH$	10	1.359	.016
" " "	40	1.382	.016	" " "	20	1.384	.018
" " "	60	1.410	..	" " "	30	1.404	.020
" " "	80	1.434	.018	Nitrate Sodium $NaNO_3$	20	1.355	.017
" " "	100	1.434?	.017?	" " "	40	1.380	.021
Alcohol, C_2H_5OH	0	1.333	.014	Sugar, $C_{12}H_{22}O_{11}$	10	1.348	.015
" "	40	1.358	.015?	" "	20	1.364	.016
" "	100	1.360	.015?	" "	30	1.381	.017

81. C. Table for preparing Mixtures of any Desired Strength.

Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B	Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B	Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B	Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B	Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B
0	0.000	100	20	25.000	80	40	66.667	60	60	150.00	40	80	400.00	20
1	1.010	99	21	26.582	79	41	69.492	59	61	156.41	39	81	426.32	19
2	2.041	98	22	28.205	78	42	72.414	58	62	163.16	38	82	455.56	18
3	3.093	97	23	29.870	77	43	75.439	57	63	170.27	37	83	488.24	17
4	4.167	96	24	31.579	76	44	78.571	56	64	177.78	36	84	525.00	16
5	5.263	95	25	33.333	75	45	81.818	55	65	185.71	35	85	566 67	15
6	6.383	94	26	35.135	74	46	85.185	54	66	194.12	34	86	614.28	14
7	7.527	93	27	36.986	73	47	88.679	53	67	203.03	33	87	669.23	13
8	8.696	92	28	38.889	72	48	92.308	52	68	212.50	32	88	733.33	12
9	9.890	91	29	40.845	71	49	96.078	51	69	222.58	31	89	809.09	11
10	11.111	90	30	42.857	70	50	100.00	50	70	233.33	30	90	900.00	10
11	12.360	89	31	44.928	69	51	104.08	49	71	244.83	29	91	1,011.1	9
12	13.636	88	32	47.059	68	52	108.33	48	72	257.14	28	92	1,150.0	8
13	14.943	87	33	49.254	67	53	112.77	47	73	270.37	27	93	1,328.6	7
14	16.279	86	34	51.515	66	54	117.39	46	74	284.62	26	94	1,566.7	6
15	17.647	85	35	53.846	65	55	122.22	45	75	300.00	25	95	1,900.0	5
16	19.048	84	36	56.250	64	56	127.27	44	76	316.67	24	96	2,400.0	4
17	20.482	83	37	58.730	63	57	132.56	43	77	334.78	23	97	3,233.3	3
18	21.951	82	38	61.290	62	58	138.10	42	78	354.55	22	98	4,900.0	2
19	23.457	81	39	63.934	61	59	143.90	41	79	376.19	21	99	9,900.0	1
20	25.000	80	40	66.667	60	60	150.00	40	80	400 00	20	100	∞	0

81. D. Coefficients of Diffusion of Saline Solutions in Water.

Hydrochloric Acid. . . .	.000, 0100	Sulphate of Potassium .	.000, 0037
Hydrate of Potassium . .	.000, 0070	Sulphate of Sodium . .	.000, 0030
Sulphuric Acid	.000, 0052	Sulphate of Magnesium .	.000, 0020
Nitrate of Potassium . .	.000, 0052	Sugar	.000, 0019
Common Salt	.000, 0046	Gum Arabic	.000, 0010
Nitrate of Sodium . . .	.000, 0040	Albumen	.000, 0002
. . . .	. . . .	Caramel	.000, 0001

31, E. Rotation in degrees of the Plane of Polarization for the Fraunhofer Lines A—H, produced by passing through 100 cm, of various solutions, containing in each case 1 gram of a given substance in 100 cu. cm.

Name and Symbol of Substance		A	B	C	D	E	F	G	H
Acid. Malic. $H_2C_4H_4O_5$+aq	—	..	..	..	0.3				
„ Tartaric $H_2C_4H_4O_6$+aq.	+	..	..	1.2	1.5	1.9	2.0		
Camphor $C_{10}H_{16}O$+alcohol.	+	..	..	3.0	4.2	6.1	8.0		
Cholesterine $C_{26}H_{44}O$+ether	—	..	2.0	2.6	3.2	4.0	4.9	6.2	
Cinchonidine $C_{20}H_{24}N_2O$+alcohol.	—	..	..	..	11				
Cinchonine $C_{20}H_{24}N_2O$+alcohol.	+	..	..	..	24				
Conchinine $C_{20}H_{24}N_2O_2 2\frac{1}{2}H_2O$+alcohol	+	..	..	..	24				
Glycocol CH_2NH_2COOH+alcohol.	+	..	..	2.2	2.9	3.8	4.9	5.7	
Malate of Ammon $(H_4N)_2C_4H_4O_5$+aq.	—	..	..	..	2.2				
„ „ Lithium $Li_2C_4H_4O_5$+aq.	—	..	..	..	1.3				
„ „ Potassium $K_2C_4H_4O_5$+aq.	—	..	..	..	0.7				
„ „ Sodium $Na_2C_4H_4O_5$+aq.	—	..	..	..	1.0				
Morphine chl. $C_{17}H_{19}NO_3HCl.3H_2O$+aq.	—	..	..	..	10				
„ sulph. $2C_{17}H_{19}NO_3.H_2SO_4.5H_2O$+aq.	—	..	..	..	10				
Quinine hydr. $C_{20}H_{24}N_2O_2.3H_2O$+alcohol	—	..	..	..	15				
„ sulph. $C_{20}H_{24}N_2O_2.H_2SO_4.7H_2O$+aq.	—	..	..	..	16				
Salicine $C_{13}H_{18}O_7$+aq.	—	..	..	..	6.5				
Santonid, para-; $C_{15}H_{18}O_3$+alcohol.	+		58	66	89	126	167		
Santonine $C_{15}H_{18}O_3$+alcohol.	—		11	12	16	22	26		
Sugar (Cane-) $C_{12}H_{22}O_{11}$+aq.	+	3.8	4.8	5.3	6.65	8.5	10.1	13.2	15.7
„ grape „	+	..	..	..	5				
„ milk „	+	..	..	..	5				
„ maltose „	+	..	..	..	14				
„ lactose „	+	..	..	..	8				
„ inverted „	—	..	..	..	2.9				

31, F. Rotation in degrees of the Plane of Polarization for Fraunhofer Lines A—H produced by plates of various substances 1 cm. thick.

Name and Symbol of Substance		A	B	C	D	E	F	G	H
Benzil, $C_{14}H_{10}O_2$		..	..	..	248				
Bromate of Sodium $NaBrO_3$		..	..	..	28				
Chlorate „ $NaClO_3$		..	24	25	32	40	46	59	69
Cinnabar, HgS		..	3000	?					
Diacetylphenolphthaleine		..	168 ?	..	197	246	?		
Ethylenediaminesulphate		..	..	..	155				
Guanidine Carbonate		..	123 ?	..	146	178	?		
Hyposulphate of Calcium $CaS_2O_6.4H_2O$		..	..	..		21			
„ Lead $PbS_2O_6.4H_2O$		..	..	41	55	73	89		
„ Potassium $K_2S_2O_6.2H_2O$		..	..	62	84	105	123		
„ Strontium $SrS_2O_6.4H_2O$		..	..	..	..	16			
Iodate sodium, per- $NaIO_4$		..	..	194	233	285	342	471	
Nicotine (liquid) $C_{10}H_{14}N_2$	—	..	..	..	16				
Quartz (ordinary right handed) SiO_2	+	127	157	173	217	275	327	425	511
Strychnine (sulphate) $2C_{21}H_{22}N_2O_2.H_2SO_4$		..	108	?					
Tartaric Ether (liquid) $(C_2H_5)_2C_4H_4O_6$	+	..	..	..	0.8				
Turpentine right handed $C_{10}H_{16}$	+	..	..	..	14.1				
„ (liquid) left handed $C_{10}H_{16}$	—	..	..	..	37.0				

81, G. Rotation of the Plane of Polarization caused by a Unit Magnetic Field (C. G. S.) in Unit Thicknesses of Different Substances.

Bisulphide of Carbon (sodium light) 0°.0070	Water (white light) . . . . 0°.0001
" " " (thallium ") 0°.0086	Coal gas . . . . 0°.000,000,2

Note. In these, and in nearly all cases, the rotation is with the current producing the magnetic field. A solution of ferric chloride in methyl alcohol is mentioned as one of the exceptions to this rule (Deschanel, § 839).

81, H. Magnetic Moment of 1 cu. cm. of various substances (C. G. S.)

Name of Substance	Magnetization induced by Unit Field	Maximum Magnetization	Maximum Permanent Magnetization	Name of Substance	Magnetization induced by Unit Field
Iron	300?	1400	..	Nickel Oxide	+0.1?
Steel	70?	1400	< 800	Water	−0.01?
Cobalt	300?	800?	..	Bismuth . . .	−0.01?
Nickel	140?	500	..	Phosphorus .	−0.004?
Iron Oxide .	0.2?	..	..		

81, I. Coefficients of Friction (f) for water corresponding to Velocities (v) in centimetres per second (From Weisbach).

v	*f*	*v*	*f*	*v*	*f*	*v*	*f*	*v*	*f*
0	∞	100	.00299	200	.00264	300	.00249	400	.00239
10	.00554	110	.00293	210	.00262	310	.00248	410	.00239
20	.00445	120	.00288	220	.00260	320	.00247	420	.00238
30	.00396	130	.00284	230	.00258	330	.00246	430	.00238
40	.00368	140	.00280	240	.00256	340	.00245	440	.00236
50	.00347	150	.00276	250	.00255	350	.00244	450	.00236
60	.00333	160	.00274	260	.00254	360	.00243	460	.00235
70	.00321	170	.00271	270	.00253	370	.00242	470	.00235
80	.00312	180	.00269	280	.00251	380	.00241	480	.00234
90	.00305	190	.00266	290	.00250	390	.00240	490	.00234
100	.00299	200	.00264	300	.00249	400	.00239	500	.00233

81, J. Coefficients of Friction of Solids on Solids.

	Oak	Hard Wood	India Rubber	Leather	Hemp.	Bronze	Iron	Cast Iron
Oak	.2—.5	.38	...	.30	.52	.48	..	.49
" soaped	.16	..	...	...	...	.16	..	.19
Bronze	.48	..	...	...	...	.20	.18	.21
Iron (cast, smooth) .	.49	..	.56	.2	.08	.2	.2—.4	.2—.4
" " wet . . .	.24	..	.36	.36	..	.31	..	.31
" " greased .	.08	..	.20	.15	..	.15	..	.15

81, K. Action of Plates (1 cm thick and bounded by plane surfaces) upon normally incident Radiant Heat.

	Substance	Reflects	Absorbs	Transmits		Substance	Reflects	Absorbs	Transmits
White Heat	Lampblack. . . .	0%	100%	0%	White Heat	Water	4%	86%	10%
	India Ink	5	95	0		AqueousSolutions	4	86	10
	Ice	4	91	5?		Alcohol	5	82	13
	Alum.	6	86	8?		Ether.	5	75	20
	White Lead . . .	44?	56	0		Oils	6	73	21
	Glass.	8	62	30		Chloroform . . .	6	69?	25?
	Shellac.	8	47	45		Turpentine. . . .	6	64	30
	Polished Metals .	80?	20?	0		Bisulphide Carbon	12	35	53
	Rock Salt	8	0	92		Mercury	75?	25?	0
Steam Heat	Lampblack. . . .	0	100	0	Steam Heat	Shellac.	8	72	20
	White Lead . . .	0	100	0		Mercury	75?	25?	0
	Ice	4	96	0		Polished Metals .	80?	20?	0
	Alum.	6	94	0		Rock Salt	8	0?	92
	Glass.	8	92	0		Vapors at 1 cm .	0	0—10?	90?—100
	India Ink.	10	90	0		Perm. Gases 76 cm	0	0—.02?	99.98+

81, L. Estimates of the number of Units of Heat radiated in 1 sec. by 1 sq. cm. blackened surface in space at 0°.

Temp.	Rad.	Temp.	Rad.	Temp.	Rad.	Temp.	Rad.	Temp.	Rad.	Temp.	Rad.	Temp.	Rad.
−273°	−.019?	+100°	+.012? *	500°	.1?	900°	.5?	1300°	2? **	1700°	5?	2500°	60? ✝*
−200°	−.015?	200°	.028? *	600°	.2? †	1000°	.7? //	1400°	2? **	1800°	7? ††	3000°	270? ✝*
−100°	−.009?	300°	.05? *	700°	.2? §*	1100°	.9? //	1500°	3?	1900°	10?	3500°	1200? §§
0°	.000	400°	.08? *	800°	.3? §	1200°	1.2? †§	1600°	4?	2000°	13?	4000°	5400? §§

* Dark. † Dull red. §* "Red Heat". § Cherry Red. // Orange. †§ Yellow. ** "White Heat". †† Flame. ✝* Voltaic Arc Light. §§ Sunlight.

32a. Heats of Combustion in Oxygen.

Name of Substance Consumed.	Chemical Reaction involving 16.0 grams of Oxygen in each case.	Grams of Substance consumed.	Grams of Product formed.	Units of Heat developed.	Units of Heat per gram consumed.	Megergs per milligram consumed.	Electromotive force in volts.
Acetylene	$2\,C_2H_2 + 5\,O_2 = 4\,CO_2 + 2\,H_2O$	5.2	21.2	62,000	12,000	500	
Alcohol	$C_2H_6O + 3\,O_2 = 2\,CO_2 + 3\,H_2O$	7.7	23.7	54,000	7,000	290	
Arsenic	$As_4 + 3\,O_2 = 2\,As_2O_3$	50.0	66.0	51,500	1,030	43	
"	$As_4 + 5\,O_2 = 2\,As_2O_5$	30.0	46.0	44,000	1,460	61	
Barium	$2\,Ba + O_2 = 2\,BaO$	136.8	152.8	130,000	950	40	
Bismuth	$Bi_4 + 3\,O_2 = 2\,Bi_2O_3$	138.7	154.7	13,300	96	4.0	
Calcium	$2\,Ca + O_2 = 2\,CaO$	39.9	55.9	130,000	3,280	138	
Carbon	$C + O_2 = CO_2$	6.0	22.0	48,000	8,000	334	
Carbonic Oxide	$2\,CO + O_2 = 2\,CO_2$	28.0	44.0	67,000	2,400	100	
Chlorine	$2\,Cl_2 + O_2 = 2\,Cl_2O$	70.7	86.7	− 18,000	− 250	−10	
Copper	$4\,Cu + O_2 = 2\,Cu_2O$	126.2	142.2	40,000	320	13	
"	$2\,Cu + O_2 = 2\,CuO$	63.1	79.1	38,000	600	25	
Ethane	$2\,C_2H_6 + 7\,O_2 = 4\,CO_2 + 6\,H_2O$	4.3	20.3	54,000	12,500	520	
Ether	$C_4H_{10}O + 6\,O_2 = 4\,CO_2 + 5\,H_2O$	6.2	22.2	56,000	9,000	375	
Ethylene	$2\,C_2H_4 + 6\,O_2 = 4\,CO_2 + 4\,H_2O$	4.7	20.6	56,000	12,000	500	
Hydrogen	$2\,H_2 + O_2 = 2\,H_2O$	2.0	18.0	69,000	34,500	1440	1.49
Iodine	$2\,I_2 + 5\,O_2 = 2\,I_2O_5$	50.6	66.6	9,000	177	7.4	
Iron	$2\,Fe + O_2 = 2\,FeO$	55.9	71.9	75,000	1350	56	
"	$3\,Fe + 2\,O_2 = Fe_3O_4$	41.9	57.9	66,000	1575	66	
Lead	$2\,Pb + O_2 = 2\,PbO$	206.4	222.4	50,000	243	10	
Magnesium	$2\,Mg + O_2 = 2\,MgO$	24.0	40.0	146,000	6,100	255	
Mercury	$4\,Hg + O_2 = 2\,Hg_2O$	399.6	415.6	42,000	105	4.4	
"	$2\,Hg + O_2 = 2\,HgO$	199.8	215.8	30,000	153	64	
Methane	$CH_4 + 2\,O_2 = CO_2 + 2\,H_2O$	4.0	20.0	52,000	13,100	550	
Nitrogen	$2\,N_2 + O_2 = 2\,N_2O$	28.0	44.0	18,000	− 650	−27	
"	$N_2 + O_2 = 2\,NO$	14.0	30.0	− 22,000	−1550	−65	
"	$N_2 + 2\,O_2 = 2\,NO_2$	7.0	23.0	− 1,000	− 150	-6.3	
Phosphorus	$P_4 + 5\,O_2 = 2\,P_2O_5$	12.4	28.3	71,000	5750	240	
Potassium	$2\,K_2 + O_2 = 2\,K_2O$	78.1	94.1	136,000	1745	73	
Selenium	$Se_2 + 2\,O_2 = 2\,SeO_2$	39.4	55.4	29,000	730	31	
Silver	$2\,Ag_2 + O_2 = 2\,Ag_2O$	215.4	231.4	5,800	27	1.1	
Sodium	$2\,Na_2 + O_2 = 2\,Na_2O$	46.0	62.0	152,000	3,300	138	
Spermaceti	— — — — — — —	—	—	—	10,300	—	
Stearine	— — — — — — —	—	—	—	9,700	—	
Strontium	$2\,Sr + O_2 = 2\,SrO$	87.3	103.3	131,000	1,500	63	
Sulphide Carbon, Bi-	$CS_2 + 3\,O_2 = CO_2 + 2\,SO_2$	12.7	28.6	43,000	3,400	142	
Sulphur	$S_2 + 2\,O_2 = 2\,SO_2$	16.0	32.0	36,000	2,250	94	
Thallium	$2\,Tl_2 + O_2 = 2\,Tl_2O$	408.0	424.0	42,400	104	4.3	
Tin	$2\,Sn + O_2 = 2\,SnO$	118.0	134.0	68,000	575	24	
"	$2\,Sn + 2\,O_2 = 2\,SnO_2$	59.0	75.0	72,600	1,230	51	
Turpentine	$C_{10}H_{16} + 14\,O_2 = 10\,CO_2 + 8\,H_2O$	4.9	20.8	52,000	10,700	446	
Wax	— — — — — — —	—	—	—	10,500	—	
Wood	(50% C)	—	—	—	4,000	—	
Zinc	$2\,Zn + O_2 = 2\,ZnO$	64.9	80.9	84,400	1,300	54	

32b. Heats of Combustion in Chlorine.

Name of Substance Consumed.	Chemical Reaction involving 70.7 grams of Chlorine in each case.	Grams of Substance consumed.	Grams of Product formed	Units of Heat developed.	Units of Heat per gram consumed.	Megergs per milligram consumed.	Electromotive force in volts.
Antimony	$Sb_4 + 6\,Cl_2 = 4\,SbCl_3$	80.7	151.4	57,000	707	30	1.23
Arsenic	$As_4 + 6\,Cl_2 = 4\,AsCl_3$	49.9	120.6	50,000	994	42	1.08
Copper	$Cu + Cl_2 = CuCl_2$	63.1	133.8	61,000	960	40	1.32
Hydrogen	$H_2 + Cl_2 = 2\,HCl$	2.0	72.7	47,000	23,500	980	1.01
Iron	$2\,Fe + 3\,Cl_2 = Fe_2Cl_6$	37.3	108.0	65,000	1,750	73	1.40
Potassium	$K_2 + Cl_2 = 2\,KCl$	78.1	148.8	207,000	2,650	110	4.48
Tin	$Sn + 2\,Cl_2 = SnCl_4$	59.0	129.7	64,000	1,080	45	1.38
Zinc	$Zn + Cl_2 = ZnCl_2$	64.9	135.6	99,000	1,530	64	2.15

33. Heats of Combination.

Name of Substance Acted upon.	Chemical Reaction involving 16.0 grams of Oxygen or its equivalent.	Grams of Substance consumed.	Grams of Product formed.	Units of Heat developed.	Units of Heat per gram consumed.	Megergs per milligram consumed.	Electromotive force in volts.
Copper	$2\,Cu + O_2 + 2\,SO_3 + Aq. = 2\,CuSO_4 \cdot Aq.$	63.1	159.1	54,200	860	36	1.17
Nitric Oxide	$4\,NO + O_2 + 2\,H_2O + Aq. = 4\,HNO_3 \cdot Aq.$	60.0	94.0	36,300	600	25	0.78
Nitrous Acid	$2\,HNO_2 \cdot Aq. + O_2 = 2\,HNO_3 \cdot Aq.$	47.0	63.0	18,300	390	16	0.40
Zinc	$2\,Zn + O_2 + 2\,SO_3 + Aq. = 2\,ZnSO_4 \cdot Aq.$	64.9	160.9	108,500	1,670	70	2.35

84. Contact differences of Potential in Volts.*

	60% Sulph. Zinc $ZnSO_4.7H_2O.Aq.$	45% " "	20% " "	12% Alum. $Al_2K_2S_4O_{16}.24H_2O,Aq.$	29% Sulph. Copper $CuSO_4.5H_2O.Aq.$	Amalgamated Zinc	Zinc	Lead	Tin	Iron	Brass	Copper	Platinum	Carbon	Mercury	Water (pure)	Nitric Acid (concentrated)
Chloride Ammon.† 25%							—.637	—.189	—.364	—.652	—.348	—.396	.057				
Salt, NaCl, Aq. 24%							—.565	—.267	—.334	—.605	—.435	—.475	—.856				
Sulphate of Copper 29%	095	.102										.070					
$CuSO_4.5H_2O$, Aq. 14%			.090									.103					
Amalgamated Zinc	284					0	—.144	—.357	—.463	—.744	—.822	—.894	—1.125	—1.208		—.100	
Zinc	430	.444	.238	.536		.144	0	—.210	—.281	—.600	—.679	—.750	—.981	—1.096		.0±?	
Lead				.139		.357	.210	0	—.099	—.401	—.472	—.542	—.771	—.858		—.171	
Tin				.225		.463	.281	.099	0	—.313	—.372	—.456	—.690	—.795		—.177	
Iron				.653		.744	.600	.401	.313	0	—.064	—.146	—.369	—.485	—.502	—.148	
Brass				.014		.822	.679	.472	.372	.064	0	—.087	—.287	—.414		—.231	
Copper				.127	—.070	.894	.750	.542	.456	.146	.087	0	—.238	—.370	—.308	—.2	
Platinum				—.246		1.125	.981	.771	.690	.369	.287	.238	0	—.113	—.156	—.3	—.672
Carbon						1.208	1.096	.858	.795	.485	.414	.370	.113	0	—.092	—.1	
Mercury										.502		.308	.156	.092	0		
Water (pure)	.18				.0?	.100	.0±	.171	.177	.148	.231	.2	.3	.1		0	
Sulphuric Acid H_2SO_4, Aq. — trace							—.241										.078
" 5%							—.344										
" 9%						—.358								—.035			
" 17%						—.429											
" 83%								—.120	—.25		.016			.2			
" 100%	1.699			1.456	1.269	.848		1.				1.113	1.5	.7		1.298	
Sulphate of Mercury §															.475		

* Each number in this table represents the potential in volts of one substance (above the number) when brought in contact with a second substance (at the left of the number) at the potential zero. † $H_4NCl + Aq.$ § $3HgSO_4 + 2H_2O + Aq = Hg_3O_2SO_4 + 2H_2SO_4 + Aq.$

85. Electromotive Force of Voltaic cells.

Name of Cell	Negative or Dissolving Pole	Solution next Negative Pole.	Solution next Positive Pole.	Positive Pole	Temperature	Electromotive Force in Volts
[Beetz]	Potas.Amalg.	Hydrate of Potassium.	Permanganates(withMnO_2)*	Sulphur?†	18°?	3.
Bunsen	Amalg. Zinc	30% Sulphuric Acid.	Pure Nitric Acid	Carbon	„	1.96
„	„	„	60% „	„	„	1.89
„	„	„	Bichromates	„	„	1.87
Clark	„	Sulphate of Zinc and	Sulphate of Mercury (paste)	„	15°	1.435
„	„	„ „ „ „	„ „ „ „	„	24°	1.425
Daniell I.	„	60% Sulphuric Acid.	30% Sulphate of Copper	Copper	18°?	1.08
„ „	„	30% „ „	„ „ „ „	„	„	0.98
„ II.	Zinc	1% Sulphate of Zinc.	„ „ „ „	„	„	1.12
„ „	„	5% „ „ „	„ „ „ „	„	„	1.14
„ „	„	10% „ „ „	„ „ „ „	„	„	1.12
„ „	„	20% „ „ „	„ „ „ „	„	„	1.11
„ III.	„	30% Sulphuric Acid.	50% Nitrate of Copper	„	„	1.00
Davy	Amalg. Zinc	25% Chloride of Sodium.	Sulphate of Mercury (paste)	Carbon	„	1.41
Grove	„	30% Sulphuric Acid.	Nitric Acid	Platinum	„	1.93
Leclanché	„	25% Chloride of Ammonium.	Powdered oxides Manganese	Carbon	„	1.32
Silver Chloride	Zinc	„ „ „	Silver Chloride	Silver	„	1.03

* Ganot, § 814. † Daniell, page 553. § Or water. Ganot § 812.

86. Electromotive Force in Volts and Striking Distance in Millimetres.

mm.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0	500	1000	1470	1920	2360	2780	3190	3580	3960
1	4340	4700	5050	5400	5740	6070	6390	6700	7000	7300
2	7600	7890	8170	8450	8730	9000	9270	9540	9810	10070
3	10320	10580	10830	11080	11320	11560	11800	12040	12270	12500

The values in this table are subject to a probable error of about 100 volts.

87a. Specific Electrical Resistances of Conductors at 0°.

Name of Substance	Resistance of a centimetre-cube in microhms.*	Resistance in ohms of a wire 1 m. long 1 mm. diam.	Resistance in ohms of a wire 1 m. long weighing 1 g.	Per Cent of Increase per degree centigrade
Silver, annealed	1.50*	0.019	0.16	0.377
„ hard drawn	1.60	0.020	0.17	..
Copper, annealed	1.58	0.020	0.14	0.388
„ hard drawn	1.61	0.020	0.14	..
Gold, annealed	2.0	0.026	0.39	0.365
„ hard drawn	2.1	0.026	0.41	..
Aluminum annealed	2.8	0.036	0.07	..
Brass	5.5	0.070	0.46	..
Zinc, pressed	5.6	0.073	0.40	0.365
Platinum, annealed	9.0	0.115	1.90	..
Iron, annealed	9.5	0.121	0.74	..
Gold(2) Silver(1) alloy	10.9	0.138	1.65	0.065
Nickel, annealed	12.4	0.157	1.10	..
Tin, pressed	13.0	0.166	0.95	0.365
Lead, „	19.0	0.242	2.16	0.387
German Silver	20.8	0.265	1.77	0.044
Platinum(2) Silver(1) alloy	24.0	0.306	3.3?	0.031
Antimony	35.2	..	2.36	0.389
Mercury (liquid)	94.2	..	..	0.072
Bismuth	130.0	1.656	12.7	0.354
Electric Light Carbons (about)	6000.	..	..	..

* These results must be multiplied by 1000 to reduce them to the C. G. S. system

87b. Specific Electrical Resistances of Insulators.

Name of Substance	Resist. in Ohms of a centimetre-cube †	% increase per °.
Selenium	(about) 60,000.	+ 1.
Gutta Percha	„ 7,000,000,000,000,000.	— 10?
Shellac	„ 9,000,000,000,000,000.	..
Ebonite	„ 30,000,000,000,000,000.	..
Paraffine	„ 30,000,000,000,000,000.	..
Glass	Greater than any above.	great, negative.
Air and other Gases	Practically Infinite.	..

† These results must be multiplied by 1,000,000,000 to reduce them to the C. G. S. System

88. Specific Electrical Resistance in Ohms, of a Centimetre-cube of different Electrolytes (see Table 31).

Per Cent. %	Hydrochloric Acid, HCl	Nitric Acid. HNO_3	Sulphuric Acid H_2SO_4	Sulphate of Copper $CuSO_4$	Sulphate of Zinc $ZnSO_4$	Chloride Ammonium H_4NCl	Chloride Sodium NaCl
5	2.6	3.8	5.0	56.0	55.0	11.6	16.0
10	1.6	2.2	2.6	33.0	33.0	6.0	9.0
15	1.4	1.6	1.9	25.0	26.0	4.0	6.5
20	1.3	1.4	1.5	20.0	23.0	3.2	5.5
25	1.4	1.3	1.4	..	22.5	2.8	5.0
30	1.5	1.3	1.4	..	25.0		
35	1.7	1.3	1.4		30.0		
40	2.0	1.4	1.5				
45		1.5	1.6				
50		1.6	1 9				
60		2.0	2.7				
70		2.5	4.8				
80		3.7	9.0				
90			10.0				
100			12.5				

Note. The results in this table must be multiplied by 1,000,000,000 to reduce them to the C. G. S. System. They are intended to be accurate at about 18°, but are subject to a probable error of about 10%. See Table 31.

39 — Fahrenheit and Centigrade Thermometers.

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
—125	—193.0	0	32.0	25	77.0	50	122.0	75	167.0	100	212.0	225	437.0	350	662
120	184.0	1	33.8	26	78.8	51	123.8	76	168.8	105	221.0	230	446.0	400	752
115	175.0	2	35.6	27	80.6	52	125.6	77	170.6	110	230.0	235	455.0	450	842
110	166.0	3	37.4	28	82.4	53	127.4	78	172.4	115	239.0	240	464.0	500	932
105	157.0	4	39.2	29	84.2	54	129.2	79	174.2	120	248.0	245	473.0	550	1022
100	148.0	5	41.0	30	86.0	55	131.0	80	176.0	125	257.0	250	482.0	600	1112
95	139.0	6	42.8	31	87.8	56	132.8	81	177.8	130	266.0	255	491.0	650	1202
90	130.0	7	44.6	32	89.6	57	134.6	82	179.6	135	275.0	260	500.0	700	1292
85	121.0	8	46.4	33	91.4	58	136.4	83	181.4	140	284.0	265	509.0	750	1382
80	112.0	9	48.2	34	93.2	59	138.2	84	183.2	145	293.0	270	518.0	800	1472
75	103.0	10	50.0	35	95.0	60	140.0	85	185.0	150	302.0	275	527.0	850	1562
70	94.0	11	51.8	36	96.8	61	141.8	86	186.8	155	311.0	280	536.0	900	1652
65	85.0	12	53.6	37	98.6	62	143.6	87	188.6	160	320.0	285	545.0	950	1742
60	76.0	13	55.4	38	100.4	63	145.4	88	190.4	165	329.0	290	554.0	1000	1832
55	67.0	14	57.2	39	102.2	64	147.2	89	192.2	170	338.0	295	563.0	1050	1922
50	58.0	15	59.0	40	104.0	65	149.0	90	194.0	175	347.0	300	572.0	1100	2012
45	49.0	16	60.8	41	105.8	66	150.8	91	195.8	180	356.0	305	581.0	1200	2192
40	40.0	17	62.6	42	107.6	67	152.6	92	197.6	185	365.0	310	590.0	1300	2372
35	31.0	18	64.4	43	109.4	68	154.4	93	199.4	190	374.0	315	599.0	1400	2552
30	22.0	19	66.2	44	111.2	69	156.2	94	201.2	195	383.0	320	608.0	1500	2732
25	13.0	20	68.0	45	113.0	70	158.0	95	203.0	200	392.0	325	617.0	1600	2912
20	—4.0	21	69.8	46	114.8	71	159.8	96	204.8	205	401.0	330	626.0	1700	3092
15	+5.0	22	71.6	47	116.6	72	161.6	97	206.6	210	410.0	335	635.0	1800	3272
10	+14.0	23	73.4	48	118.4	73	163.4	98	208.4	215	419.0	340	644.0	1900	3452
5	+23.0	24	75.2	49	120.2	74	165.2	99	210.2	220	428.0	345	653.0	2000	3632
—0	+32.0	25	77.0	50	122.0	75	167.0	100	212.0	225	437.0	350	662.0	2100	3812

40. Hydrometer Scales.

Reading	Baumé heavy liquids	Baumé light liquids	Beck heavy liq.	Beck light liq.	Cartier	Twaddell.
0	1.000		1.000	1.000		1.000
5	1.035		1.030	.971		1.025
10	1.073	1.000	1.062	.944		1.050
15	1.114	.967	1.097	.919	.970	1.075
20	1.158	.936	1.133	.895	.936	1.100
25	1.205	.907	1.172	.872	.905	1.125
30	1.257	.880	1.214	.850	.876	1.150
35	1.313	.854	1.259	.829	.849	1.175
40	1.375	.830	1.308	.810	.824	1.200
45	1.442	.807	1.360	.791		1.225
50	1.517	.785	1.417	.773		1.250
55	1.599	.764	1.478	.756		1.275
60	1.691	.745	1.545	.739		1.300
65	1.795		1.619	.723		1.325
70	1.912		1.700	.708		1.350
75	2.045		1.790			1.375
80						1.400
100						1.500

41. Wave-lengths in Air.

Fraunhofer Line	Designation	Element	Color	Bunsen's scale	Kirchoff's scale	Wave-Length cm.
—	$K\alpha$	K	—	17	383	.00007685
A	—	—	—	18	404	.00007605
B	—	—	Red	28	593	.00006870
—	$Li\alpha$	Li	—	32	645	.00006708
C	$H\alpha$	H	—	34	694	.00006563
D_1	Na	Na	Yellow	50	1003	.00005896
D_2					1007	.00005890
—	—	Tl	Green	68	—	.00005350
E	—	—	—	71	1523	.00005270
F	$H\beta$	H	—	90	2080	.00004862
—	$Sr\delta$	Sr	Blue	105	2386	.00004607
f	$H\gamma$	H	—	127	—	.00004341
G	—	—	—	128	2854	.00004309
g	—	Ca	—	135	2870	.00004227
—	$H\delta$	H	Violet	151	—	.00004102
—	$K\beta$	K	—	153	—	.00004060
H_1	H	Ca	—	162	—	.00003969
H_2	—	—	—	166	—	.00003934

42. a. English Board of Trade (Imperial) Wire Gauge.

No. of Wire on Gauge	Diameter of Wire in cm.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam
		1	0.762	11	0.295	21	.0813	31	.0295	41	.0112
		2	.701	12	.261	22	.0711	32	.0274	42	.0102
		3	.610	13	.234	23	.0610	33	.0254	43	.0091
7/0	1.270	4	.589	14	.203	24	.0559	34	.0234	44	.0081
6/0	1.179	5	.538	15	.183	25	.0508	35	.0213	45	.0071
5/0	1.097	6	.488	16	.163	26	.0457	36	.0193	46	.0061
4/0	1.016	7	.447	17	.142	27	.0417	37	.0173	47	.0051
3/0	.945	8	.406	18	.122	28	.0376	38	.0152	48	.0041
2/0	.884	9	.366	19	.102	29	.0345	39	.0132	49	.0031
0	.823	10	0.325	20	0.091	30	.0315	40	.0122	50	.0025

42 b. Birmingham Wire Gauge (B. W. G.)

No. of Wire on Gauge	Diameter of Wire in cm	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.
		1	0.80	10	0.35	19	0.110	28	0.037
		2	.74	11	.32	20	.091	29	.034
		3	.68	12	.28	21	.083	30	.031
		4	.62	13	.25	22	.073	31	.026
		5	.57	14	.22	23	.065	32	.023
0000	1.2	6	.53	15	.19	24	.057	33	.021
000	1.1	7	.48	16	.17	25	.051	34	.018
00	1.0	8	.43	17	.15	26	.046	35	.013
0	0.9	9	0.39	18	0.13	27	0.041	36	0.010

43. Musical Pitch (Tempered Scale—complete Vibrations per second).

Physical Pitch	32 foot Octave	16 foot Octave	Great Octave	Little Octave	2 foot Octave	1 foot Octave	6 inch Octave	3 inch Octave	Concert Pitch (approx.)
C	16.0	32.0	64.0	128.0	256.0**	512.0	1024	2048	
	16.5	32.9	65.9	131.8	263.5**	527.0	1054	2108	
C#	17.0	33.9	67.8	135.6	271.2**	542.4	1085	2170	C
	17.4	34.9	69.8	139.6	279.2	558.3	1117	2233	
D	18.0	35.9	71.8*	143.7	287.4†	574.7	1149	2298	C#
	18.5	37.0	73.9*	147.9	295.8†	591.5	1183	2366	
D#	19.0	38.1	76.1*	152.2	304.4†	608.9	1218	2436	D
	19.6	39.2	78.3	156.7	313.4	626.7	1253	2507	
E	20.2	40.3	80.6	161.3	322.5	645.1	1290	2580	D#
	20.7	41.5	83.0	166.0	332.0	664.0	1328	2656	
F	21.4	42.7	85.4	170.9	341.7	683.4	1367	2734	E
	22.0	44.0	88.0	175.9	351.7	703.5	1407	2814	
F#	22.6	45.3	90.5	181.0	362.0	724.1	1448	2896	F
	23.3	46.6	93.2	186.3	372.6	745.3	1491	2981	
G	24.0	47.9	95.9	191.8	383.6	767.1§	1534	3068	F#
	24.7	49.4	98.7	197.4	394.8	789.6§	1579	3158	
G#	25.4	50.8	101.6	203.2	406.4	812.8§	1626	3251	G
	26.1	52.3	104.6	209.1	418.3	836.6	1673	3346	
A	26.9	53.8	107.6	215.3	430.5††	861.1	1722	3444	G#
	27.7	55.4	110.8	221.6	443.2††	886.3	1773	3545	
A#	28.5	57.0	114.0	228.1	456.1††	912.3	1825	3649	A
	29.3	58.7	117.4	234.8	469.5	939.0	1878	3756	
B	30.2	60.4	120.8	241.6	483.3	966.5	1933	3866	A#
	31.1	62.2	124.4	248.7	497.4	994.8	1990	3979	
	32.0	64.0	128.0	256.0	512.0	1024.0	2048	4096	B

Note. The Paris Conservatoire standard of pitch, recently adopted by the International Congress at Vienna, is 435 vibrations per second for the note A of the treble staff. This gives C=261 on the natural scale. American instruments tuned to "Concert Pitch" give C = 270 +.

* Lowest D of Bass Voice. ** Middle C of Piano. † Lowest D of Flute. †† Violin A. § Highest G of Treble Voice.

44 A. Reduction of Minutes (′) and seconds (″) to thousandths of a degree ($\frac{1}{1000}$)°.

			Thousandths of degrees.
0′	°.0	0".0	00
		3".6	01
6′	°.1	7".2	02
		10".8	03
12′	°.2	14".4	04
		18".0	05
18′	°.3	21".6	06
		25".2	07
24′	°.4	28".8	08
		32".4	09
30′	°.5	36".0	10
		39".6	11
36′	°.6	43".2	12
		46".8	13
42′	°.7	50".4	14
		54".0	15
48′	°.8	57".6	16
		61".2	17
54′	°.9		

			Thousandths of degrees.
1′	°.0	1".2	17
		4".8	18
7′	°.1	8".4	19
		12".0	20
13′	°.2	15".6	21
		19".2	22
19′	°.3	22".8	23
		26".4	24
25′	°.4	30".0	25
		33".6	26
31′	°.5	37".2	27
		40".8	28
37′	°.6	44".4	29
		48".0	30
43′	°.7	51".6	31
		55".2	32
49′	°.8	58".8	33
		62".4	34
55′	°.9		

			Thousandths of degrees.
2′	°.0	2".4	34
		6".0	35
8′	°.1	9".6	36
		13".2	37
14′	°.2	16".8	38
		20".4	39
20′	°.3	24".0	40
		27".6	41
26′	°.4	31".2	42
		34".8	43
32′	°.5	38".4	44
		42".0	45
38′	°.6	45".6	46
		49".2	47
44′	°.7	52".8	48
		56".4	49
50′	°.8	60".0	50
56′	°.9		

			Thousandths of degrees.
3′	°.0	0".0	50
		3".6	51
9′	°.1	7".2	52
		10".8	53
15′	°.2	14".4	54
		18".0	55
21′	°.3	21".6	56
		25".2	57
27′	°.4	28".8	58
		32".4	59
33′	°.5	36".0	60
		39".6	61
39′	°.6	43".2	62
		46".8	63
45′	°.7	50".4	64
		54".0	65
51′	°.8	57".6	66
		61".2	67
57′	°.9		

			Thousandths of degrees.
4′	°.0	1".2	67
		4".8	68
10′	°.1	8".4	69
		12".0	70
16′	°.2	15".6	71
		19".2	72
22′	°.3	22".8	73
		26".4	74
28′	°.4	30".0	75
		33".6	76
34′	°.5	37".2	77
		40".8	78
40′	°.6	44".4	79
		48".0	80
46′	°.7	51".6	81
		55".2	82
52′	°.8	58".8	83
		62".4	84
58′	°.9		

			Thousandths of degrees.
5′	°.0	2".4	84
		6".0	85
11′	°.1	9".6	86
		13".2	87
17′	°.2	16".8	88
		20".4	89
23′	°.3	24".0	90
		27".6	91
29′	°.4	31".2	92
		34".8	93
35′	°.5	38".4	94
		42".0	95
41′	°.6	45".6	96
		49".2	97
47′	°.7	52".8	98
		56".4	99
53′	°.8	60".0	100
59′	°.9		

Dif.																		
	0".2	0".4	0".6	0".8	1".0	1".2	1".4	1".6	1".8	2".0	2".2	2".4	2".6	2".8	3".0	3".2	3".4	3".6
	6	11	17	22	28	33	39	44	50	56	61	67	72	78	83	89	94	100

44 B. Correction to be added to Dates in Different years to compare them with Tables for 1891.

Jan. Feb.	h. m. s.	days	Mch.-Dec.	h. m. s.	days	year	h. m. s.	days	year	h. m. s.	days	year	h. m. s.	days
1892	−5 48 49	−.212	1892	+18 11 11	+.758	1893	+12 22 22	+.516	1894	+6 33 33	+.273	1895	+0 44 44	+.031
1896	−5 4 5	−.211	1896	+18 55 55	+.789	1897	+13 7 6	+.547	1898	+7 18 17	+.304	1899	+1 29 28	+.062

44 C. Gain of Sidereal Time during different Intervals of Mean Solar Time.

Days	Gain	Days	Gain	Days	Gain	Days	Gain	Hours	Gain	Hours	Gain	Minutes	Gain	Minutes	Gain
½	1m. 58s.	3½	13m. 48s.	6½	25m. 38s.	9½	37m. 27s.	1	0m. 10s.	7	1m. 9s.	0—3	0 sec.	34—39	6 sec.
1	3 57	4	15 46	7	27 36	10	39 26	2	0 20	8	1 19	4—9	1	40—45	7
1½	5 55	4½	17 45	7½	29 34	10½	41 24	3	0 30	9	1 29	10—15	2	46—51	8
2	7 53	5	19 43	8	31 32	11	43 22	4	0 39	10	1 39	16—21	3	52—57	9
2½	9 51	5½	21 41	8½	33 31	11½	45 20	5	0 49	11	1 48	22—27	4	58—63	10
3	11 50	6	23 39	9	35 29	12	47 19	6	0 59	12	1 58	28—33	5	64—70	11

44 D. Sidereal Time at Greenwich mean Noon on Different Dates in the year 1891.

Jan. 0	18h. 39m. 27s.	March 0	22h. 32m. 4s.	May 0	2h. 32m. 34s.	July 0	6h. 33m. 4s.	Sept. 0	10h. 37m. 31s.	Nov. 0	14h. 34m. 4s.
10	19 18 53	10	23 11 30	10	3 12 0	10	7 12 30	10	11 16 56	10	15 17 26
20	19 58 19	20	23 50 55	20	3 51 25	20	7 51 55	20	11 56 22	20	15 56 51
Feb. 0	20 41 41	April 0	0 34 17	June 0	4 34 47	Aug. 0	8 35 17	Oct. 0	12 35 47	Dec. 0	16 36 17
10	21 21 6	10	1 13 43	10	5 14 13	10	9 14 43	10	13 15 13	10	17 15 43
20	22 0 32	20	1 53 9	20	5 53 38	20	9 54 8	20	13 54 48	20	17 55 8

44 E. Semidiameter of the Sun in Thousandths of a Degree for the First Day of Each Month in 1891.

Jan. 0.272; Feb. 0.271; Mar. 0.269; Apr. 0.267; May 0.265; June 0.263; July 0.263; Aug. 0.263; Sept. 0.265; Oct. 0.267; Nov. 0.269; Dec. 0.271.

44, F. Declination of the Sun in Degrees at Greenwich Mean Noon for 1891.

Day	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Day
	—	—	±	+	+	+	+	+	±	—	—	—	
0	23.091	17.378	—7.958	4.157	14.772	21.920	23.191	18.293	+8.664	2.791	14.117	21.661	0
1	23.011	17.096	7.579	4.543	15.077	22.060	23.127	18.044	8.303	3.180	14.440	21.820	1
2	22.925	16.809	7.199	4.928	15.377	22.193	23.057	17.790	7.938	3.568	14.759	21.972	2
3	22.831	16.516	6.816	5.312	15.673	22.319	22.980	17.531	7.572	3.956	15.074	22.118	3
4	22.729	16.219	6.432	5.694	15.966	22.439	22.897	17.268	7.203	4.343	15.385	22.255	4
5	22.619	15.918	—6.046	6.074	16.253	22.553	22.806	16.999	+6.833	4.729	15.692	22.386	5
6	22.502	15.611	5.659	6.453	16.536	22.660	22.710	16.726	6.461	5.114	15.994	22.509	6
7	22.378	15.301	5.271	6.830	16.815	22.760	22.606	16.449	6.087	5.498	16.292	22.626	7
8	22.246	14.986	4.881	7.205	17.089	22.854	22.496	16.167	5.711	5.881	16.585	22.734	8
9	22.108	14.666	4.491	7.578	17.358	22.941	22.380	15.881	5.334	6.262	16.873	22.836	9
10	21.961	14.343	—4.099	7.949	17.622	23.021	22.257	15.590	+4.955	6.643	17.157	22.929	10
11	21.808	14.016	3.707	8.318	17.882	23.095	22.128	15.296	4.575	7.021	17.436	23.015	11
12	21.648	13.684	3.314	8.684	18.136	23.162	21.993	14.997	4.194	7.398	17.710	23.094	12
13	21.481	13.350	2.920	9.048	18.386	23.222	21.851	14.694	3.811	7.773	17.978	23.165	13
14	21.306	13.011	2.525	9.409	18.630	23.275	21.703	14.388	3.427	8.147	18.242	23.228	14
15	21.125	12.669	—2.131	9.768	18.869	23.321	21.548	14.078	+3.043	8.518	18.500	23.283	15
16	20.938	12.324	1.736	10.124	19.102	23.360	21.388	13.763	2.657	8.888	18.752	23.331	16
17	20.743	11.975	1.341	10.477	19.331	23.393	21.221	13.446	2.271	9.255	18.999	23.371	17
18	20.542	11.623	0.945	10.828	19.553	23.419	21.049	13.124	1.883	9.620	19.241	23.403	18
19	20.335	11.269	0.550	11.175	19.770	23.437	20.871	12.800	1.495	9.983	19.476	23.428	19
20	20.121	10.911	—0.155	11.519	19.982	23.449	20.687	12.472	+1.107	10.343	19.706	23.444	20
21	19.901	10.551	+0.240	11.860	20.188	23.454	20.497	12.141	0.718	10.701	19.930	23.453	21
22	19.675	10.188	0.635	12.198	20.388	23.452	20.301	11.806	+0.328	11.056	20 148	23.454	22
23	19.443	9.822	1.029	12.532	20.582	23.444	20.100	11.468	—0.061	11.409	20.359	23.447	23
24	19.204	9.454	1.423	12.863	20.771	23.428	19.893	11.128	0.451	11.758	20.565	23.432	24
25	18.960	9.083	+1.816	13.190	20.953	23.405	19.680	10.784	—0.841	12.105	20.764	23.409	25
26	18.710	8.710	2.208	13.514	21.130	23.376	19.463	10.437	1.232	12.449	20.957	23.379	26
27	18.455	8.335	2.600	13.834	21.300	23.340	19.239	10.088	1.622	12.789	21.143	23.341	27
28	18.194	7.958	2.991	14.151	21.464	23.297	19.011	9.736	2.012	13.127	21.323	23.294	28
29	17.927	[7.579]	3.380	14.463	21.623	23.247	18.777	9.381	2.401	13.460	21.495	23.241	29
30	17.655		3.769	14.772	21.775	23.191	18.538	9.024	2.791	13.791	21.661	23.179	30
31	17.378		4.157		21.920		18.293	8.664		14.117		23.109	31

44, G. Equation of Time in Minutes and Seconds at Greenwich Mean Noon for 1891.

Day	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Day
	+	+	+	±	—	±	+	+	±	—	—	±	
	m s	m s	m s	m s	m s	m s	m s	m s	m s	m s	m s	m s	
0	3 16	13 40	12 45	+4 17	2 51	—2 35	3 20	6 10	+0 15	9 59	16 18	—11 14	0
1	3 45	13 48	12 33	3 59	2 59	2 27	3 32	6 7	—0 4	10 18	16 20	10 52	1
2	4 13	13 56	12 21	3 41	3 6	2 18	3 44	6 3	0 23	10 37	16 21	10 29	2
3	4 41	14 2	12 8	3 23	3 13	2 8	3 55	5 59	0 42	10 56	16 21	10 5	3
4	5 8	14 8	11 55	3 5	3 19	1 58	4 6	5 54	1 1	11 14	16 20	9 41	4
5	5 35	14 13	11 42	+2 48	3 25	—1 48	4 17	5 48	—1 21	11 32	16 19	— 9 17	5
6	6 2	14 18	11 28	2 30	3 30	1 37	4 27	5 42	1 41	11 50	16 16	8 51	6
7	6 28	14 21	11 14	2 13	3 34	1 20	4 37	5 35	2 1	12 7	16 13	8 26	7
8	6 54	14 24	10 59	1 56	3 38	1 15	4 47	5 28	2 21	12 24	16 9	7 59	8
9	7 19	14 26	10 44	1 39	3 41	1 3	4 56	5 20	2 42	12 40	16 5	7 33	9
10	7 44	14 27	10 29	+1 23	3 44	—0 51	5 5	5 12	—3 3	12 56	15 59	— 7 6	10
11	8 8	14 28	10 13	1 7	3 46	0 39	5 13	5 3	3 23	13 12	15 53	6 38	11
12	8 32	14 27	9 57	0 51	3 48	0 27	5 21	4 53	3 44	13 27	15 45	6 10	12
13	8 54	14 26	9 40	0 35	3 49	0 15	5 28	4 43	4 5	13 41	15 37	5 42	13
14	9 17	14 24	9 24	0 20	3 49	— 0 2	5 35	4 32	4 27	13 55	15 28	5 13	14
15	9 38	14 22	9 7	+0 5	3 49	+0 10	5 42	4 21	—4 48	14 9	15 19	— 4 45	15
16	9 59	14 18	8 50	—0 10	3 48	0 23	5 48	4 9	5 9	14 22	15 8	4 16	16
17	10 19	14 14	8 32	0 24	3 47	0 36	5 53	3 56	5 30	14 34	14 57	3 46	17
18	10 38	14 9	8 15	0 38	3 45	0 49	5 58	3 43	5 52	14 46	14 44	3 17	18
19	10 57	14 3	7 57	0 52	3 43	1 2	6 2	3 30	6 13	14 58	14 31	2 47	19
20	11 15	13 57	7 39	—1 5	3 40	+1 15	6 6	3 16	—6 34	15 8	14 17	— 2 18	20
21	11 32	13 50	7 21	1 18	3 37	1 28	6 9	3 1	6 55	15 18	14 3	1 48	21
22	11 48	13 43	7 3	1 30	3 33	1 40	6 12	2 46	7 16	15 27	13 47	1 18	22
23	12 4	13 34	6 44	1 42	3 29	1 53	6 14	2 31	7 37	15 36	13 31	0 48	23
24	12 19	13 26	6 26	1 53	3 24	2 6	6 15	2 15	7 58	15 44	13 13	— 0 18	24
25	12 33	13 16	6 8	—2 4	3 18	+2 19	6 16	1 59	—8 19	15 51	12 55	+ 0 12	25
26	12 46	13 6	5 49	2 15	3 12	2 31	6 17	1 43	8 39	15 57	12 37	0 42	26
27	12 58	12 56	5 31	2 25	3 06	2 44	6 16	1 26	8 59	16 3	12 17	1 12	27
28	13 10	12 45	5 12	2 34	2 59	2 56	6 16	1 9	9 19	16 8	11 57	1 41	28
29	13 21	[12 33]	4 54	2 43	2 52	3 8	6 14	0 51	9 39	16 12	11 36	2 11	29
30	13 31		4 35	2 51	2 44	3 20	6 12	0 33	9 59	16 15	11 14	2 40	30
31	13 40		4 17		2 35		6 10	0 15		16 18		3 9	31

44, H. Solar System.

Names	Time of Sidereal Revolution in Mean Solar Days	Relative distance from Sun Earth = 1	Relative Mass Earth = 1	Distance in Mega-Kilom. 10^{11} cm	Diameter in Megametres 10^8 cm	Mass in tetra-Mega-Kilos 10^{27} grams	Mean Density g. per cu. cm.
Sun . . .	...	...	320,000	...	1.392	2,000,000	1.4
Mercury .	87.97	.387	0.07?	58.	4.8	0.4?	6.?
Venus . .	224.70	.723	0.8?	108.	12.2	5.?	6.?
Earth . .	365.26	1.000	1.00	149.	12.74	6.1	5.6
Moon . .	27.32	.0026*	0.012	*0.39	3.48	0.07	3.4
Mars . . .	686.98	1.524	0.11	227.	8.	0.7	4.
Jupiter . .	4332.53	5.203	310.	777.	142.	1900.	1.3
Saturn . .	10759.22	9.539	93.	1424.	119.	570.	0.7
Uranus .	30686.82	19.18	14.	2864.	50.	85.	1.3
Neptune .	60126.71	30.05	17.	4487.	60.	100.	0.9

* Distance from the Earth.

45. Mean Position of Fixed Stars, Jan. 0 1891.

Names	Designation	Magnitude	Right Ascension	Yearly Change	Declination	Yearly Change
			h m s	s	°	°
Sirrah	α Andromedae	2	0 2 45.2	+3.09	+28.489	+.0055
Polaris	α Ursae Minoris	2	1 18 53.4	2.36	+88.727	+.0053
—	α Arietis	2	2 1 1.7	3.37	+22.947	+.0048
Aldebaran	α Tauri	1	4 29 39.9	3.44	+16.289	+.0021
Capella	α Aurigae	1	5 8 38.2	4.43	+45.886	+.0011
Rigel	β Orionis	1	5 9 17.9	2.88	− 8.328	+.0012
Beteigeuze	α Orionis	1	5 49 16.2	3.25	+ 7.386	+.0003
Canopus	α Argus	1	6 21 31.9	1.33	−52.636	−.0005
Sirius	α Canis Majoris	1	6 40 20.6	2.64	−16.567	−.0013
Castor	α² Geminorum	2-1	7 27 38.7	3.84	+32.127	−.0021
Procyon	α Canis Minoris	1	7 33 35.7	3.14	+ 5.504	−.0025
Pollux	β Geminorum	1-2	7 38 38.7	3.68	+28.289	−.0023
Regulus	α Leonis	1-2	10 2 34.0	3.20	+12.500	−.0049
Denebola	β Leonis	2	11 43 30.0	3.06	+15.181	−.0056
—	α Crucis	1	12 20 32.6	3.30	−62.495	−.0056
Spica	α Virginis	1	13 19 27.0	3.15	−10.592	−.0053
—	β Centauri	1	13 56 8.0	4.18	−59.847	−.0049
Arcturus	α Bootis	1	14 10 41.3	2.73	+19.750	−.0053
—	α² Centauri	1	14 32 12.5	4.04	−60.383	−.0042
Antares	α Scorpii	1-2	16 22 43.4	3.67	−26.189	−.0023
Vega	α Lyrae	1	18 33 14.8	2.03	+38.683	+.0009
Altair	α Aquilae	1-2	19 45 27.9	2.93	+ 8.581	+.0026
Deneb	α Cygni	2-1	20 37 42.9	2.04	+44.891	+.0035
Formalhaut	α Piscis Aust.	1-2	22 51 37.6	3.32	−30.200	+.0053
Markab	α Pegasi	2	22 59 19.8	+2.98	+14.619	+.0054

Note. Tho yearly precession of the equinoxes is about 50″.25, or 0°.00245+. The mean (not apparent) obliquity of the ecliptic for 1891 is about 23°, 27′, 13″, or 27°.453. The mean obliquity decreases annually by 0″.8, or 0°.0002.

46. Latitudes and Longitudes Measured from Greenwich.

Place		Latitude °		Longitude h	m	s		Elevation Metres
Aberdeen . . .	O	57.149	N	0	8	23	W	
Amsterdam . .	T	52.371	N	0	19	39	E	
Antwerp . . .	T	51.221	N	0	17	37	E	
Athens	O	37.972	N	1	34	55	E	
Baltimore . . .	T	39.298	N	5	6	28	W	55
Belfast		54.66	N	0	23	..	W	
Berlin	O	52.505	N	0	53	35	E	40
Bonn		50.729	N	0	28	23	E	50
Boston	T	42.358	N	4	44	15	W	63
Brussels . . .		50.853	N	0	17	29	E	90
Calcutta . . .	T	22.557	N	5	53	19	E	39
Cambridge U. S.	O	42.380	N	4	44	31	W	
Cambridge Eng. .	O	52.215	N	0	0	23	E	
Cape of Good Hope	O	33.934	S	1	13	55	E	
Christiania . .	O	59.912	N	0	42	54	E	42
Copenhagen . .	O	55.687	N	0	50	19	E	53
Cork	T	51.90	N	0	33	51	W	
Dublin	O	53.387	N	0	25	21	W	T 24
Edinboro . . .	O	55.956	N	0	12	43	W	T 139
Geneva		46.200	N	0	24	37	E	
Genoa	T	44.419	N	0	35	41	E	
Glasgow . . .	O	55.879	N	0	17	11	W	
Göttingen . . .		51.530	N	0	39	46	E	130
Greenwich . . .	O	51.477	N	0	0	0		T 64
Heidelburg . .		49.40	N	0	34	32	E	100
Leipzic		51.335	N	0	49	34	E	100
Lisbon	O	38.705	N	0	36	34	W	
Liverpool . . .	O	53.401	N	0	12	17	W	
Magnetic Pole .		77.83	N	4	14		W	
London . .		51.514	N	0	0	23	W	50
Madrid . .	O	40.408	N	0	14	45	W	663
Manchester .		53.48	N	0	9	..	W	
Melbourne .		37.831	S	9	39	54	E	
Montreal . .	T	45.52	N	4	54	13	W	44
Munich . .		48.146	N	0	46	26	E	525
Naples . .	O	40.863	N	0	57	1	W	
New Orleans .	T	29.963	N	6	0	14	W	43
New York . .	O	40.730	N	4	55	57	W	T 86
Paris . . .	O	48.836	N	0	9	21	E	60
Philadelphia .	T	39.953	N	5	0	39	W	50
Quebec . .	O	46.805	N	4	44	49	W	T 108
Queenstown .	T	51.85	N	0	33	6	W	
Rio de Janeiro	O	22.907	S	2	52	41	W	T 69
Rome . . .		41.898	N	0	49	54	E	29
Rotterdam .	T	51.908	N	0	17	55	E	28
San Francisco	O	37.790	N	8	9	43	W	T 111
Savannah . .	T	32.081	N	5	24	21	W	42
St. John (N.S.)	T	45.262	N	4	24	15	W	38
St Petersburg	O	59.942	N	2	1	14	E	11
Stockholm .	O	59.343	N	1	12	14	E	20
Strassburg .	O	48.582	N	0	31	2	E	150
Sydney . .	O	33.861	S	10	4	50	E	T 65
Triest . . .	O	45.643	N	0	55	2	E	T 17
Venice . .	O	45.430	N	0	49	25	E	
Vienna . .		48.210	N	1	5	32	E	182
Washington .	O	38.894	N	5	8	12	W	T 63
Wellington .	T	41.288	S	11	39	11	E	T 18

[*Note.* T = Time Signal. O = Observatory.

47. Acceleration of Gravity in Different Latitudes (cm. per sec. per sec.).*

Lat.	+0°	+1°	+2°	+3°	+4°	+5°	+6°	+7°	+8°	+9°	Dif
0°	978.10	978.10	978.11	978.12	978.13	978.14	978.16	978.18	978.20	978.23	1
10°	978.25	978.29	978.32	978.36	978.40	978.44	978.48	978.53	978.58	978.63	4
20°	978.69	978.75	978.81	978.87	978.93	979.00	979.06	979.13	979.21	979.28	7
30°	979.35	979.43	979.51	979.59	979.67	979.75	979.83	979.92	980.00	980.09	8
40°	980.17	980.26	980.34	980.43	980.52	980.61	980.69	980.78	980.86	980.95	9
50°	981.04	981.13	981.21	981.30	981.38	981.46	981.54	981.62	981.70	981.78	8
60°	981.86	981.93	982.01	982.08	982.15	982.21	982.28	982.34	982.41	982.47	7
70°	982.52	982.58	982.63	982.68	982.73	982.77	982.82	982.86	982.89	982.93	4
80°	982.96	982.99	983.01	983.03	983.05	983.07	983.08	983.09	983.10	983.11	1

48. Length of Seconds-Pendulum in Different Latitudes (cm.).*

Lat.	+0°	+1°	+2°	+3°	+4°	+5°	+6°	+7°	+8°	+9°	Dif
0°	99.103	99.103	99.103	99.104	99.105	99.106	99.108	99.110	99.112	99.115	1
10°	99.118	99.121	99.125	99.128	99.132	99.137	99.141	99.146	99.151	99.156	4
20°	99.162	99.168	99.174	99.180	99.187	99.193	99.200	99.207	99.214	99.222	7
30°	99.229	99.237	99.245	99.253	99.261	99.269	99.278	99.286	99.295	99.303	8
40°	99.312	99.321	99.330	99.338	99.347	99.356	99.365	99.374	99.383	99.391	9
50°	99.400	99.409	99.418	99.426	99.435	99.443	99.451	99.459	99.467	99.475	8
60°	99.483	99.491	99.498	99.505	99.512	99.519	99.526	99.532	99.539	99.545	7
70°	99.550	99.556	99.561	99.566	99.571	99.576	99.580	99.584	99.588	99.591	4
80°	99.594	99.597	99.600	99.602	99.604	99.606	99.607	99.608	99.609	99.010	1

* These values are calculated for the sea level. A deduction of 0.03 % should be made for each kilometre of elevation above the ground and a deduction of 0.02 % should be made for each kilometre of elevation of the ground above the sea.

49a. Reduction of Measures to and from the C. G. S. System.

	Equivalent	Logarithm	Reciprocal
Lengths in centimetres			
1 inch . . . . =	2.53997	0.40483	.393705
1 link = 7.92 in. . . . . =	20.1165	1.30355	0497103
1 foot = 12 in. . . . . =	30.4796	1.48401	.0328088
1 yard = 3 ft. . . . . =	91.4389	1.96113	.0109363
1 fathom = 6 ft. . . . . =	182.878	2.26216	00546813
1 rod = $16^1/_2$ ft. . . . . =	502.914	2.70149	.00198841
1 chain = 100 links = 66 ft. . . =	2011.65	3.30355	.000497103
1 statute mile = 5280 ft. . . . =	160,932	5.20664	6.21378×10^{-6}
1 nautical mile . . . . =	185,200(?)	5.2676	5.40×10^{-6}
Areas in square centimetres			
1 square inch. . . . . =	6.4514	0.80966	.15500
1 square foot = 144 sq. in. . . =	929.01	2.96802	.0010764
1 square yard = 9 sq. ft. . . . =	8361.1	3.92226	.00011960
1 acre = 43,560 sq. ft. . . . . =	4.0468×10^{7}	7.60711	2.4711×10^{-8}
1 square mile = 640 acres . . =	2.5899×10^{10}	10.41329	3.8611×10^{-11}
Volumes in cubic centimetres			
1 cubic inch . . . . =	16.386	1.21449	.061026
1 cubic foot = 1728 cu. in. . . =	28316	4.45203	3.5316×10^{-5}
1 cubic yard = 27 cu. ft. . . . =	764526	5.88339	$1\ 3080\times10^{-6}$
1 U. S. pint = 1.043 lbs. water =	473	2.6750	.002114
1 U. S. quart = 2 pints. . . . =	946	2.9760	.001057
1 dry quart . . . . =	1101	3.0418	.000908
1 U. S. gallon = 231 cu. in. = 4 qts. =	3785	3.5781	.0002642
1 imperial gallon = 10 lbs. water =	4541	3.6572	.0002202
Masses in grams			
1 grain . . . . =	.0647987	$\bar{2}$.81157	15.4324
1 ounce (Avoirdupois) = $^1/_{16}$ lb. =	28 3494	1.45254	.0352741
1 ounce (Troy) = 480 grains . =	31.1034	1.49281	.0321509
1 pound (Troy) = 12 oz. Troy. =	373.240	2.57199	.00267924
1 pound (Avoir) = 7000 grains. =	453 590	2.65666	2.20463×10^{-3}
1 English ton = 2240 lbs. . . . =	1.01604×10^{6}	6.00691	9.84210×10^{-7}
Times in mean solar seconds			
1 year (tropical) = 365.24222 days =	31,556.928	7.49809	3.16888×10^{-8}
1 sidereal year = 365.25637 days =	31,558,150	7.49811	3.16875×10^{-8}
1 (mean solar) day. . . . . . . =	86,400	4.93651	.000011574074
1 hour . . . . =	3,600	3.55630	.00027777778
1 minute . . . . =	60	1.77815	.016666667
1 so-called sidereal second . . =	0.9972695666	$\bar{1}$.99881	1.0027379091
1 true sidereal second . . . . =	0.9972696721	$\bar{1}$.99881	1.0027378030
Velocities in centimetres per second			
1 kilometre per hour. . . . . =	27 7778	1.44370	.0360000
1 foot per second . . . . . . =	30 4796	1.48401	.0328088
1 mile per hour. . . . . =	44.7033	1.65034	.0223696
1 nautical mile per hour . . . =	51.44	1.7113	.01944
1 kilometre per minute . . . . =	1666.67	3.22185	.0006000000
1 mile per minute . . . . . . =	2682.20	3.42849	.000372827
Accelerations in cm. per sec. per. sec.			
1 foot per sec. per sec. . . . . =	30.4796	1.48401	.0328088
Densities in grams per cu. cm.			
1 grain per cubic inch. . . . . =	.0039544	$\bar{3}$.59708	252.88
1 lb. per cubic foot . . . . . =	.016019	$\bar{2}$.20463	62.426
Heat Units in ergs.			
1 unit of heat = 1 gram-degree C. =	4.17×10^{7}	7.620	2.40×10^{-8}
1 lb.-degree Fahrenheit . . . . =	1.051×10^{10}	10.022	9.52×10^{-11}
1 lb.-degree Centigrade . . . . =	1.89×10^{10}	10.277	5.29×10^{-11}
1 Calorie = 1000 g° . . . . . . =	4.17×10^{10}	10.620	2.40×10^{-11}

49 b. Continuation. Reduction of Measures to and from the C. G. S. System.

Values marked with an asterisk (*) are independent of the acceleration of gravity (g).

		Equivalent		Logarithm		Reciprocal	
		g = 980	g = 981	g = 980	g = 981	g = 980	g = 981
Force in dynes							
1 grain weight, in dynes	=	63.50	63.57	1.80279	1.80324	.01575	.01573
1 gram " " "	=	980.	981.	2.99123	2.99167	.0010204	.0010194
1 oz. (avoir) " " "	=	2.778×10^4	2.781×10^4	4.44377	4.44421	3.599×10^{-5}	3.596×10^{-5}
1 lb. (avoir) " " "	=	4.445×10^5	4.450×10^5	4.64789	5.64833	2.250×10^{-6}	2.247×10^{-6}
1 kilogram " " "	=	980,000	981,000	5.99123	5.99167	1.0204×10^{-6}	1.0194×10^{-6}
1 tonne " " "	=	980,000,000	981,000,000	8.99123	8.99167	1.0204×10^{-9}	1.0194×10^{-9}
1 English ton " "	=	9.957×10^8	9.967×10^8	8.99814	8.99858	1.004×10^{-9}	1.003×10^{-9}
1 poundal* " "	=	13825*		4.14067*		7.2331×10^{-5}*	
Pressure in dynes per sq. cm.							
1 lb. per sq. ft. in dynes per sq. cm.	=	478.5	479.0	2.67987	2.68031	.002090	.002088
1 gram per sq. cm. " " " "	=	980	981	2.99123	2.99167	.0010204	.0010194
1 kilo. per sq. decim. " " " "	=	9,800	9,810	3.99123	3.99167	.00010204	.00010194
1 cm. mercury at 0° " " " "	=	13,324	13,338	4.12464	4.12508	.00007505	.00007498
1 lb. per sq. in. " " " "	=	68,902	68,973	4.83823	4.83868	1.451×10^{-5}	1.450×10^{-5}
1 kilo per sq. cm. " " " "	=	980,000	981,000	5.99123	5.99167	1.0204×10^{-6}	1.0194×10^{-6}
1 atmosphere 30 in. mercury at 62½° F.	=	1,012,200	1,013,200	6.00526	6.00571	9.880×10^{-7}	9.870×10^{-7}
" 76 cm. " " 0° C.	=	1,012,630	1,013,660	6.00545	6.00589	9.875×10^{-7}	9.865×10^{-7}
" 30 in. " " 0° C.	=	1,015,300	1,016,300	6.00659	6.00703	9.849×10^{-7}	9.839×10^{-7}
1 kilo per sq. mm.	=	98,000,000	98,100,000	7.99123	7.99167	1.0204×10^{-8}	1.0194×10^{-8}
1 megadyne per sq. cm.*	=	1,000,000*		6.00000*		1.000000×10^{-6}*	
Work in ergs							
1 gram-centimetre in ergs	=	980	981	2.99123	2.99167	1.0204×10^{-3}	1.0194×10^{-3}
1 kilogram metre " "	=	98,000,000	98,100,000	7.99123	7.99167	1.0204×10^{-8}	1.0194×10^{-8}
1 foot-pound " "	=	13,550,000	13,560,000	7.13190	7.13234	7.381×10^{-8}	7.373×10^{-8}
1 foot-poundal* " "	=	421390*		5.62468*		2.3731×10^{-6}*	
1 joule*	=	10,000,000*		7.00000*		.0000001*	
Power in ergs per second							
1 horse-power {33,000 ft. lbs. per min.	=	7.452×10^9	7.459×10^9	9.87226	9.87270	1.342×10^{-10}	1.341×10^{-10}
1 horse-power {75 kilogrammetres per sec.	=	7.350×10^9	7.357×10^9	9.86629	9.86673	1.361×10^{-10}	1.359×10^{-10}
1 man's power (approx) in ergs per sec.	=	1×10^9		9.0		1×10^{-9}	
1 unit of heat per sec. " " " "	=	4.17×10^7*		7.620*		2.40×10^{-8}*	
1 watt in ergs per sec.	=	1×10^7*		7.00000*		1×10^{-7}*	

50. Numbers Frequently Required in Calculation.

	Number	Logarithm	Reciprocal.
Mathematical Constants.			
Ratio of circumference to diameter . . . . π =	3.1415927	.049715	.3183099
Square of Ditto . . . . π^2 =	9.8696044	0.99430	.1013212
Square Root of Ditto . . . . $\sqrt{\pi}$ =	1.7724539	0.24857	.5641896
Square Root of 2 . . . . $\sqrt{2}$ =	1.4142136	0.15051	.7071068
Square Root of 3 . . . . $\sqrt{3}$ =	1.7320508	0.23856	.5773503
Square Root of 10 . . . . $\sqrt{10}$ =	3.1622777	0.50000	.3162278
Logarithmic Base . . . . e =	2.7182818	0.43429	.3678794
Logarithmic Modulus . . . . M =	0.4342945	$\bar{1}$.63778	2.302585
1 degree in circular measure . . . . $1°$ =	.01745329	$\bar{2}$.24188	57.29578
1 minute in circular measure . . . . $1'$ =	.00029089	$\bar{4}$.46373	3437.747
1 second in circular measure . . . . $1''$ =	.00000485	$\bar{6}$.68557	206264.8
Probable error (mean error = 1)	0.67449	$\bar{1}$.82898	1.4826
Astronomical Constants.			
Sidereal time in Mean time	0.9972696	$\bar{1}$.99881	1.0027379
1 (tropical) year — See Table 49 — in days	365.24222	2.56258	0.0027379
Annual precession of equinoxes (50''.25) in days . .	0.01415	$\bar{2}$.1509	70.6
Aberration constant (20''.45) in degrees	0.00568	$\bar{3}$.754+	176.
Sun's mean angular semidiameter (16'2'') in degrees	0.267	$\bar{1}$.427	3.74
Solar parallax (8''.83?)	.00245	$\bar{3}$.390	408.
Earths equatorial radius in kilom.	6378.	3.8047	.0001568
Earths polar „ „ „	6356.+	3.8032	.0001573
Gravity. Attraction between two unit masses (1 g) at unit distance (1 cm.) in dynes . . .	6.5×10^{-8}	$\bar{8}$.813	1.54×10^{7}
Seconds-pendulum (lat 45°) in cm.	99.356	1.99719	.010065
1 gram (lat. 45°) in dynes	980.61	2.99149	.0010198
Atmospheric mean molecular weight	28.86	1.4603	.03465
„ Pressure (76 cm. Paris) in megadynes per sq. cm.	1.01360	0.00587	.98658
Density of Air, (0°, 76 cm. Paris)	0.0012932	$\bar{3}$.11167	773.3
„ „ „ (0°, 1 megadyne per sq. cm.)	0.0012759	$\bar{3}$.10581	783.8
„ „ Hydrogen (0°, 76 cm. Paris)	0.00008957	$\bar{5}$.95216	11164.
„ „ „ (0°, 1 megadyne per sq. cm.) .	0.00008837	$\bar{5}$.94630	11316.
„ „ Water at 4°	1.00001	0.000004	0.99999
„ „ „ apparent at 21°	0.997	$\bar{1}$.9987	1.003
„ „ Crown glass. about	2.5	0.400	0.40
„ „ Brass „	8.4	0.924	.119
„ „ Mercury at 19°	13.550	1.13194	0.73800
Sound Velocity in dry air at 0° in cm. per sec. 33,220 or	33,200	4.521	.000301
1 mean semitone involves ratio $\sqrt[12]{2}$	1.059463	0.02509	.94387
Light. Velocity in cm. per sec	3.00×10^{10}	10.477	3.33×10^{-11}
„ sodium, wave length in air cm.)	.00005893	$\bar{5}$.77033	16970
Refractive Index of water for ditto	1.333	0.1248	.750
Dispersive „ „ „ „ „	0.014+	2.15	70
Rotation of ditto by Quartz plate 1 cm. thick . . .	217°	2.3365	.00461
Candle power of 1 sq. cm. melted platinum (2 Carcels)	20	1.30	.05
Heat. Conductivity of Copper C. G. S.	0.9+	$\bar{1}$.96	1.1
Coefficient of Expansion of glass (cubical)	.000025	$\bar{5}$.40	40,000.
„ „ „ „ steel (linear)	.000012	$\bar{5}$.08	83,000.
„ „ „ „ brass „	.000019	$\bar{5}$.28	53,000.
„ „ „ „ mercury (cubical 20°) .	.000180+	$\bar{4}$.255	55,000
„ „ „ „ gases „ . . .	.00367	$\bar{3}$.564+	273
Latent Heat of Water	79	1.900	.0127
„ „ „ Steam (100°, 76 cm.)	536	2.729	.00187
Specific Heat of Brass	0.094	$\bar{2}$.973	10.6
„ „ „ Glass	0.19	$\bar{1}$.279	5.3
„ „ „ Water (0°—100°)	1.005	0.002	.995
„ „ „ Air (0°—100°, 76 cm) 0.2383 or . . .	0.238	$\bar{1}$.377	4.20
„ „ „ Gases, (ratio of 2 Sp. Hts)	1.408	0.1486	.710
Mechanical Equivalent of			
1 unit of heat (1 g°) in ergs, 4.166×10^7 or.	4.17×10^7	7.620	2.40×10^{-8}
1 kilogrammetre (lat. 45°)	9.8061×10^7	7.99149	1.0198×10^{-8}
1 foot-pound („ „)	1.3557×10^7	7.13217	7.376×10^{-8}
Electro-Chemical equivalent of Hydrogen in grams per ampère per sec.	.00001038	$\bar{5}$.0162	9634
1 Electrostatic Unit of E. M. F. in volts	300	2.477	.00333
Electromotive Force of Daniell cell in volts . . .	1.0 to 1.2	0.00 to 0.08	1.0 to 0.8
Internal Resistance of Quart „ „ „ ohms . . .	1 to 2	0.0 to 0.3	1.0 to .5
1 B. A. unit in legal ohms	0.9889	$\bar{1}$.9952	1.011
1 Siemens „ „ „	0.9434	$\bar{1}$.9747	1.060
Specific Electrical Resistance of Mercury (C. G. S.)	0.942×10^5	4.974	1.061×10^{-5}
Magnetic susceptibility of iron	300?	2.5?	.003?
Total Intensity of Earths Field	.3 to .7	$\bar{1}$.5 to $\bar{1}$.8	3 to 1.5

With the Compliments of the Author.

PHYSICAL MEASUREMENT.

A SHORT COURSE

OF

EXPERIMENTS

IN

PHYSICAL MEASUREMENT.

BY HAROLD WHITING,

INSTRUCTOR IN PHYSICS AT HARVARD UNIVERSITY.

In Four Parts.

PART II.

SOUND, DYNAMICS, MAGNETISM, AND ELECTRICITY.

CAMBRIDGE:

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TABLE OF CONTENTS.

Part Second.

MEASUREMENTS RELATING TO

SOUND, DYNAMICS, MAGNETISM, AND ELECTRICITY.

SOUND (continued.)

MEASUREMENT OF TIME.

DYNAMICS.

MAGNETISM.

ELECTRICITY.

MEASUREMENT OF ELECTRICAL RESISTANCE.

MEASUREMENT OF ELECTROMOTIVE FORCE.

PHYSICAL MEASUREMENT.

Part Second.

MEASUREMENTS IN SOUND, DYNAMICS, MAGNETISM, AND ELECTRICITY.

SOUND — Continued.

EXPERIMENT LI.

VELOCITY OF SOUND.

¶ 135. **Determination of the Velocity of Sound.** — (1) Two data are required for the determination of the velocity with which sound passes from one point to another: 1st, the distance between two stations (see ¶ 136); and 2d, the time occupied in traversing this distance (see ¶ 137). To make use of the results, the temperature of the air must be found at various points between the two stations (see Part I. ¶ 15); and if precision is required, the humidity of the air should also be determined.[1] The velocity of sound is not affected by barometric pressure.

[1] At ordinary summer temperatures (20° to 30°) the effect of humidity upon the velocity of sound may amount to one half of 1 %. See Table 15, B.

(2) If the path traversed by the sound is at right-angles with the direction of the wind, the velocity of sound will not be perceptibly affected by any ordinary atmospheric disturbance. It is, however, increased by the velocity of the wind when the two move in the same direction, or diminished by the same amount when they move in opposite directions.[1] When the directions are oblique, the velocity of sound is always more or less affected. It is therefore best to arrange an experiment so as to find the time occupied by sound in traversing a given distance first in one, then in the opposite direction. In this case, if the velocity of the wind is small and tolerably constant, the *average* result will not be perceptibly affected by it.

(3) Two or more determinations of the velocity of sound should be made between stations at different distances. Any constant error in the estimation either of distance or of time will be shown by a disagreement of the several results. The true velocity of sound is to be calculated in such a case from the *difference in time* required to traverse two given distances (see formula II. below).

(4) Let d be the distance traversed by sound in the time t; then the velocity of sound, v, is to be calculated by the equation

$$v = \frac{d}{t}. \qquad \text{I.}$$

[1] A velocity of the wind amounting to 10 metres per second, or about 22 miles per hour, would affect the velocity of sound by about 3 %.

Distinguishing by subscript numerals 1 and 2 the results in the two cases, we should have

$$v = \frac{d_1}{t_1} = \frac{d_2}{t_2};$$

hence,

$$\frac{d_1}{d_2} = \frac{t_1}{t_2}.$$

Subtracting 1 from both sides of the equation we have

$$\frac{d_1}{d_2} - 1 = \frac{t_1}{t_2} - 1;$$

or, reducing to a common denominator,

$$\frac{d_1 - d_2}{d_2} = \frac{t_1 - t_2}{t_2};$$

whence

$$\frac{d_1 - d_2}{t_1 - t_2} = \frac{d_2}{t_2}.$$

Finally, substituting equals for equals, we find

$$v = \frac{d_1 - d_2}{t_1 - t_2}. \qquad \text{II.}$$

By the use of this formula, constant errors (§ 24) are eliminated.

¶ 136. **Measurement of Long Distances.** — The measurement of long terrestrial distances is in general a problem for which the student must be referred to works on surveying. No particular difficulty will, however, be found in measuring approximately a distance along a moderately straight path; for even variations as great as 8° (nearly 1 foot in 7), either in the direction or in the slope of the path, will introduce an error of less than one per cent in the result.

Distances may also be determined indirectly by means of a sextant. To measure a distance, for example, across a valley, from an observing station, A, (Fig. 123) to an object B, we place (or select) an object C, so that the lines joining B with A and with C

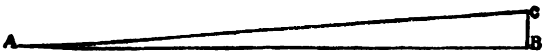

FIG. 123.

may be approximately at right-angles. The distance BC is then measured directly, and the angle CAB is determined from the observing station. Since (by definition)

$$BC \div AB = tangent\ CAB,$$

we have

$$AB = \frac{BC}{tan\ CAB}.$$

To obtain with an ordinary sextant (see ¶ 124) results accurate within 1 per cent, the distance BC actually measured should be at least a hundredth part as great as the distance AB to be determined. In regard to the direction of C from B, great accuracy is not required. If the corner of a square be

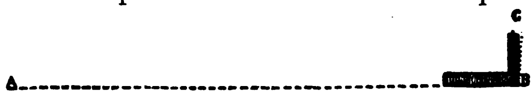

FIG. 124.

placed at B (Fig. 124) with one side directed towards A, any object, C, nearly in range with the other side of the square, will answer for our purpose. An error of 8° in the angle ABC will introduce an error of only 1 % in the result. The object C may

be on a level with B or above it, as may be more convenient. The distance BC and the angle CAB must be accurately measured.

In one part of the experiment the distance AB should be as great as possible considering the space at the disposition of the observer, and the distance through which the signals at his command can be seen or heard. If the method of difference is to be employed (¶ 135, 3), it is necessary, in a second part of the experiment, to make use of a much shorter distance. The second distance should be in no case greater than half of the first, and always as small as is consistent with the accurate determination of the time occupied by sound in traversing it. When the time is to be found by an ordinary watch (¶ 137, I.), the smaller distance should be several hundred, the greater several thousand metres. In the pendulum method (¶ 137, IV.), distances of 300, 600, and 900 metres may conveniently be employed. When sound signals are to be sent back and forth between two stations (¶ 137, III.), the minimum distance may be reduced to about 150 metres. The velocity of sound has been determined by the use of echoes (¶ 137, II.) between the Jefferson Physical Laboratory and the Lawrence Scientific School, the walls of which are about 80 metres apart. Long corridors, tunnels, and conduits of various sorts frequently give rise to echoes suitable for the determination of the velocity of sound.

It must be remembered that in the time required for a signal to go from one station to another, then

back to the first, the distance traversed is twice that between the stations. When the sound is reflected back to the observer the distance traversed is twice that of the observer from the object causing the reflection. Care must be taken to identify the object in question. In the interval between two successive echoes, sound must obviously traverse twice the distance between two objects which reflect it, as for instance two parallel walls or the two ends of a conduit.

¶ 137. **Measurement of Short Intervals of Time.** — I. One of the oldest methods of estimating the time required for sound to traverse a given distance is to count the ticks of a watch which occur between the flash and the report of a cannon discharged at that distance from the observer (see ¶ 138). When, owing to obstructions in the field of view, it is impossible to see the flash, an electric telegraph may serve in the place of light to inform the observer of the exact moment of the discharge.[1] Instead of counting ticks, a "stop-watch" may be used, or a chronograph may be employed (¶ 266). Amongst various ingenious devices for the measurement of small intervals of time may be mentioned the use of a stream of mercury from a Mariotte's bottle (see Fig. 275, ¶ 250), which may be directed into a receptacle at the beginning of the interval, and diverted at the

[1] The velocity of light is about 30,000,000,000 *cm. per sec.*; hence the time lost in traversing terrestrial distances may generally be disregarded. An electric current is practically instantaneous in its action; but an allowance must be made for the slowness of telegraphic instruments to respond to the current, unless a method of difference be employed. See ¶ 135, 3.

end of the interval. The quantity of mercury collected serves to estimate very precisely the interval of time in question.

II. In certain localities the velocity of sound may be similarly determined by timing the interval between a sound and its echo. When a series of echoes may be heard, the interval between them may be determined by adjusting a pendulum or a metronome so as to keep time with the echoes while they last, then afterward finding the rate of the pendulum or metronome, by timing 100 or more oscillations. Again, a method of multiplication may be used (§ 39). When the last audible echo reaches the observer, a new sound may be made; so that the interval of time to be measured may be indefinitely increased. One of the earliest determinations of the velocity of sound is said to have been made by a monk, who made use of the echo in a cloister caused by clapping his hands. The sounds thus produced were, it is said, so timed as to alternate regularly with the echoes.

III. The effects of an echo may be imitated by a series of sound signals interchanged between two stations. Let us suppose that two observers, each provided with a hammer and a plank, place themselves at suitable distances (see ¶ 136). The first gives a blow with his hammer, then the second returns the signal as soon as the sound reaches him. When the first hears the response, he gives another blow, etc. As in the last method (II.), the interval of time to be measured may be indefinitely multiplied.

With practice, each observer will learn to anticipate the return signal, so that very little time will be lost in the act of repetition. The time thus lost is to be eliminated by making two experiments, as has been suggested above (¶ 135, 3).

IV. Another method[1] is to station two observers let us say 300 or 350 metres apart, and to provide each with a telescope, if necessary, so that he may watch a pendulum, or any other object having a periodic motion, in sight of both observers. Either the length of the pendulum, or the distance between the observers is then varied until a sharp sound made by A, when the pendulum is at the middle point of its swing, is heard by B at the moment when the pendulum, after completing one or more oscillations, again passes the middle point. The distance is then measured, and the time of the pendulum determined. Measurements must also be taken in which sounds made by B are heard by A as the pendulum passes its middle point. The experiment is then repeated with a distance between the observers (¶ 135, 3) two or three times as great as before.

Other methods of measuring short intervals of time will be considered in experiments which follow.

¶ 138. **Proper Methods of Counting.** — In counting the ticks of a watch (which usually occur at intervals of one-fifth of a second), it will be found difficult, if not impossible, to repeat, even mentally, the names of numbers which contain more than one

[1] See Ex. 30, Elementary Physical Experiments published by Harvard University.

syllable.[1] In the following method of counting, this difficulty is avoided: —

1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	**2**
1	2	3	4	5	6	7	8	9	**3**

By counting the ticks which actually occur *within* a given interval of time, the length of that interval will on the whole be fairly estimated. There is, however, a tendency in most persons to count one too many ticks. When a given interval contains a whole number of ticks, one occurring at the beginning of the interval should be counted "nought," or not counted at all. Obviously the first and last tick should not both be counted.

With intervals of time (as with intervals of space), care must be taken to distinguish the number of intervals from the number of divisions between which they lie. In the same way that the zero of a scale should not be counted "one," the beginning of an interval of time should not be called one second or one-fifth of a second. A miscount may generally be avoided by pronouncing the word "now" at the beginning of the interval, then beginning the count immediately afterward.

An accurate method of counting is important in a great variety of measurements, especially those which involve rates of vibration or revolution. The student should consider carefully what habits he has formed

[1] The difficulty is greatly lessened by counting every other tick; but on account of the greater inaccuracy, this method of counting is not generally recommended.

in this respect, and if they are not good, whether it is preferable to change them, or to make an allowance for "personal error" in each separate determination.

EXPERIMENT LII.

GRAPHICAL METHOD.

¶ 139. **Determination of Rates of Vibration by the Graphical Method.**[1] — A tuning-fork (*ae*, Fig. 125)

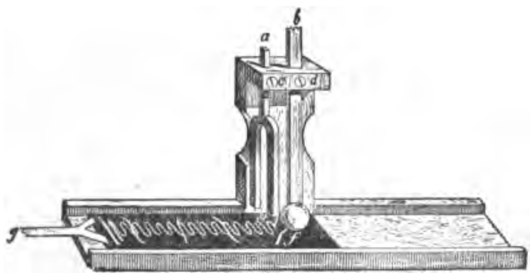

FIG. 125.

making from 100 to 300 vibrations per second, and a pendulum (*bf*), made of an ounce bullet (*f*) and a piece of clock-spring (*b*), are mounted as in the figure, so that when the tuning-fork and pendulum are in vibration, two short and fine brass wires attached one to each may make marks (*h* and *i*, Fig. 126) as close together as possible on a piece of smoked glass.

[1] The experiment here described is essentially the same as that given in Exercise 81, Elementary Physical Experiments, Harvard University. This application of the graphical method is due to Prof. Hall.

The tuning-fork and the spring are then firmly clamped by the screws *c* and *d*.

The smoked glass is now drawn slowly out from under the pendulum and the tuning-fork. The points of the wires *e* and *f* should draw a single line (*h i x*, Fig. 126) upon the surface of the glass. If they do not, the wires should be bent, or their relative position otherwise adjusted. The smoked glass is now to be replaced, and both the pendulum and the tuning-fork are to be set in vibration,—the latter by drawing a violin-bow across one of the prongs. The bow must be drawn slowly at first, and always *in a*

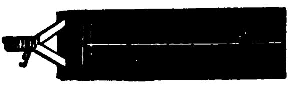

FIG. 126.

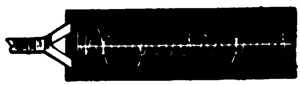

FIG. 127.

direction nearly parallel to the vibration which it is desired to create. That is, the bow should be held at right-angles to the prongs, but nearly parallel to the plane containing them. The smoked glass is again drawn out from under the pendulum and the fork, with a slow but uniform velocity.

The wire attached to the tuning-fork, partaking of its vibration, will trace upon the glass a series of waves. The wire attached to the pendulum would similarly trace a series of much longer waves, were it not that owing to the amplitude of its oscillation, the wire usually leaves the glass at the extreme points of a swing. The result is a series of marks (*j*, *k*, *l*, etc., Fig. 127).

19

The time required for one complete oscillation of the pendulum is represented by the distance between alternate marks (*j* and *l*, or *k* and *m*, Fig. 127). The number of complete vibrations made by the tuning-fork in the same length of time is to be found by counting the waves executed in the same distance. Thus between *j* and *l* there are (in the figure) about 6¼ complete, or 12½ half-waves; and between *l* and *n* there are similarly about 7 waves. In practice, a much greater number would be counted.

If the waves are perceptibly closer together at *k* or at *l* than at *m* or at *n* (or the reverse), the glass has not been drawn with sufficiently uniform velocity. In this case, instead of depending upon the marks (*j*, *k*, *l*, etc.) actually made by the pendulum, it is necessary to draw a line at a distance from each mark equal to that between *h* and *i* (Fig. 126), and at the left or at the right of it, according to whether *h* is at the left or at the right of *i*. The new lines show where the wire attached to the pendulum *would have crossed* the glass, provided that it could have been made absolutely coincident with the wire attached to the pendulum. By the use of lines drawn as above, we may in counting the waves avoid errors due to irregularity in the speed of the glass. The number of whole waves included between two alternate lines should be recorded in each case, together with an estimate of the fractions of a wave left over at each end of the series. This fraction should be expressed in tenths § (26).

To find the rate of vibration of the tuning-fork,

the time occupied by one complete oscillation of the pendulum must now be determined. This is done by timing, let us say, one hundred complete oscillations. Having given a signal, one observer begins to count the oscillations of the pendulum, while a second observer, as soon as the signal is perceived, begins to count the ticks of a watch (see ¶ 138). When the pendulum has completed a given number of oscillations, the first observer signals to the second to stop counting.

The number of complete oscillations of the pendulum per second is found from the time required for 100 or 200 oscillations (as the case may be), by simple division, and the result is multiplied by the average number of waves made by the fork during one of these complete oscillations to find the "vibration number," or "pitch" of the fork, — that is, the number of complete vibrations made in one second.

EXPERIMENT LIII.

BEATS.

¶ 140. **Theory of Beats.** — When two musical notes, nearly but not quite in unison, are sounded together with about the same degree of loudness, the effect upon the ear is by no means uniform. At regular intervals the sound swells out, and these intervals are separated by moments of comparative silence. Each rise and fall of the sound constitutes a "*beat.*"

The increase is due to the mutual re-enforcement of the two sets of vibrations communicated to the air; the decrease is caused by the interference of these vibrations.

Let us suppose that two tuning-forks, one making 256, the other 255 vibrations per second, are started at a given instant by forcing their prongs together and suddenly releasing them. The prongs of both forks will spring apart simultaneously, and each fork will cause a slight condensation of the air on each side of it. This condensation will be followed by a rarefaction when the prongs rebound, then by several alternate condensations and rarefactions, nearly though not quite synchronously performed. The result is that the vibrations reaching the ear at the same distance from both forks are very much greater than if one fork were sounding alone. At the end of half a second, however, the first fork will have made $256 \div 2$, or 128, complete vibrations; so that, as at the start, its prongs will be springing apart; but the second fork will have made only $255 \div 2$ or $127\frac{1}{2}$ vibrations, so that its prongs will be approaching each other. The condensation produced by one fork will tend to offset the rarefaction produced by the other. The effect on the ear will accordingly be less than if one of the forks were sounding alone. This interference of the vibrations will evidently continue as long as the forks are vibrating in opposite ways. At the end of a second, the first fork will have made just 256, the second fork just 255 complete vibrations, and the direction in which the prongs

are moving will be in each case the same as at the start, and hence the same for both forks. The sounds will therefore re-enforce each other as at first. It is evident that, with the forks in question, periods of re-enforcement must occur every second, separated by intervals of interference. In other words, two forks making 256 and 255 vibrations per second must give rise to 1 "beat" per second when sounded together.

In the same way it may be shown that two forks differing by *n* vibrations per second give rise to *n* beats per second. In other words, when two musical notes are nearly in unison, *the number of beats per second is equal to the difference between the vibration numbers* corresponding to the two notes in question.

¶ 141. **Determinations of Pitch by the Method of Beats.** — The special apparatus required for this experiment consists of a series of tuning-forks with differences of from three to five vibrations per second, covering an interval of one octave (¶ 134). The first and the last of the series are to be sounded together, to make sure that the musical interval is exact. If the forks are nearly but not quite an octave apart, faint beats may be heard. In this case one of the forks must be loaded with small bits of wax near the end of its prongs until the beats disappear. If the wrong fork is loaded the beats will become more frequent than before. The same effect may be produced if *too much* weight is added to *either fork;* hence care must be taken at first to add very little weight at one time.

The simplest way in general to tell whether a fork is higher or lower than may be required for the purposes of harmony is by the method of loading suggested above. The effect of the additional weight is to lower the rate of vibration of the fork to which it is attached. Whenever by loading a fork it may be brought into harmony with a given musical note, we know that fork to have a higher rate of vibration than the purposes of harmony require.

If, for instance, the first fork in the series gives 61, and the last 120 vibrations per second, the first will have to be loaded until it gives 60 vibrations per second, in order to be in harmony with the other fork. Again, if the second fork gives 64 vibrations per second, it will have to be loaded to bring it in unison with the first fork. We may generally assume that the forks are arranged by the instrument-maker in an ascending series.

The experiment consists in a determination of the number of beats produced in a given length of time by sounding together each pair of consecutive forks in the series, that is, the first and second, the second and third, the third and fourth, etc. The student will do well to begin counting with one of the beats which happens to occur when the second-hand of his watch indicates a round number. The *beginning* of this beat should not be counted (see ¶ 138). One hundred beats should be timed if possible. The time of the last beat should be observed to a fraction of a second. The number of beats per second should be calculated in each case.

The results represent differences between each pair of consecutive forks in the series; hence when added together we have the difference between the first and the last in the series, for the whole difference in question must be equal to the sum of all its parts.

Now two notes an octave apart are to each other, in respect to their vibration numbers, as 2 is to 1 (¶ 134); hence the last number in the series is twice the first. It follows that the difference between the first and last numbers is equal to the first number in the series. The result of adding together the numbers of beats per second is therefore to find the number of vibrations executed by the first fork in one second.

By adding to this number the number of beats per second between the first fork and the second fork we find the pitch of the second fork; and in the same way, successively, the pitch of each fork in the series can be calculated.

EXPERIMENT LIV.

LISSAJOUS' CURVES.

¶ 142. **Theory of Lissajous' Curves.** — We have seen, in Experiment 52, that when a piece of smoked glass is drawn beneath a pointed wire attached to a vibrating tuning-fork, a wave-line is traced upon it. If instead of drawing the glass completely away from the tracer, the motion be suddenly reversed, we shall evidently obtain a double wave which will re-

semble one of the figures below (Fig. 128, 1, 2, and 3) according to the point (*a*) in the curve at which the reversal takes place. In the first curve the two waves happen very nearly to coincide. We may imagine the reversal to take place so that there should be a perfect coincidence.

Now let us suppose that when the tracer reaches a certain point, *b*, a second reversal takes place, and a third reversal occurs when the tracer returns to the former point, *a*. Evidently, if the reversals are properly timed, the tracer will follow the same path over and over.

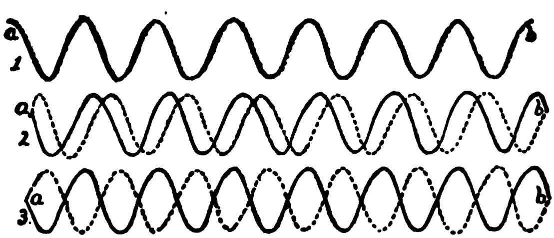

FIG. 128.

In practice we obtain a similar result by attaching a small piece of smoked glass to the larger of two tuning-forks. When the larger fork makes one vibration in the same time that the smaller fork makes for instance 8, we obtain tracings as in Fig. 129, 1, 2, or 3, according to the relation which happens to exist between the forks at the start.

These are examples of Lissajous' curves. The reversal of the smoked glass is not sudden, as in the case previously supposed, and its velocity is greatest

when the middle of the figure is being drawn. This accounts for the difference in appearance between these curves and those represented in Fig. 128.

It may be shown that whenever two vibrations at right-angles are compounded graphically, as in Fig. 129, unless the times of the vibrations are incommensurate, a Lissajous' curve results. Each musical interval (¶ 134) has, accordingly, its characteristic curves. These curves are in general too complicated to be discussed in an elementary work. We shall confine ourselves to such cases as are represented in

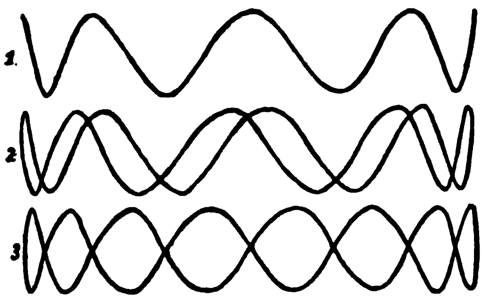

FIG. 129.

Fig. 129, where one fork makes a certain whole number of vibrations while the other makes one.

To find in such cases the musical interval between the forks, we have to experiment until a figure like the third is obtained (Fig. 129, 3). If this figure contains n lobes, then the higher fork makes n times as many vibrations as the lower fork.

It has been so far assumed that the two forks are separated by an exact musical interval, so that at the end of a certain period they find themselves in

exactly the same mutual relation as at the start. If this is not the case, it is evident that the tracer will not follow the same path in all cases, but that this path will be continually changing.

Let us suppose that the tracer reaches its highest point, as seen in the figure, when the glass reaches its extreme right-hand or left-hand turning-point. Then the curve traced will be represented as in Fig. 129, 1. If the small fork is a little behind-hand we shall have a tracing as in Fig. 129, 2; and if the small fork has only reached the middle of its course when the glass turns, we shall have a tracing like

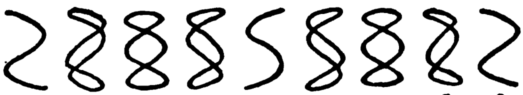
1. 2. 3. 4. 5 6 7. 8 9.

FIG 130.

Fig. 129, 3. Evidently, if the small fork starts as in (1) and falls slowly behind the other, we shall have a series of tracings represented by (1), (2), and (3). It is not until the higher fork has fallen one complete vibration behindhand that the same figure will be repeated.

If the smaller fork is gaining instead of losing, a similar series of changes will be produced. There is in fact no way to tell which fork is too high for the musical interval in question, except as in the last experiment, by loading it and observing the result. A complete cycle of changes in the case of two forks one octave and one fifth apart (¶ 134) is shown in Fig. 130.

The symmetrical lobed figures (3 and 7) appear twice in a cycle; the serpentines appear also twice; but one of them is left-handed (1), the other right-handed (5). The interval between two left-handed (or that between two right-handed) serpentines always represents one complete cycle, and is accordingly equal to the time in which the higher fork makes one whole vibration more or less than would be required to give a perfect musical interval.

Let p be the pitch of the lower fork, that is, the number of vibrations it makes in one second, and let n denote the approximate musical interval between the forks; then the pitch of the higher fork, which we will call P, must be equal to np, nearly. If, however, we observe c cycles per second, the true pitch of the higher fork is $np \pm c$. Here c is positive if by loading the higher fork the musical interval may be made perfect; if on the other hand the lower fork must be loaded, c will be negative. With this understanding we have

$$P = np + c. \qquad \text{I.}$$

and

$$p = \frac{P - c}{n}. \qquad \text{II.}$$

These formulæ apply only to cases in which, as we have supposed, n is a whole number.

¶ 143. **Determination of Pitch by Lissajous' Curves.** —A tuning-fork of known pitch (Exps. 52 and 53) and one approximately an octave above or below it are to be mounted, as in Fig. 131, with their prongs at right-angles. The prongs of one fork (A) are to be coated with lampblack, except at a small point

where, by the touch of a pin, the bright metallic surface is made visible. Opposite this point on the other fork (B) a lens, C, of about 1 inch focus, is to be attached with sealing-wax, at such a distance that a highly magnified image of the point may be seen

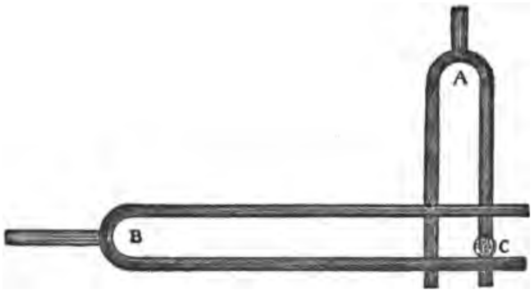

FIG. 131.

through the lens. When a violin-bow[1] is drawn across the fork A, the bright spot partaking of the vibration will be apparently extended into a horizontal line, Fig. 132.

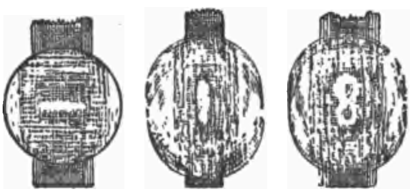

FIG. 132. FIG. 133. FIG. 134.

When the fork B is set in vibration, the motion of the lens will cause the spot to be apparently elongated into a vertical line, as in Fig. 133. When both

[1] In practice, it will be found convenient that one or both of the forks should be maintained in vibration by electrical means.

forks vibrate simultaneously the vertical and horizontal motions will be combined, and if the forks are seprated by an exact octave, one of Lissajous' curves will be formed, as for instance in Fig 134.

If this curve is permanent in form, the experiment is now finished; but if, as is generally the case, it passes through a series of cycles, as in Fig. 130, ¶ 142, it becomes necessary to count the number of complete cycles which take place in a given length of time. It is also necessary to load one of the forks, as in ¶ 141, until the changes in the cycles become less frequent.[1]

We thus find whether c is positive or negative in the formulæ of ¶ 142. The pitch of one of the forks is finally to be calculated by one of the formulæ in question from the pitch of the other fork, previously determined.

EXPERIMENT LV.

THE TOOTHED WHEEL.

¶ 144. **Construction of a Toothed-Wheel Apparatus.** —A toothed-wheel apparatus capable of giving fairly accurate results is represented in Fig. 135, as seen from above. A vertical cross-section is shown also in Fig. 136. The works (e) of an ordinary eight-day

[1] It is possible to load a fork so that a figure of a certain class (see Fig. 130, 1–9) may preserve its characteristics until the vibration dies away.

spring clock, from which the escapement has been removed, are mounted on a piece of wood, and a disc of cardboard (*a*) is attached to the axle usually carrying the second hand. Two pieces of watch-spring are also attached to this axle at *b*, and bent into loops so that two small loads (*c* and *d*) which they bear may hang quite close together when the wheel is at rest. The friction which the springs exert against the air acts as a governor upon the speed of the machine.

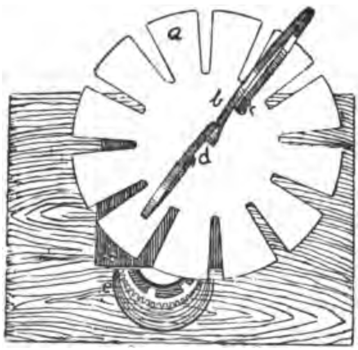

Fig. 135.

The velocity of rotation will be found to vary very little as the force of the main-spring grows less and less. To make the wheel turn faster, the loads (*c* and *d*) may be decreased; or a *slight* change may be produced by winding up the main-spring. To make the wheel go slow, the load may be increased; or a slight decrease in speed may be had either by waiting for the main-spring to unwind itself, or by applying friction to one of the more slowly moving wheels. The upper surface of the disc, *a*, should be painted black. The number of revolutions which it makes in a given time may be counted by watching a white spot upon it, or still better by listening to the sound

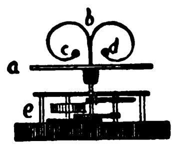

Fig. 136.

made by an object striking lightly against a projection from the wheel or from the axle upon which it is mounted. At equal distances around the circumference of the wheel, narrow radial slits should be cut out. The number of slits must be made with reference to the usual speed of the machine and the number of vibrations per second which the toothed wheel is intended to measure. The wheel represented in Fig. 135 makes about 8 revolutions per second without any load, — the speed being reduced to 4 revolutions per second by a load of a few grams at *c* and *d*. With twelve notches in the disc, this apparatus affords from 48 to 96 nearly instantaneous views of objects seen through the rim of the wheel. The instrument is accordingly suited to the determination of the pitch of tuning-forks making from 48 to 96 vibrations per second. It may also be used for much higher forks, as will be presently explained.

¶ 145. **Theory of the Toothed Wheel.** — By the apparatus just described we are able to obtain at regular intervals a series of instantaneous views of a vibrating object. If the intervals between the views correspond to the period of vibration in question, the same view will evidently repeat itself over and over. If the intervals are sufficiently short, the effect will be a continuous impression upon the eye. Thus when the eye is held close behind the rim of the rotating disc (Fig. 135), the speed of which is properly adjusted, we may obtain a series of views of a tuning-fork, in all of which the prongs are, for in-

stance, at their greatest elongation. The result is that the fork appears to be at rest. To obtain this result the number of slits which pass in front of the eye in one second must be equal to the number of vibrations executed by the fork in the same time. If the wheel is moving a little too fast or too slow, the successive views of the fork will not be exactly the same.

FIG. 137

The position of the prongs will seem to change as if the fork were executing a very slow vibration. When the fork is held close behind the rim of the disc, as in Fig. 137, a different effect is produced.

Let us first consider the effect of a single slit moving along the fork. Let 1, 2, 3, 4, 5, 6, 7, 8, Fig. 138,

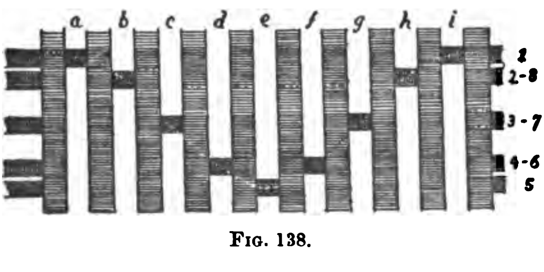

FIG. 138.

be views of the fork seen through such a slit when occupying the successive positions *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, and *i*. These views are evidently situated along the dotted line *ai*. Let us now supply the intermediate views. We shall evidently have the curve shown in

Fig. 137, or in *ab*, Fig. 139. Now let another slit pass along the fork. We shall have similarly a curve, *cd* or *ef* (Fig. 139), which may or may not coincide with *ab*. If it does not coincide with *ab*, we shall probably not see either of the curves, since the light reflected through the slits will hardly have time to affect the eye. If, however, several such curves coincide, the joint effect will be similar to that shown in Fig. 137.

In order that successive curves may coincide, it is necessary that successive slits should reach a given point in the curve (as *a*, Fig. 138) at the same instant that the prong of the tuning-fork reaches that point.

FIG. 139.

In other words, the interval of time between the arrivals of successive slits must correspond with the period of the tuning-fork.

It will be found, if a toothed wheel is adjusted so as to show waves, as in Fig. 137, that when the speed is increased the waves will seem to follow the direction in which the wheel is moving, while if the speed is lessened, the waves will move in the opposite direction. This is the result of a series of wave images (see Fig. 139), each of which is situated in a *slightly* different place from the one preceding it. The direction in which the waves seem to move is a valuable guide in adjusting the speed of the wheel.

It is easy to trace out in a similar manner the appearance of a vibrating fork for any speed of the wheel. Usually it will appear blurred, as if looked at in the ordinary manner. If, however, the wheel is moving twice as fast as it ought, a double wave will be visible, as in Fig. 140. If, again, the fork makes in one second a number of vibrations twice as great as the number of slits which pass a given point, the appearance of the fork will be as in Fig. 141. Care must be taken not to mistake this curve for the double curve of Fig. 140, nor for the regular curve of Fig. 137. We notice that in Fig. 141 there are two complete waves in the distance between two successive slits (*a* and *b*).

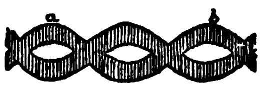

FIG. 140.

FIG. 141.

In the same way this distance will be divided into *n* waves if the fork executes *n* vibrations between successive views from a given point.

By this principle we may find the rate of a fork too high to be measured by the ordinary method.

¶ 146. **Determination of Pitch by means of a Toothed Wheel.** — The experiment consists simply in adjusting the speed of a toothed wheel (Fig. 135, ¶ 144) so that a fork held behind the rim of a wheel (as in Fig. 137, ¶ 145), and making about 64 vibrations per second, will be apparently thrown into simple stationary waves, the lengths of which will be equal to the distance between the teeth of the wheel, then finding

how many teeth pass by a given point in one second. We have already considered (¶ 144) the manner in which the speed of the wheel may be adjusted and how the number of revolutions may be counted.[1] The number of revolutions made in one second multiplied by the number of teeth gives the number of teeth per second. This is (see ¶ 139) the "pitch" of the tuning-fork.

[1] If it is found impossible to adjust the speed exactly, or to keep it adjusted, accurate results may still be obtained by counting the number of waves which in one second traverse the field of view. This number is to be added to the number of slits passing a given point in one second if the motion of the waves is opposite to that of the wheel; if both move in the same direction the first number is to be subtracted from the second.

DYNAMICS.

¶ 147. **Different Methods of Measuring Velocity in Dynamics.**—When a body is moving so slowly that it is possible to make a series of observations of its position at different points of time, no particular difficulty is met in the measurement of its velocity. Thus in Exp. 60, to find the average velocity of a ring rotating about its axis, we observe the distance traversed between two ticks of a clock, and divide it by the interval of time in question. Such slow motions are, however, the exception in dynamics. In certain cases instantaneous photography has been employed for the study of rapid motions. The estimation of velocity generally requires, however, special devices, such as have been employed for the velocity of sound (Exp. 51).

(1) In rough measurements, we frequently make use of the sounds produced by a moving body when it strikes different obstacles in its course. A familiar example of this method consists in the determination of the speed of a railway train by counting the number of rails crossed in a given length of time. To find the velocity of a marble rolling in a groove, small tacks may be driven into the groove at such distances that the successive sounds made by the marble in crossing them correspond with the ticks of a clock. The regular increase of velocity caused by a steady

incline is then easily demonstrated by measuring the distances between the tacks.

(2) By substituting for a series of tacks a series of electrical connections which are made or broken by a moving body, we may make use of any of the devices by which time is measured by electrical agency.[1]

The velocity of a rifle bullet has been measured by the interval of time between the rupture of two wires a known distance apart. The time of rupture is usually recorded "*graphically*" by means of a chronograph (see ¶ 266). Curves traced simultaneously by the armature of an electrical sounder and by a tuning-fork (see Exp. 52) enable us to estimate precisely exceedingly small intervals of time.

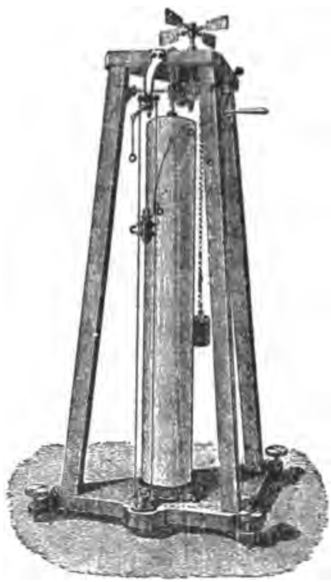

FIG. 142.

(3) There are various devices in which the motion of a body may be directly recorded by the graphical method. Thus, in Morin's Apparatus (Fig. 142), a pencil (*c*) attached to a falling body marks directly upon a revolving cylinder covered with paper. If the rate

[1] See Trowbridge's New Physics, Exp. 71, 72, 73.

of revolution is known, we may obviously infer the position of the body at different points of time from the tracing (*ab*) made by the pencil.

Another device in which the vibrations of a tuning-fork attached to a falling body may be made to indicate its position, will be found in Trowbridge's New Physics, Exp. 74.

A simple instrument illustrating the graphical method of measuring velocity will be described in the next section.

(4) In studying the motion of fluid streams, the velocity is frequently calculated from the size of a tube or orifice, and from the volume which flows through this tube or orifice in a given time. Thus if a stream

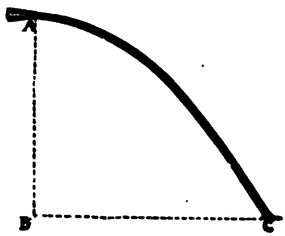

FIG. 143.

FIG. 144.

of water issues from an orifice $\frac{1}{4}$ *sq. cm.* in cross-section at the rate of 25 *cu. cm. per sec.*, its velocity at the orifice must be 100 *cm. per sec.* This principle has been applied to illustrate the law of falling bodies. A stream of water projected horizontally with a known velocity must traverse a known horizontal distance (*BC*, Fig. 143) in a known time; hence the time required for gravity to deflect the stream through a known vertical distance (*AB*) is determined.

(5) The pressure of a stream of gas has been applied to the determination of the mass of the gas when its velocity is known, and conversely for a determination of its velocity when the mass is known. If, for instance, a mass of gas m, impinging with the velocity v, on a scale-pan (a, Fig. 144) causes a force, f, to be exerted for a time t, we have from the general formula (§ 106)

$$m = \frac{ft}{v}; \quad v = \frac{ft}{m}.$$

(6) The laws of falling bodies are frequently made use of for indirect measurements of velocity. Thus since a body is known to fall 4.9 metres in 1 second, the velocity of a stream of water projected horizontally at a distance of 4.9 metres above a certain level will be equal numerically to the horizontal distance traversed before reaching that level, the time in question being 1 second. Again, the velocity of a pendulum when it passes its central point may be estimated by the distance it has fallen in reaching that point, or by the distance it rises after reaching that point (see § 109).

(7) The law of *action and reaction* enables us to make comparisons of velocity. Thus if a bullet of mass m, striking a log of mass M, suspended as in Fig. 145, gives it a velocity V (see § 106), the velocity of the bullet (v) may be found by the equation,

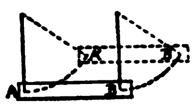
FIG. 145.

$$v = \frac{(m + M)}{m} V.$$

Changes in velocity may be measured by the same principle. If two billiard balls, A and B (Fig. 146), are suspended by cords of equal length so as to just touch each other without pressure, and if the greater, A, is drawn aside to a position A' (Fig. 147) and allowed to strike B while resting at B', the latter will reach a position B'', while the former reaches A''. The velocity acquired by A in falling from A' to A will be proportional to the straight line $A'A$ (§ 109); the velocity after impact will be proportional to AA'' and in the same direction as before; hence the loss

FIG. 146.

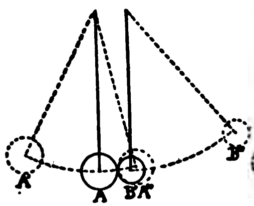

FIG. 147.

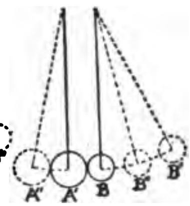

FIG. 148.

will be proportional to $A'A - AA''$. At the same time B gains a velocity represented by $B'B''$.

If on the other hand B strikes A from a position B' (Fig. 148), it will rebound to B'' in the opposite direction; hence its change of velocity will be $B'B + B''B$. The corresponding gain of velocity by A will be represented by $A'A''$.

It is easy to show by experiment that the products of the masses and their respective changes of velocity are equal, whether the balls are elastic or inelastic.[1]

[1] See Ex. 20 of the Descriptive List of Elementary Physical Experiments published by Harvard University.

A comparison of the changes of velocity in question gives a simple means of estimating the relative masses of the balls.

EXPERIMENT LVI.

FALLING BODIES.

¶ 148. **Determination of Distances traversed by Falling Bodies in Different Lengths of Time.** — A wooden rod, *jp* (seen edgewise in Fig. 149), about 25 *cm.* in length, 3 *cm.* in breadth, and 1 *cm.* in thickness, is suspended from the edge, *f*, of a bracket, *ef*, by a strap of paper forked at *h*, so that the rod, when free, may hang in a vertical position. An ounce bullet is next suspended by a thread from the peg, *c*, and lowered to a position, *q*, near the bottom of the rod. The bracket is then moved (by loosening the screws *d* and *g*) so that the rod may barely touch the bullet. Then the bullet is removed, and either the rod is smoked at *j* and at *p*, or pieces of smoked paper are attached to it at these points.

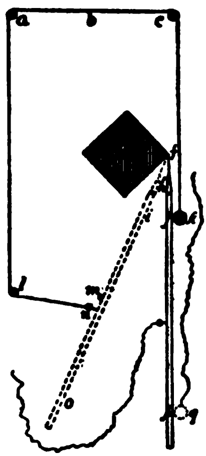

Fig. 149.

The bullet is now suspended at a point, *k*, near the top of the rod, by a thread passing over the smooth round pegs *c*, *a*, and *l*, to a screw-eye, *n*, near the

middle of the rod. The rod is drawn one side by the pull on the thread, due to the weight of the bullet. Care must be taken to ease the thread round the pegs, so that the true position of equilibrium may be found. A pin *m* may then be placed so as to mark this position of equilibrium.

To find the height of the bullet a finger is laid upon the thread at *a*, and the thread is slipped off the peg *l*, so that the rod may strike the bullet. A mark will thus be made on the smoked surface at *j*. The thread is now carefully replaced on the peg *l*, so that the tension may be the same as before. When the finger is finally removed from *a*, there should be no slipping of the thread. If there is, the experiment must be repeated, until the bullet, having made a mark on the rod, remains unchanged in position.

Any oscillation of the bullet must now be arrested by lightly pushing the thread, just below *c*, in a direction always opposite to that in which the bullet is swinging, or simply by allowing time enough for it to come to rest. The thread is then burned at *b* by holding a lighted match under it. The rod and the bullet will thus be released at very nearly the same instant. When the rod reaches its vertical position, *jp*, it will strike the bullet at some point, *q*, where the bullet will make a mark on the smoked surface.

The distance between the two marks, one near *j*, the other near *p*, is now to be measured. This distance is equal to that through which the bullet falls while the rod is reaching its vertical position; that is, in half the time it takes the rod to swing from one side

to the other. To determine the time in question, we set the rod once more in oscillation and find how long it takes it to complete 100 or more swings.[1]

To obtain the best results, the oscillations should be timed as will be explained in the next experiment. The time of a single oscillation (either from left to right or from right to left) is then calculated and divided by 2, to find the time occupied by the rod in reaching its vertical position in the middle of one swing. This gives the time occupied by the bullet in falling through the observed distance.

The experiment should be repeated with the same apparatus until results are obtained agreeing within 2 or 3 per cent. The experiment should be then varied by using rods of different lengths. The results should be entered as follows: in the first column, the distance through which the bullet falls; in a second column, the corresponding times of falling; in a third column, the squares of these times; in a fourth column, the ratios of the distances to the squares of the times. Thus:—

1. Distance Fallen.	2. Time Occupied.	3. Square of Time.	4. Ratio of 1 to 3.
19.2 *cm.*	0.20 sec.	0 040	480
80.0	0.40	0.160	500
etc.	etc.	etc.	etc.

It will be seen by the formula $d = \frac{1}{2} gt^2$ (§ 108) that the ratio of the distance to the square of the time must be equal to $\frac{1}{2} g$, which is the distance a body

[1] The student should notice that though the swings grow shorter and shorter in length, there is little or no perceptible change in the rate of oscillation (see § 111). A more exact method of testing this point will be met incidentally in Exp. 58.

falls in one second. The numbers in the fourth column may be considered, therefore, as different estimates of this distance, founded on observations lasting through different intervals of time. These estimates should evidently show an approximate agreement; but the results are modified somewhat by the fact that we are not experimenting with a body which is perfectly free to fall. A device, similar in many respects to that shown in Fig. 149, will be found described in Exp. 20 of the Descriptive List of Experiments in Physics, published July, 1888, by Harvard University. A device in which two electromagnets are used to set free a pendulum and a falling body will be found in Trowbridge's New Physics, Exp. 67.

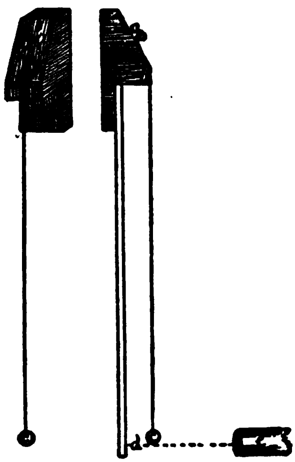

FIG. 150. FIG. 151.

EXPERIMENT LVII.

LAW OF PENDULUM.

¶ 149. **Determination of Times of Oscillation.** — An ounce bullet (*c*, Fig. 150) is to be suspended by a waxed silk thread, passing through a notch (*b*) in the edge of a bracket to and round a pin, *a*, by which the thread can be lengthened or shortened. The lower surface of the bracket must be horizontal (see *b*, Fig. 151), and the groove must be

deep enough to reach this surface. It is now required to find the length of the pendulum thus constructed; that is, the distance from its point of suspension, in the surface, *b*, to the middle of the bullet, *c*. This is done by means of a wooden rod, *bd*, graduated in millimetres. The rod is held parallel to the thread (and hence vertical) with its zero at *b*. The height of the centre of the bullet is found from that of the top and bottom by taking the mean. To avoid parallax (§ 25) these heights are sighted through a telescope (*e*), on the same level with them. We thus find the length of the pendulum in question. The time occupied by a hundred or more consecutive[1] oscillations of the pen-

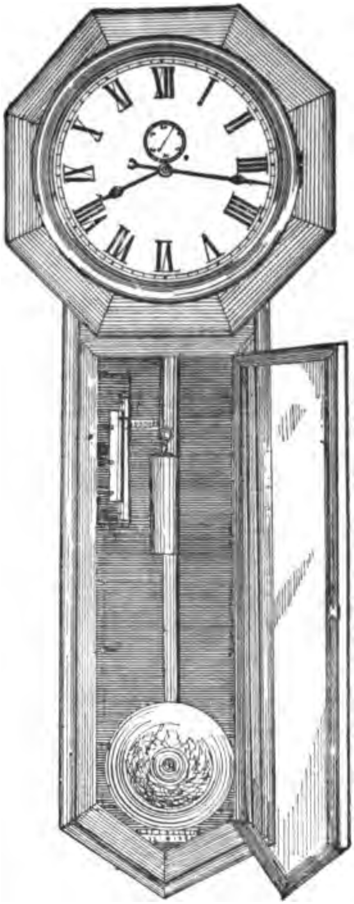

FIG. 152.

[1] The importance of observing long series of consecutive observations must not be overlooked. A student is apt to imagine that 10

with that obtained for falling bodies in Exp. 56, we discover a curious relation. The length of a pendulum which makes one swing in one second is about 99 cm. The distance a body falls in one second is about 490 cm. The latter is nearly 5 times as great as the former. Again, the length of a half-second pendulum is not quite 25 cm.. the distance a body falls in half a second is about 122 cm., that is, nearly 5 times as great as the corresponding length of the pendulum. This proportion will be found to exist in every case.

It is obvious that if this proportion is known,[1] we may calculate the distance through which a body falls in a given time from the length of a pendulum making one swing in the same time. We shall make use of this principle in the next experiment.

EXPERIMENT LVIII.

METHOD OF COINCIDENCES.

¶ 150. **Adjustment of a Pendulum of Peculiar Construction.** — A serviceable device, which conforms approximately to the conditions required of a simple pendulum, is represented in Fig. 153 as seen from in front, and in Fig. 154, in profile. It consists of a cylinder (gj) suspended by two vertical loops of silk

[1] The law of falling bodies gives (§ 108) $d = \frac{1}{2} gt^2$; the theory of the pendulum gives (see Appendix) $l = \frac{gt^2}{\pi^2}$; hence we have $d : l :: \pi^2 : 2 \cdot 4.935 : 1$, nearly. This ratio is not affected by the value of g, but is slightly affected by the resistance of the air.

thread passing around the horizontal pins *ab* and *hi*. The diameter of these pins should be exactly the same, and not over 1 *cm*. Their length should be about 10 *cm*. The upper pin (*ab*) is driven through a fixed support; the lower pin should pass as nearly as possible through the centre of gravity of the cylinder. The ends of the thread, after passing over the pin *ab*, are carried each to one of the pins *c*, *d*, *e*, and *f*, by turning which the threads may be lengthened or shortened. A disc is

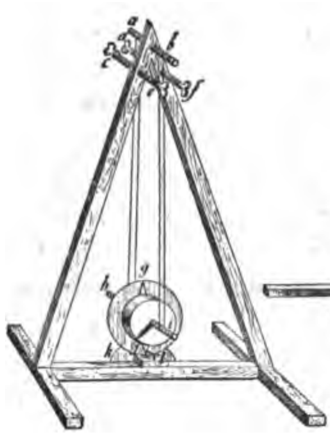

FIG. 153.

FIG. 154.

also attached to the cylinder, and in this disc are made two *V* shaped holes (*g* and *j*). Opposite the lower hole (*j*) may be placed an opening (*k*), in a shield, through which instantaneous views of objects behind the pendulum may be obtained at regular intervals. A small wire loop may be attached to the pendulum so as to complete an electrical connection between two drops of mercury at *l* when the pendu-

lum is at rest or in the middle of a swing. The length of the pendulum thus constructed is found by measuring the distance between these pins from centre to centre. In the absence of a cathetometer (¶ 262) or other device by which the distance in question may be accurately measured, it is well to adjust it by turning the pins *c*, *d*, *e*, and *f* until a metre rod fits without looseness or pressure between the pins *ab* and *hi*, so as to subtend the vertical distances either between *a* and *h* or between *b* and *i*. The diameters of the pins at *a*, *b*, *h* and *i* are now measured by a vernier gauge (Part I. ¶ 50). The average diameter added to the length of the metre rod gives the distance between the pins from centre to centre.

In regard to the working of this pendulum, it may be pointed out that the cords (*ah* and *bi*) keep the pins (*ab* and *hi*) parallel, hence horizontal, and always the same distance apart. The centre of the pin *hi* swings, therefore, in a vertical plane about the middle point of *ab* as a centre. Now equal parallel forces applied by the cords (*ah* and *bi*) on each side of the pins (*ab* and *hi*) act in all cases like single forces applied at the centres of these pins (see Experiment 61, ¶ 159, 1). If the centre of gravity of the cylinder and disc is in the axis of *hi*, we have, as in the simple pendulum, a weight acting as if it were applied at a single point (in *hi*), and made by forces also applied at the same point (in *hi*) to oscillate about another point (in *ab*) as the centre. There is no rotation either of the cylinder or of the disc to complicate the result, as in the case of an ordinary

compound pendulum. Evidently no such rotation can exist, unless the cords (*ah* and *bi*) slip on the pins (*ab* and *hi*). There is, moreover, no tendency to produce such rotation; because forces acting at the centre of gravity of a body (in *hi*) can cause only a linear motion of that centre of gravity. A line in the disc or cylinder which is vertical in one position of the pendulum, remains accordingly vertical in all positions. Here lies an essential distinction between this and other compound pendula.[1]

¶ 151. **Determination of Times of Oscillation by the Method of Coincidences.** — A pendulum between 100 and 101 *cm.* in length, adjusted and measured as in ¶ 150, is placed, let us say, in front of the pendulum of a regulator (Fig. 152, ¶ 149) and set in vibration in an arc not exceeding 10 *cm.* in length (that is, 5 *cm.* on each side of the vertical — see Table 3, *g*). Each swing will occupy a little over a second; hence the first pendulum will fall slowly behind the second. The two pendula will be moving now the same way, now opposite ways. The ticks of the regulator will occur when the first pendulum is now at its furthest right-hand or left-hand point, and now when it is at the middle point of its swing. Every such corres-

[1] The student may notice that the time of oscillation of the stick used in Exp. 56 is considerably greater than that of a simple pendulum (see Table, ¶ 149) equal in length to the distance between the centre of gravity of the stick and its point of suspension. This is owing to the fact that gravity has not only to move the centre of the stick through a certain angle about its point of suspension, but also to turn the stick through the same angle. For a similar reason all ordinary compound pendula are somewhat retarded.

pondence involves a "coincidence" of some sort. The object of this experiment is to find the average interval of time between two coincidences of a given kind. The student will be surprised to find in the reduction of different results (¶ 152) how large an error may be committed in the method of coincidences without introducing any considerable error into the result.

I. OCULAR METHOD. — When the pendula are apparently swinging the same way, the time is to be read by the clock in hours, minutes, and seconds; and again the time is to be noted when the pendula seem to be moving in opposite ways. This should be continued for half an hour or more, according to the length of time that the pendulum may continue to swing perceptibly. The two pendula will probably seem to coincide for a long time in each case. Every effort must be made to determine the middle of such periods of coincidence.

II. EYE AND EAR METHOD (§ 28). — The times may be noted when the ticks of the regulator are heard just as the pendulum under observation reaches its furthest point to the right or to the left; or better, when it reaches the middle point of its swing. In the latter method, the time of coincidence may be generally found within 10 seconds. It may be convenient in some cases to connect an electrical telegraph instrument with a break-circuit in the clock (Fig. 152, *a*) so that the ticks may be re-enforced or reproduced at a distance.

III. OPTICAL METHOD. — Instantaneous views of

the pendulum of the regulator may be obtained through the opening, *k*, in a fixed shield (Fig. 153), and an opening, *j*, in the disk of the pendulum. The regulator should be illuminated so that these views may produce a sufficient impression upon the eye. The times are to be noted when the pendulum of the regulator is seen at the middle point of its swing. Times of coincidence may thus be determined within a few seconds.

IV. ELECTRICAL METHOD. — An electrical current is sent first through the break-circuit of the clock (Fig. 152, ¶ 149), then through the break-circuit *lmno* (Fig. 156) attached to the pendulum (see Pick-

FIG. 155.

FIG. 156.

FIG. 157.

ering, Physical Manipulation I. § 41). The ends of these wires should be amalgamated by dipping them first in nitric acid, then in mercury in order to make good electrical connections. The two hollows, *n* and *o* (Fig. 157), must be filled with mercury and raised by thin wedges so that the mercury may touch the wires (*lm*) in the middle point of the swing (*m*, Fig. 155).

When the swings of the two pendula come into a certain mutual relation, an electrical connection will be made by both break-circuits at the same time, and the sounder will respond. After a certain time this relation will cease, and the sounder will become

silent. The beginning and end of each period of response should be noted, and the middle of the period found by calculation. This method, though more complicated in detail, requires much less effort than the optical method, and is in general equally accurate.

The experiment is to be repeated with a *hollow* cylinder of sheet zinc, instead of the solid zinc cylinder represented in *gj*, Fig. 153; then again repeated with this hollow cylinder filled with sand or lead shot. The weights of the empty cylinder and its contents should be noted.

¶ 152. **Reduction of Results obtained by the Method of Coincidences.** — The reduction of results obtained by the method of coincidences will be best explained by an example. The times of coincidence should be arranged (see § 61) in three columns of about equal length. These columns should contain an odd number of observations, and should be averaged, thus: —

	min.	sec.		min.	sec.		min.	sec.
1st	13	41	6th	24	0	11th	34	32
2d	15	44	7th	26	3	12th	36	34
3d	17	51	8th	28	9	13th	38	39
4th	19	56	9th	30	15	14th	40	46
5th	21	58	10th	32	23	15th	42	49
Average 3d	17	50	8th	28	10	13th	38	40

The first average corresponds in the example to the time of the 3d observation; the second average corresponds similarly to the 8th observation, and the last average corresponds to the 13th observation. For reasons stated in § 51, these averages are probably more accurate than the single observations to

which they correspond. The difference between the first and second averages is 620 seconds; and since between the 3d and 8th observations, to which they correspond, there are 5 intervals, the average for each interval must be 124 seconds. It appears, therefore, that in 124 seconds the first pendulum loses just one swing with respect to the regulator; that is, it makes 123 swings while the regulator makes 124. Assuming that 124 swings of the regulator occupy as many seconds, one swing of the first pendulum must occupy $\frac{1}{123}$ of 124 seconds, or 1.0081 sec. In the same way, between the 8th and 13th observations, we find coincidences on the average 126 seconds apart; hence the average time of one swing is $\frac{1}{125}$ of 126 seconds, or 1.0080 sec. The student should note that the time occupied by one swing (1.0081 sec.) in the first part of the experiment differs very slightly from that (1.0080 sec.) in the last part of the experiment. The difference, due to a decrease in the arc of the pendulum, is in fact only about $\frac{1}{10000}$ of a second (see Table 3, *g*). He should also notice that this small difference in the result corresponds to a comparatively large difference (2 seconds) in the average interval between coincidences. Even with rough methods (¶ 151, I. and II.) such a difference could hardly fail to be observed when sufficiently multiplied by a *long series* of observations. If, conversely, the average interval between coincidences can be found within 2 seconds, the time of oscillation must be accurate within $\frac{1}{10000}$ of a second.

A comparison of results obtained with a solid and

with a hollow cylinder of a given size and shape should show that the resistance of the air (which must exert a relatively greater influence in one case than in the other) is slight. A comparison of results obtained with a hollow pendulum filled with *different materials* should show that the time of oscillation of a pendulum of given length is independent of the nature of the substance of which it is composed.

¶ 153. **Relation between the Length and Time of Oscillation of a Pendulum and the Acceleration of Gravity.** — We have already seen (¶ 149) that a relation must exist between the length of a pendulum and the distance traversed by a falling body while the pendulum is making one swing. To find the distance which a body falls in 1.0081 sec. we have only to multiply the length of the pendulum, let us say 100.8 *cm.* by a certain number (4.935) already determined. From the distance which a body falls, and from the time occupied, we may calculate the velocity imparted to the body (see § 108); and from the velocity imparted in a given length of time, we can find that imparted in 1 second (§ 108). This is called the acceleration of gravity, and is denoted by g in the formulæ of § 108. To shorten this calculation, which depends solely on the length and time of oscillation of a pendulum, the following table has been computed for simple pendula between 99 and 101 *cm.* in length: —

TIME OF OSCILLATION.

Length of Pendulum.								
99.0	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975	0.9970	0.9965
99.1	1.0006	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975	0.9970
99.2	1.0011	1.0005	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975
99.3	1.0016	1.0010	1.0005	1.0000	0.9995	0.9990	0.9985	0.9980
99.4	1.0021	1.0016	1.0010	1.0005	1.0000	0.9995	0.9990	0.9985
99.5	1.0026	1.0021	1.0015	1.0010	1.0005	1.0000	0.9995	0.9990
99.6	1.0031	1.0026	1.0020	1.0015	1.0010	1.0005	1.0000	0.9995
99.7	1.0036	1.0031	1.0026	1.0020	1.0015	1.0010	1.0005	1.0000
99.8	1.0041	1.0036	1.0031	1.0025	1.0020	1.0015	1.0010	1.0005
99.9	1.0046	1.0041	1.0036	1.0030	1.0025	1.0020	1.0015	1.0010
100.0	1.0051	1.0046	1.0041	1.0035	1.0030	1.0025	1.0020	1.0015
100.1	1.0056	1.0051	1.0046	1.0040	1.0035	1.0030	1.0025	1.0020
100.2	1.0061	1.0056	1.0051	1.0045	1.0040	1.0035	1.0030	1.0025
100.3	1.0066	1.0061	1.0056	1.0050	1.0045	1.0040	1.0035	1.0030
100.4	1.0071	1.0066	1.0061	1.0056	1.0050	1.0045	1.0040	1.0035
100.5	1.0076	1.0071	1.0066	1.0061	1.0055	1.0050	1.0045	1.0040
100.6	1.0081	1.0076	1.0071	1.0066	1.0060	1.0055	1.0050	1.0045
100.7	1.0086	1.0081	1.0076	1.0071	1.0065	1.0060	1.0055	1.0050
100.8	1.0091	1.0086	1.0081	1.0076	1.0070	1.0065	1.0060	1.0055
100.9	1.0096	1.0091	1.0086	1.0081	1.0075	1.0070	1.0065	1.0060
101.0	1.0101	1.0096	1.0091	1.0086	1.0080	1.0075	1.0070	1.0065
$g =$	977	978	979	980	981	982	983	984

The length of the pendulum is to be found in the left-hand column; then in line with it the number nearest the time of oscillation is to be selected. Beneath this number, at the bottom of the column will be found the value of g.

EXAMPLE I. Given the length, 100.8 *cm*, and the time, 100.81 sec., required g. We find the time of oscillation, 1.0081, in the 4th column in line with 100.8 in the left-hand column and at the bottom of the 4th column we find the number 979, which represents the acceleration of gravity in question.

EXAMPLE II. Given the length, 100.84, and the time, 100.81, required g. We notice that the times increase by the amount .0005 when the length increases by 0.1 *cm.*; hence 0.04 *cm.* corresponds to .0002 sec.

If, therefore, the length had been 100.8 instead of 1.0084 the time would have been 1.0079 instead of 1.0081. Now 1.0079 comes between two numbers opposite 1.008, namely 1.0081 and 1.0076. Under the first we find 979, under the second we find 980. Since 1.0079 differs from 1.0081 by .0002 sec., and a difference of .0005 sec. makes a difference of 1 unit in g, we must add $.0002 \div .0005$ or $\frac{2}{5}$ of a unit to 979 to find the value of g. We have, therefore, $g =$ 979.4.

The object of this calculation is not so much to determine the value of g, which is already known with sufficient accuracy for all latitudes (see Table 47), and is believed to be the same for all materials, but rather to obtain a check upon the standards and methods hitherto employed for the measurement of length and time.

EXPERIMENT LIX.

INERTIA, I.

¶ 154. **Determinations of Mass by the Method of Oscillations.** — A small glass beaker (d, Fig. 158) is to be suspended from a support, a, by a coiled spring of steel wire, bc, as long and as flexible as may be convenient. A substance whose mass is to be determined is placed in the beaker. The beaker is then pulled downward to a position d', vertically beneath d, then released. It will spring up to a

position d'', nearly as far above d as d' is below it. Then it will return nearly to d', and thus make a considerable number of oscillations before it comes to rest. The oscillations should not displace the load in the beaker; if they do, the load must be rearranged, or the oscillations must be diminished in amplitude. The time of oscillation is now to be found as in ¶ 149.

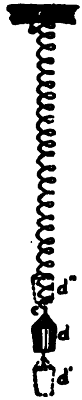

FIG. 158.

The load is next removed from the beaker, and in its stead weights from a set are placed there, sufficient in quantity to stretch the balance to the same point as before. The time of oscillation is again determined. If it is less than before, more weights are added, if greater, weights are removed; and thus by trial (§ 35) the weight is adjusted until the time of oscillation is the same with the weights as with the substance, the mass of which is to be determined.

The student should notice that the time of oscillation is nearly independent of the amplitude of oscillation as in an ordinary gravity pendulum. It should be pointed out, however, that in the vertical oscillation shown in Fig. 158, gravity has nothing to do with the time of oscillation in question, except in so far as it may affect the elasticity of the spring by stretching it to a greater or less extent. When a spring is already loaded the force required to stretch it 1 *cm.* further may be taken as a measure of the stiffness of the spring under the load in question.

The time of oscillation of a load suspended by a

spring depends (1st) on the stiffness of the spring and (2d) on the *mass* to be set in oscillation. When two loads give the same time of oscillation under the same circumstances, their masses are necessarily equal.

Having adopted as our standard of mass a certain piece of platinum in the French Archives (§ 6), we should theoretically use platinum weights in this experiment. It has been found, however, that two quantities which have equal masses, estimated as above by the dynamical method, have also equal weights (*in vacuo*); that is, gravity exerts the same acceleration upon them, without regard to the substances of which they are composed (see Exp. 58.) The use of brass weights will not, therefore, in *practice*, introduce any error.

The results of Exp. 59 are to be expressed in grams like results obtained by an ordinary balance. Strictly, however, the word *mass* should be written before or after these results instead of the word *weight* (§§ 152, 153).

¶ 155. **Relation between Weight and Mass.** — The student must not assume that weight and mass are *necessarily* the same. We do not know *why* a body is attracted by the earth, neither do we know *why*, being attracted, it does not move instantly, under that attraction, from one place to another. The former phenomenon we attribute to *gravity* (§ 150), the latter to *inertia* (§ 151).

By the weight in grams of a body we mean the number of grams of platinum to which the body is equal in respect to weight proper (§ 153), or the

force exerted upon it by gravity. By the mass in grams of a body we mean the number of grams of platinum to which it is equal in respect to *inertia*, or the necessity of force to set it in motion (§ 152).[1] In the absence of any explanation of gravity and inertia, no reason can be assigned why any proportion should exist between them. There is no proportion between electrical or magnetic forces and the masses upon which they act. The existence of such a proportion between mass and weight is simply an inference from the results of experiment (see Exp. 58). It is possible, so far as we know, that a new substance may be discovered, the mass of which may be disproportional to its weight. It is also possible that if masses could be measured with the same accuracy as weights, slight variations might be discovered which have hitherto escaped observation. We have several instances of physical laws which are approximately but not exactly fulfilled; as for instance the law connecting the molecular weights and specific heats of elementary substances (§ 86, note). At the same time that such variations are *possible*, as far as we know, in the case of gravity and inertia, it is by no means *probable* that any such will ever be discovered. It is much more probable that gravity and inertia are both manifestations of a single principle, according to which, for reasons unknown to us, one must be proportional to the other.

[1] See Hall's Elementary Ideas, published by C. W. Sever, Cambridge, Mass.

EXPERIMENT LX.

INERTIA, II.

¶ 156. **Determination of Force by Observations of Mass, Length, and Time.** — A metallic ring about 20 *cm.* in diameter, and weighing about 500 grams (C D F E, Fig. 159) is suspended horizontally by a spring brass wire *AB*, about 0.25 *mm.* in diameter (No. 31, B.W.G.), and at least one metre long. The wire is fastened at the top and held at the bottom by a small vice, *B*. This vice, *B*, is connected by fine iron wires (about No. 31) with four points C, D, E, and F of the ring. A paper millimetre scale is attached to the ring, and the distance through which it revolves is indicated by a fixed marker (*G*).

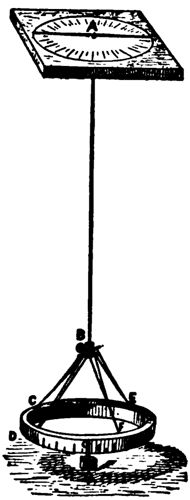

Fig. 159.

The reading of the marker is to be first observed when the ring is at rest. Then the ring is turned through nearly 360°, and released. All pendular vibration must be stopped by touching (if necessary) the wire *AB*. The ring will then have only a rotary movement, due to the "torsion" of the wire. As the ring approaches a turning-point, several readings of the marker are taken at intervals of two seconds. The intervals may be deter-

mined by the ticks of a regulator, or by an electrical sounder connected with the regulator.[1]

When the experiment has been repeated a sufficient number of times, the ring is taken down and its weight in grams determined. The vice, B, should not be weighed with the ring. It is better not to weigh the connecting wires with the ring; but their weight (which should not exceed 1 gram) will not in any case introduce a serious error into the result. The material, length, and diameter of the wire AB should be noted. The observations are then to be reduced as in ¶ 157.

¶ 157. **Calculation of Force from Observations of Mass, Length, and Time.** — The rotation of a ring about its axis presents one of the simplest cases in dynamics. The whole mass of the ring is at (nearly) the same distance from the axis in question, and hence acquires (nearly) the same velocity. To find the force exerted upon the ring in the direction of this velocity, we have to find (1) the velocity acquired, (2) the time required to attain this velocity, and (3) the mass acted upon. The force may then be calculated by the general formula (§ 106): —

$$f = \frac{mv}{t}$$

[1] If greater precision is required than can be obtained by the eye, a small bristle attached to the armature of the sounder can be made to mark the seconds on the edge of the ring, which must be previously smoked for this purpose. By employing two such markers on opposite sides of the ring, slight errors due to swinging of the ring can be eliminated.

In practice we make this calculation as in the example below. The observations are numbered and arranged as follows: —

	mm.	Difference in 2 sec.	Mean Velocity.	Difference in 2 sec.	Acceleration
1	552			—	—
		+33	+16.5		
2	585			8.0	4.0
		+15	+ 7.5		
3	600			10.0	5.0
		− 5	− 2.5		
4	595			7.5	3.8
		−20	−10.0		
5	575			10.0	5.0
		−40	−20.0		
6	535			—	—

The differences in the 3d column show the distance passed over in 2 seconds; hence these are divided by 2 to find the distance passed over in 1 second, or the mean velocity for a period of 2 seconds. The velocity is called positive if the ring is turning away from its position of equilibrium, otherwise negative. The 5th column shows the algebraic differences in these velocities; that is, the change of velocity in 2 seconds. To find the acceleration, or change of velocity in one second, the numbers in the 5th column must be divided by 2. This gives the numbers in the 6th column, the average of which is 4.5, nearly. Since we have used *mm.* throughout, the change of velocity in one second amounts to 4.5 *mm. per sec.*, or 0.45 *cm. per sec.*

This is the acceleration strictly of the outer surface of the ring. Let us suppose that the outside diameter is 20.5 *cm.* and the inside 19.5 *cm.*, so that the mean diameter is 20.0 *cm.*; then the average acceleration will be less than 0.45 in the ratio of 20.0 to 20.5. The average acceleration will be, therefore, about 0.44 *cm. per sec.* If now a mass of 500 *g.* receives this

acceleration, the force exerted upon it must be 500 × .44, or 220 dynes (§ 12). The angle through which the steel wire is twisted is given in circular measure by the ratio of the arc to the radius. Since the latter is 10 *cm.* (nearly), the minimum deflection (53.5 *cm.*) corresponds to 5.35 units of angle. The maximum deflection (60.0 *cm.*) corresponds similarly to 6.00 units of angle. The mean deflection is accordingly not far from 5.7 units of angle. Since one unit of angle in circular measure is equal to 57°.3, nearly, the mean deflection of the ring is about 57°.3 × 5.7, or 327°.

We note, therefore, that a piece of steel wire of given length and diameter, when twisted 327°, exerts at a distance of 10 *cm.* from its axis a force of about 220 dynes.

The use which is to be made of this result will be explained in ¶ 165 in connection with a method by which a force similar to the one in question may be directly balanced by gravitation. A more accurate method of reducing results obtained by the "torsion pendulum" will be given in the Appendix (Part IV).

EXPERIMENT LXI.

COMPOSITION OF FORCES.

¶ 158. **Correction of Spring Balances.** — A spring balance consists of a spiral spring, *cd* (Fig. 160), contained in a hollow metallic case, *bh*, to which it is

fastened at *c*. The spring is connected by a rod, *di*, with a hook, *ij*, from which weights are hung. A slit, *eg*, is made in the case so that a pointer, *f*, attached to the rod, *di*, may indicate the elongation of the spring on a scale outside of the case. In measuring vertical forces with a spring balance, the instrument is generally suspended by the ring, *a*. When forces in other directions are to be determined, the case (*bh*) should also be supported, so as not to bear against the index, *f*. If this precaution is not observed, large errors from friction may be introduced into the results. Spring balances are usually graduated so as to indicate the weight of a body either in kilograms or in pounds. It must be remembered that such indications are affected by the force of gravity. Thus a spring balance, graduated correctly in England, would give, in Brazil, readings too low by about $\frac{1}{3}$ of 1 %. Obviously spring balances, however sensitive, cannot serve everywhere as standards of *mass* (§ 6). The readings depend, not directly upon the masses suspended, but upon the forces which they exert on the instrument. A spring balance once graduated correctly in *megadynes*[1] should, however, give *forces* correctly (in megadynes) irrespective of locality. A

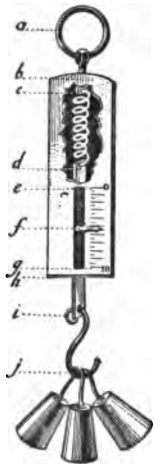

FIG. 160.

[1] The student may be interested to cut a scale of megadynes by the side of the ordinary scale. In latitude 40°–45°, 1 megadyne = 1.02 kilos. = 2¼ lbs. nearly.

spring balance is essentially an instrument for measuring force, and it is only ***in a given latitude*** that it may be employed for estimating weights either in kilograms or in pounds. A pair of 10-kilo. (or 24-lb.) spring balances will be suitable for the experiments which follow.

The reading of a spring balance may be corrected by hanging known weights upon it, as in Fig. 160. Weights provided with a ring, a hook, or an eye will be found convenient for this purpose. The reading of the balance should be tested with weights of 1, 2, 3, etc., up to 10 kilos. (or 2, 4, 6, up to 24 lbs.). The zero-reading of the spring balance should also be found, both in a vertical and in a horizontal position. The weights used may be compared by an ordinary balance with standards if it is thought necessary. From these results we are to calculate the corrections to be added to the reading of the spring balance under different loads, in order to find the true load. Thus if the indication with a 4 lb. weight is 3 lbs. 14 oz., the correction is +2 oz. The results should be arranged in tabular form, either in kilos. or in pounds, as follows: —

FIRST TABLE OF CORRECTIONS.

(1) Load in kilos.	Correction in kilos.	(2) Load in lbs.	Correction in oz.
0	−0.10	0	−3
1	−0.05	2	−1
2	+0.08	4	+2
3	+0.25	6	+6
. . .	. . .	. . .	. . .
10	+0.05	24	+1

One of the weights is now to be attached to the spring balance by a light but strong cord (*ac*, Fig.

161) passing over a pulley (b) made to run as freely as possible. The readings of the balance are to be carefully compared in different positions (a', a'', etc.). To eliminate the effects of the friction of the pulley, the readings are to be made in each case (1) when the weight is being slowly raised, and (2) when it is being slowly lowered. If the two readings differ perceptibly, the mean is to be taken.

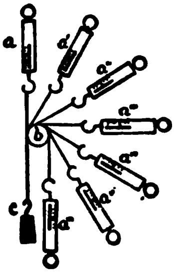

Fig. 161.

The object of testing a spring balance in different positions is to eliminate the effects due to the weight of the hook and spring.[1] From the results we are to calculate the corrections to be added to the readings under different inclinations in order to find the reading in the vertical position. Thus if a 2 lb. weight weighs apparently 2 lbs. 1 oz. in the vertical position, and 1 lb. 11 oz. in the horizontal position, the correction for an inclination of 90° is +6 oz. These corrections should be the same for all weights, and should be entered in a second table, as follows: —

SECOND TABLE OF CORRECTIONS.

(1) Inclination of Balance.	Correction in kilos.	(2) Inclination of Balance.	Correction in oz.
30°	+0.02	30°	1
60°	0 08	60°	8
90°	0.16	90°	6
120°	0 24	120°	9
150°	0.30	150°	11
180°	0.32	180°	12

[1] This method was suggested to the author by a similar one employed by Mr. Forbes of the Roxbury Latin School. See also Elementary Physical Experiments, published by Harvard University, page 11, footnote.

¶ 159. **Determinations of Weight by the Composition of Forces.** — It is frequently inconvenient to measure the weight of a body directly, either by ordinary scales, or by a single spring balance, as when the weight of the body exceeds the capacity of such instruments, or when the body forms an inseparable part of a combination. In such cases, we may sometimes make use of principles involved in the composition and resolution of forces.

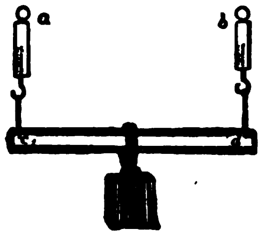

FIG. 162.

(1) To find the force of gravity on a "28-lb." weight with two spring balances, each of 10 kilograms' capacity, we hang the weight (*e*, Fig. 162) at the middle of a stick (*cd*) so that it may bear about equally upon the spring balances (*a* and *b*) while hanging in a vertical position. The reading of each balance is to be noted; then the weight is to be removed, and the readings again taken with the stick alone. The difference between the two readings of a given balance, with and without the weight, corrected if necessary by Table I., ¶ 158, gives the part of the load borne by that balance. The sum of the two parts is of course equal to the whole load.

(2) To find the force of gravity on a "56-lb." weight with a single spring balance of 10 kilograms' capacity, we suspend a lever (*cd*, Fig. 163) as before, except that a cord, *bd*, takes the place of the spring balance (*b*, Fig. 162). The weight is then hung at a

point, *e*, let us say one-fourth the distance from *d* to *c*, and the reading of the spring balance is observed. Care must be taken that the cords *fg* and *hi*, by which the weight is suspended, swing free of the side of the lever as in the cross-section (Fig. 163). A similar precaution should be observed in respect to the cords by which the spring balance, *a*, is attached to the lever at *c*.

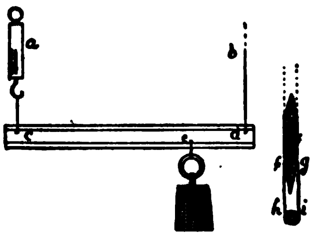

FIG. 163.

The cords should both be vertical. The horizontal distances *cd* and *ed* are to be accurately measured. The weight is now to be removed, and the reading of the spring balance again noted. If F and f are the forces indicated by the spring balance with and without the weight, both being corrected by the first table of ¶ 158, the force (w) exerted by the weight at c is evidently equal to $F - f$. If we call the whole weight W, then since the *couple* (§ 113) produced by W (equal to $W \times de$) is balanced by the couple produced by the spring balance (equal to $w \times cd$), allowing for the weight of the lever, it follows that —

$$W = (F - f) \times cd \div ed.$$

(3) Another method of suspension is represented in Fig. 164. It is assumed that the weight will be able to lift the lever, so that the balance must be applied from under the lever. The reading of the

balance in this position must be corrected both by the first and by the second table of ¶ 158. Thus since the inclination of the balance is 180° (compare Figs. 164 and 161), we must add 0.32 kilos according to the second table (¶ 158), *besides* the ordinary correction for the observed reading from the first table (¶ 158). In addition to the force exerted by the spring balance, we have that part of the weight of the lever which is felt at a, helping to balance the 56-lb. weight. To allow for the weight of the lever, we remove the 56-lb. weight, and apply the spring balance as in Fig. 163, so as to sustain the lever at a. The reading of the balance in this position needs to be corrected simply by the first table (¶ 185), and gives the force (f) exerted by the lever at a. This is to be *added* accordingly to the force (F) exerted by the spring balance with the weight (e) to find the total force which balances this weight. Calling this force w, and the load W, we have $w \times ab = W \times bc$, or —

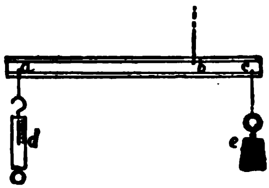

FIG. 164.

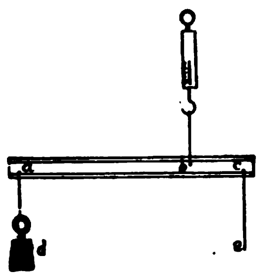

FIG. 165.

$$W = (F + f) \times ab \div bc.$$

(4) To test a 4-lb. weight with a 10-kilogram spring balance, we fasten one end of a lever (c, Fig. 165) to

the ground by means of a vertical cord, *ce*, and suspend the lever from a spring balance by a cord *b*, not far from *c*. The force, *f*, indicated by the balance is to be observed. The weight, *d*, is then hung from the free end of the lever, and the force (F) indicated is again observed. Allowing as before for the weight of the lever we find the force ($F - f = w$) exerted by the spring which balances the load W at *d*. Then since $W \times ac = w \times bc$, we have $W = (F - f) \times bc \div ac$.

If the distance *bc* is one fourth of *ac*, every ounce at *a* will produce an effect at *b* equal to 4 oz. We might therefore weigh a small object to ounces with a balance graduated only to 4 oz. (or $\frac{1}{4}$ lb.).

(5) Another method of weighing small objects is to hang two spring balances, *A* and *B* (Fig. 166), from

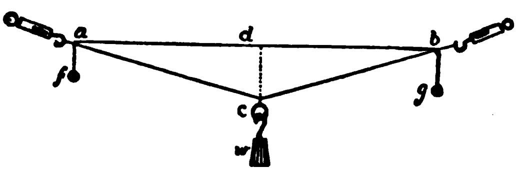

FIG. 166.

nails in the wall, 2 or 3 metres apart, then to connect them by a cord *acb*. At the middle of the cord (*c*) a ring (*C*) is hung so that the weight, *W*, may be readily attached. Two pins are driven into the wall opposite points *a* and *b*, on the cords at equal distances (let us say just 1 metre) from *c*. A cord, *ab*, is stretched between them by means of two small weights, *f* and *g*. The perpendicular distance, *cd*, between *c* and *ab* is then measured.

The vertical component of the force A registered by the spring balance near a, is by the triangle of forces (§ 105) equal to $A \times cd \div ac$. The vertical component of the force, B, due to the spring balance near b, is similarly $B \times cd \div bd$. The total sum of these components must balance the combined weight of the ring (C) and of the load (W). That is,

$$W + C = A \times cd \div ac + B \times cd \div bc.$$

To eliminate the weight of the ring, the load (W)

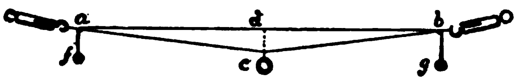

FIG. 167.

is removed, and the experiment is repeated with the ring alone, as in Fig. 167. We have, similarly,

$$C = A \times cd \div ac + B \times cd \div bc.$$

Hence subtracting the last value (C) from the first ($W + C$) we find the weight of the load (W) in question.

We will assume, for simplicity, that a and b are on the same level. A slight difference in level will, however, have no appreciable effect upon the result. The sagging of the cord ab will probably be very small, and will be eliminated in the method of difference by which the result is calculated.

The same method may be employed for the measurement of large weights. If the angle acb is small (see Fig. 168), it will be more accurate to calculate cd from a measurement of ab, than to measure cd di-

rectly. Let us suppose that the cords bB and aA have been lengthened or shortened so that the line ab is horizontal. The vertical line cd will then be at right angles with ab; and since $ac = bc$, $ad = bd = \frac{1}{2}ab$. Knowing ad, we may calculate cd by the Pythagorean proposition —

$$cd = \sqrt{(ac)^2 - (ad)^2},$$

and hence find the load C or W as before.

This method would be adopted in practice if for any reason it were inconvenient to obtain a point of

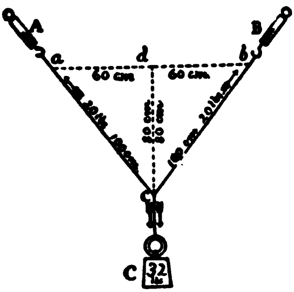

FIG. 168.

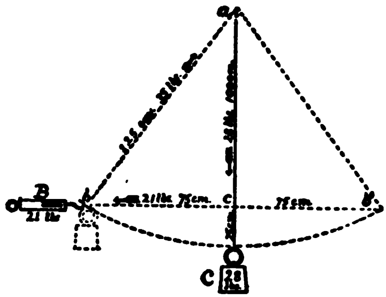

FIG. 169.

suspension directly above the weight. We should prefer, however, to employ a lever long enough to reach, as in (1) or (2), between two available points of suspension, A and B, if it were possible to obtain one of suitable weight and strength.

(6) To measure a weight (C, Fig. 169) when suspended by a cord (ac) we may pull it one side by a spring balance applied horizontally in the direction cb. The reading of the balance (corrected by *both* tables of ¶ 158) gives the force B acting in the di-

rection cb. This with the force of the cord acting in the direction ba produces a resultant which balances the weight of the body C. The direction in which the weight C acts must be parallel to that of the cord ac before the weight was disturbed. Since three forces in equilibrium are proportional (§ 105) to the sides of a triangle to which they are respectively parallel, we have $B : C = bc : ac$, or

$$C = B \times bc \div ac.$$

Instead of measuring bc directly, we may pull the cord ac first one side to a point b, then in the opposite direction to a point b' at a (nearly) equal distance from c. These points may be marked by pins, b and b' driven into the wall or into some other support behind the cord. The distance between b and b' is then measured and divided by 2 to find the distance bc. The point c may be found by a thread stretched between the pins b and b'. In this case the distance ac may be directly measured. Or the distance ab may be found and ac calculated (since ab is known) by the Pythagorean proposition,

$$ac = \sqrt{(ab)^2 - (bc)^2}.$$

By the use of very small deflections, we may measure weights many times exceeding the capacity of the spring balances which we employ.

EXPERIMENT LXII.

CENTRE OF GRAVITY.

¶ 160. **Location of the Centre of Gravity.** — A flat board,[1] *bcde* (Fig. 170), is suspended by a thread *abb'a'* (Fig. 170, 1) passing through a fine hole *bb'* in the board, and over a peg *aa'*. A plumb line, *af*, is also suspended from the same side of this peg, so as to hang as close to the board as possible. A projection of this line upon the board is to be traced in pencil (Fig. 170, 2). The eye must be held in this process

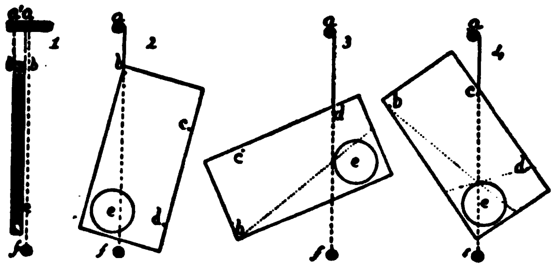

FIG. 170.

so as to look perpendicularly upon the board (§ 25). The board is then to be hung by another point, *d* (Fig. 170, 3), and another line drawn upon it. Then the board is to be suspended from a third point, *c* (Fig. 170, 4), and a third line traced. All three lines

[1] To lend interest to this experiment the board may be made of two thicknesses glued together, with a space (*e*, Fig. 170) between them which has been hollowed out and filled with lead. An irregularly shaped board may also be employed.

should intersect at a point in the surface of the board directly in front of the centre of gravity. If they do not, the experiment must be repeated.

¶ 161. **Determination of Weight by Displacement of the Centre of Gravity.** — A weight (w, Fig. 171) is attached at a to one end of a board whose centre of gravity (c) has been located (¶ 160); and the board is balanced upon a triangular piece of wood (d) or upon a pencil. The line of the support (bb' Fig. 172) is then marked upon the board, and two lines, ab and cb' are drawn from a and c perpendicular to bb'. These lines are then carefully measured. If W is

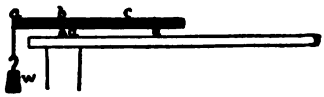

Fig. 171.

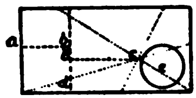

Fig. 172.

the weight of the board, which we may consider as if concentrated at c (§ 112), we have $W \times b'c = w \times ab$; whence $$W = w \times (ab) \div (b'c).$$

The experiment should be repeated with different weights applied at different parts of the board, and with the line bb' not always at the same place or in the same direction. The different values calculated for the weight of the board should be averaged. From their agreement we may infer the truth of the assumption that the weight of a body acts in all cases as if applied at its centre of gravity.

It is obvious that if W and w are both known, we may calculate the distance ($b'c$) by the formula

$$(b'c) = w \times (ab) \div W.$$

To find the distance of the centre of gravity from an axis (bb') on which a body balances, it is only necessary to know the weight of the body (W), the load (w), and its distance (ab) from this axis. For an experiment (due to Prof. Hall) in which this principle is applied, see Ex. 17 of the Elementary Physical Experiments, published by Harvard University.

EXPERIMENT LXIII.

BENDING BEAMS.

¶ 162. **Determination of the Stiffness of a Beam.** — A square steel rod, *ag* (Fig. 173), is mounted on two triangular supports with steel edges, *i* and *j*, 1 metre

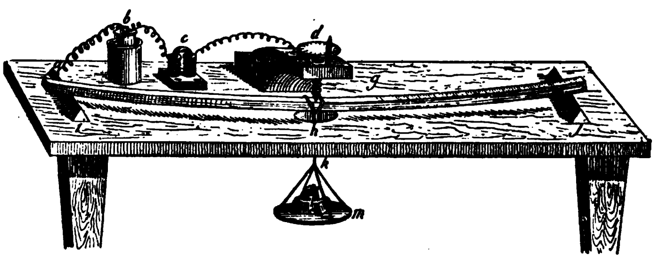

Fig. 173.

apart. A screw with a micrometer head (d) is adjusted so that its point just touches the middle of the beam when a pan, *m*, is suspended from it by the wires *hk*. The micrometer is then read. A load, *l*, is next placed in the pan, and the micrometer is once more adjusted until it touches the beam. The micrometer is again read. Its point is then withdrawn,

so as not to be injured by the recoil of the beam when the weight is removed. A new reading is then taken with the pan (m) empty. If this differs greatly from the first, the beam has probably been permanently bent, and the experiment must be repeated with a smaller load. If the reading is the same as before, a larger load may be tried. With a steel beam 100 *cm.* long and not over 1 *cm.* thick, a deflection of several centimetres should be possible without injury to its power of recovery. To discover exactly when the point of the micrometer touches the beam, we may make use of an electrical contact. One pole of a voltaic cell, b, is to be connected with one end of the beam by a wire soldered to it at a. The other pole is connected with one binding post of an electrical sounder c. The other binding post of this sounder is connected by a wire with the metallic nut e, in which the micrometer turns. The point of the micrometer and the surface of the beam beneath it are scraped bright with a file (or better, coated with platinum). When the point of the micrometer touches the beam, the electrical circuit $bceab$ is thus completed, and the armature of the sounder is attracted. A motion of one thousandth of a millimetre is sufficient, under favorable circumstances, to make or break the contact.

Care must be taken to prevent the beam from twisting or rocking under the influence of a load. The load should not bear more heavily on one side of the beam than on the other. Both sides should be supported alike at each end of the beam by the

sharp edges *i* and *j*. Various deflections under different loads are now to be determined. Each deflection requires two readings of the micrometer, one with, the other without the load. The distance between the supports *i* and *j* should be measured with a metre rod, and the breadth and thickness of the beams employed should be determined at different points with a micrometer gauge (¶ 50, II.).

(1) The deflection of a beam, let us say 1 *cm.* square, is first to be determined with the supports (*i* and *j*, Fig. 173) exactly 100 *cm.* apart, and with a load causing the greatest deflection which can be employed without permanently bending the beam, or exceeding the reach of the micrometer.

(2) The deflection due to one half this load is next to be found. The student should notice that this deflection is almost exactly half as great as before (see § 115). If it is not, the measurements in (1) and (2) should be repeated. The same should be done if the zero-reading of the micrometer is changed.

(3) To test the stiffness of the middle portion of the beam, the supports *i* and *j* are to be placed 50 *cm.* apart, — that is, with half the original distance between them. The rod is to be mounted upon them as before, but with 25 *cm.* or more at either end projecting beyond the supports. The beam is to be loaded with 4 times the weight used in (1) or 8 times that used in (2). If the beam is equally stiff in all parts, the deflection should now be the same as in (2). (See § 115.)

(4) The experiment is next to be repeated with the supports 100 *cm.* apart, with a beam twice as broad as the one first employed, but having the same thickness and bearing the same load as in (1). If the material of the beam is the same as in (1), the deflection due to a given weight should be the same as in (2), since the breadth and weight have the same relative proportion as in (2).

(5) The beam is now to be turned edgewise, and loaded as in (3). The deflection is to be determined as before. If the depth of the beam is just twice as great as in (2), and the width the same, since the force employed is eight times as great as in (2), the deflection should be the same as in (2).

¶ 163. **Calculations relating to Flexure.** — By five measurements arranged as above, we are able to test (in a single instance in each case) the application of the laws of flexure stated in § 115. These laws may be combined in a single formula. If l is the length of a beam, b its breadth, t its thickness, and d the deflection produced (all in *cm.*) by the force f (in dynes) exerted by the load; and if F is the force necessary to produce a unit deflection in a beam of unit length, breadth, and depth (supposing such a deflection to be possible), we have —

$$F = \frac{fl^3}{bdt^3}.$$

The quantity F is sometimes called the modulus of transverse elasticity. Knowing this modulus, we may evidently compute any one of the five

quantities, f, l, b, d, or t, if the other four are known. The student should calculate the value of F from at least one set of measurements. He should also find, by the rule of simple proportion, what force would be required to produce a deflection of 1 *cm.* in the case of each beam which he has employed. Thus if, with a given beam, 1 kilogram produces a deflection of 2 *cm.*, 500 grams would be the force required to produce a deflection of 1 *cm.*

The force (500 grams in this case) producing a unit deflection may be taken as a measure of the *stiffness*[1] of the beam in question. The stiffness of a beam is due to the fact that in order to bend it, the under part must be stretched and the upper part squeezed or compressed. The forces brought into play by stretching will be measured directly in Experiment 65.

EXPERIMENT LXIV.

TWISTING RODS.

¶ 164. **Effect of Couples.** — An instrument serving both to measure and to illustrate the effect of different "couples" (§ 113) is shown in Fig. 174. It con-

[1] Stiffness must not be confounded with breaking strength. A thin beam, though more easily broken than a thick one, is not so in proportion to its flexibility; for by reason of its thinness it can bend much *farther* than a thick beam without breaking. Both the strength and stiffness of a beam are proportional to its breadth; but the former depends upon the square of the ratio which the thickness bears to the length, while the stiffness depends upon the cube of this ratio. (See formula above.)

sists of a rod of ash (*ej*) 1 *cm.* square, driven into a square hole in a block (*j*) which is fastened to the floor. The rod passes through a large hole in a table to a circular disc of wood (*cg*) 20 *cm.* in diameter, at the centre of which is a square hole (*e*), into which the upper end of the rod is tightly fitted. Two markers, *b* and *g*, measure the rotation of the disc by means of a scale of degrees graduated on the edge of the disc. At certain points of the disc (*abc defgh*, Fig. 175), small screw-eyes are placed so that forces may be applied by cords attached to spring

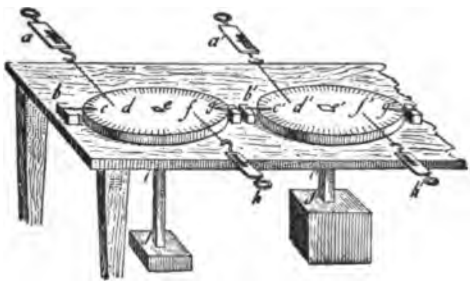

FIG. 174.

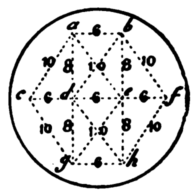

FIG. 175.

balances (*a* and *h*, Fig. 174). It is convenient that four or more of the points (*cdef*, Fig. 175) should be in the same straight line and at equal distances, let us say 6 *cm.* The points *a*, *b*, *g*, and *h* (Fig. 175) may be placed so that *ad*, *dg*, *be*, and *eh* are at right angles to *cf*, and each 8 *cm.* long. This will make the diagonal distances *ac*, *bd*, etc., each 10 *cm.*

A very slight force applied at any point of the disc will cause the rod *cj* (Fig. 174) to bend so as to touch one side of the hole in the table. To keep

the rod in the middle of this hole throughout this experiment,[1] equal and opposite forces must be applied to the disc. If these forces are applied at the same point, no effect will be observed. For instance, two equal forces applied at *d* (Fig. 175) in the directions *dc* and *de* (or in the directions *da* and *dg*) will neutralize each other. Again, if the forces and their points of application are all in the same straight line, the effect will be zero. Thus a force applied at *d* in the direction *dc* will offset an equal force applied at *e* in the direction *ef*. When, however, the lines in which the two forces act are parallel but not coincident, the couple which results (§ 113) will twist the rod. The angle through which the rod is twisted should be proportional to the magnitude of the couple acting upon the disc. The magnitude of the couple is equal (see § 113) to the product of either of the two forces which constitute it, and the "arm" or perpendicular distance between the lines in which the forces act.

The student should satisfy himself that it makes no difference where the "arm" is situated. Thus two opposite forces of 1 kilogram each applied at *a* and *b* or at *c* and *d*, at right angles to *cf*, will have the same effect as if applied in the same manner at *d* and *e*, respectively. The student will notice, moreover, that the rod is twisted but *never bent* by a pair of equal and opposite forces, whether these be applied at equal

[1] In trying this experiment, several students should work together. One may hold and read one of the spring balances, another the other spring balance, while a third observes the deflection of the disc.

or unequal distances from the *centre* of the disc. He should also satisfy himself that with a given arm (as for instance *de*), the rod is twisted through an angle which is proportional to the forces employed (let us say 1, 2, or 3 kilograms); and that the twists produced by given forces (*e. g.*, 1 kilogram each) are proportional to the arms to which they are applied. Arms of the following lengths may be most conveniently employed: 6 *cm.* (*ab*, *cd*, *de*, *ef*, or *gh*); 8 *cm.* (*ad*, *be*, *dg*, or *eh*); 10 *cm.* (*ac*, *ae*, *bd*, *bf*, *gc*, *ge*, *hd*, or *hf*); 12 *cm.* (*ce* or *df*); 16 *cm.* (*ag* or *bh*); and 18 *cm.* (*cf*). Two *equal* forces must be applied in all cases in directions at right-angles to the arms, parallel to the disc, and opposite to each other. They should be made to twist the rod sometimes to the right and sometimes to the left.

To measure accurately the angles through which the disc rotates, both markers (*b* and *g*, Fig. 174) must be observed. It is easy to calculate from a given case by simple proportion what couple would be required to twist the rod through 1°. This gives us a measure of the stiffness of the rod under torsion which may be called its coefficient of torsion.[1]

We next employ a rod, *e′j′*, of half the length of *ej* (Fig 174). This rod must be mounted on a block (*j′*) much higher than *j*. We shall find, if the material and the cross-section are the same, twice the coefficient of torsion. If we use a rod of same length, having, however, twice the diameter, we shall

[1] The coefficient of torsion must not be confounded with the strength of a rod to resist *fracture* by torsion. See note ¶ 163.

find a coefficient of torsion 16 times as great as before (see Laws of Torsion, § 116). It is therefore important to measure and note the length and diameter of the rods employed.

We shall apply the principles illustrated in this section to the determination of the coefficient of torsion of a wire.

¶ 165. **Determination of the Coefficient of Torsion of a Wire by means of a Torsion Balance.** — A hard drawn brass wire about 2 metres long and 0.25 *mm.* diameter (about No. 31, B.W.G.) is stretched horizontally between a knitting-needle (*bd*, Fig. 176) and a fixed support (*k*). The joints should be soldered both at *c* and at *k*, or made equally firm in any other manner.

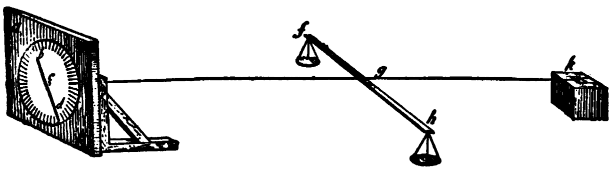

Fig. 176.

The knitting-needle is held in place by a paper protractor fixed on the surface of a board (*ae*). The board and protractor are pierced at the centre (*c*) so that the wire may pass through. A thin strip (*fh*) of some light wood, 20 *cm.* long, is attached at its central point, *g*, to the middle of the wire by sealing-wax. From the ends of this strip two paper scale-pans are suspended by threads. The "torsion" balance thus constructed should not weigh more than one or two grams.

The knitting-needle is first set so that the beam (*fh*) is horizontal. To do this, the beam must be sighted with reference to the bars of a window, or other horizontal line in the room. The reading of the needle is then found by observing both ends. This is the zero-reading of the instrument. Then a decigram is placed in one of the scale-pans, and the needle is turned until the beam is again horizontal. The decigram is then removed from the scale-pan, and the zero-reading re-determined. If any marked change has occurred, the experiment must be repeated. If the zero-reading is again disturbed, a weight smaller than 1 decigram should be employed.

The weight is to be placed first in one scale-pan, then in the other. In each case we note the angle through which the needle must be turned to the right or to the left from its zero position in order that the beam may be made horizontal. It is well to observe the zero-reading after the experiment, since the constancy of this reading is the only safeguard against slipping of the joints or permanent straining of the wires.

Since the balance beam is 20 *cm.* long, the average length of each arm must be 10 *cm.* Since the weight of 1 gram is about 980 dynes, that of 1 decigram will be about 98 dynes; hence the couple exerted by gravity is 98×10 or 980 units. This is balanced by twisting a certain portion of the wire (*cg*) through an observed number of degrees; hence the couple due to 1° is easily calculated. This couple measures a coefficient of torsion of the wire (see ¶ 164), which will be needed in experiments later on.

We notice that the portion of the wire between *g* and *k* is not twisted at the times of making our readings, because the beam *fh* remains horizontal. The torsion of this part of the wire does not, therefore, affect the result. The only use of the wire between *g* and *k* is to keep the balance in place. The length of the wire between *c* and *g* should be measured, and its diameter should be found in several places by means of a micrometer gauge (¶ 50, II.). The material should also be noted, in order that we may utilize our results in certain other experiments later on.

EXPERIMENT LXV.

STRETCHING WIRES.

¶ 166. **Young's Modulus of Elasticity.** — A fine steel wire, about 0.25 *mm.* in diameter (No. 31,

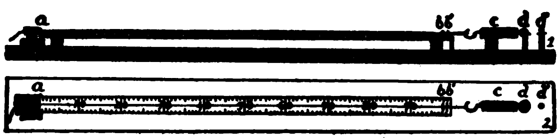

Fig. 177.

B. W. G.) and 1 metre long, may, if made of the best steel, be stretched 1 *cm.* without breaking, or losing its power of recovery. We will suppose such a wire to be held at one end by a small vice (*a*, Fig. 177) and attached at the other end (*b*) to a spring balance (*c*) held in place by a nail (*d*). Let the read-

ing of this balance be 0. Now let the wire *ab* be stretched to a point b', by placing the balance over a nail (d), and let the new reading of the balance[1] be F. Then if the length of the wire thus stretched is *ab* centimetres and the elongation is bb' *cm.*, the stretching of 1 *cm.* will be $bb' \div ab$. This is called the *strain* of the wire. When 100 *cm.* are stretched, for instance, 1 *cm.*, we have a strain of 1 per cent or $+ .01$.

Now if the diameter of the wire is measured by a micrometer gauge, and divided by 2, we have its radius, r. From this we can find the cross-section q by the ordinary formula ($q = \pi\, r^2$), or

$$q = 3.1416 \times r^2, \text{ nearly.} \qquad \text{I.}$$

The cross-section can also be determined by finding the weight, w, of a given length (l) of the wire, if its density (d) is known; for since the volume of a wire is equal to $q \times l$, we have by definition (§ 154) $d = w \div ql$, whence —

$$q = \frac{w}{ld} \qquad \text{II.}$$

We will suppose that by either of these formulæ the average cross-section of the wire *ab* has been found. Now let the force indicated by the spring balance be *reduced to dynes* by multiplying by the appropriate factor.[2] Let us call this force in dynes f.

[1] In practice a small force will be required to straighten the wire. In this case the force F, below, must be taken as the difference between the forces exerted by the balance at d and d'.

[2] Thus in latitude 50° 1 kilogram is equal to about 981,000 dynes, 1 lb. avoirdupois to 445,000 dynes, and 1 oz. to 27,800 dynes, nearly.

To find the intensity of the force per square centimetre of cross-section of the wire, we divide it by the cross-section in question. Thus if the wire had a cross-section of one 2,000th of a square centimetre (.0005 cm^2), a force of 5,000,000 dynes would represent an intensity of 10,000,000,000 dynes per square centimetre (since 5,000,000 ÷ 0005 = 10,000,000,000). The result is called the "*stress*" exerted upon the wire (§ 22).

It has been stated (§ 114) that for a given material there is always a certain proportion between the *stress* exerted upon it and the *strain* produced. The ratio of the stress to the strain in the stretching of a rod or wire is called "*Young's Modulus of Elasticity.*" If, for example, a stress of 10,000,000,000 dynes per square centimetre produces in a steel wire an elongation of one half of one per cent, that is, a strain of +.005, the Modulus of Elasticity of the steel is 10,000,000,000 ÷ .005, or 2,000,000,000,000 (two millions of millions) *dynes per square centimetre*. The Modulus of Elasticity has also been defined as the *force* necessary (under Hooke's law, § 114) to produce a unit strain in a rod of unit cross-section; that is, *to double the length of the rod.* Evidently, if 10,000,000,000 dynes are required as above to increase the length of a steel rod, 1 *cm.* square, by one part in 200, it would take 200 times as much force to double its length, provided that it kept on stretching at the same rate; hence we find 2×10^{12} for the modulus of elasticity, as before.

Few substances can be stretched one hundredth

part of their length without breaking. It is only in the case of exceedingly elastic substances, like India rubber, that the conditions suggested by the last definition can be actually attained. In the case of most substances, we can only calculate by the rules of simple proportion what stress *would* double their length, provided that fracture or other changes did not occur.

The student may notice that steel (see Table 9) has the greatest modulus of elasticity of any known substance, because it requires the greatest force to produce a given amount of stretching; or because, in other words, it yields the least. A substance like India rubber, which is in the ordinary sense particularly elastic, has for this very reason a small *modulus* of elasticity.

¶ 167. **Determination of Young's Modulus of Elasticity.** — The data necessary for a determination of Young's Modulus are, as will be seen from ¶ 166, (1) the length, (2) the cross-section of the wire to be tested, (3) the elongation produced in it by a given force, and (4) the magnitude of this force. The length of a wire may be measured, without any special difficulty, by a tape graduated in millimetres. The cross-section requires much greater care, whether it be determined (as suggested in ¶ 166) by measurements taken with a micrometer gauge at different points, or by its length, weight, and density. The principal difficulty consists, however, in measuring accurately the elongation of the wire, which is usually a very small quantity. To

make the elongation as large as possible, long wires are usually employed.

One of the chief sources of error in measuring the elongation of a wire under a given load is due to the yielding of the support to which the wire is attached. Various devices have been suggested by which this effect may be eliminated. The simplest is to measure the distance between two points on the wire. This may be easily done, when a double wire is employed, by means of two micrometers, *a* and *b* (Fig. 178, 1), attached to the wall, and adjusted so as to touch two cross-bars borne by the wires in question.[1]

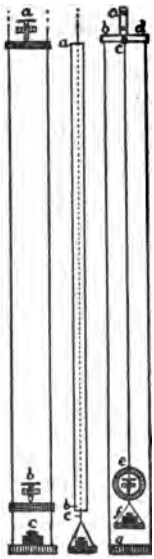

Fig. 178.

To avoid the inconvenience of making observations at a considerable height above the floor, a wire is sometimes surrounded by a tube (*ab*, Fig. 178, 2) attached to it at a point *a*. If the point *a* yields, a point *b* at the base of the tube will yield by an equal amount. The height of this point (*b*) and of a point (*c*) on the wire may be observed (¶ 262) accurately by a cathetometer. The increase of distance between *b* and *c* is evidently equal to the elongation of *ac*. In the Physical Laboratory of Harvard University the effects due to the yielding of the support are avoided by keep-

[1] This device is due to Mr. Forbes, of the Roxbury Latin School.

ing the same weight always upon it. The wires (which are nearly 6 metres long) are attached to a beam by means of a piece of iron (*abd*, Fig. 178, 3) shaped like an inverted T. At the middle of the T a split plug (*c*) driven upwards into a vertical hole firmly grasps the wire. Side wires from the arms of the T hold a small platform (*g*) just above the

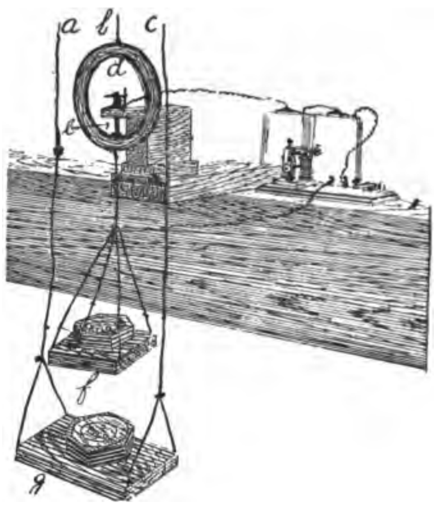

FIG. 179.

floor. The weights to be used in stretching this wire are kept on this platform when not in use. Obviously the beam and the stem of the T are subjected to the same strain whether the load be suspended from the central wire or by the side wires.

A stout ring (*de*, Fig. 179) is attached to the central wire (*b*) by a split plug (*d*). The stretching of the wire is measured by a micrometer, the

point of which touches a small level surface on the ring at *e*. The contact is determined by electrical connections, as in ¶ 162. Directly below the point of contact a platform, *f*, is suspended, for the purpose of holding the weights by which the wire is to be stretched. There are many theoretical objections to this form of apparatus, which being of no practical importance have been left out of consideration. It is obviously necessary that the wire should be straight before the stretching forces are applied. For this purpose, a small load is always kept on it. In the apparatus shown in Fig. 179, the weight of the ring (*de*) and platform (*f*) should be sufficient to straighten the wire. In calculating Young's Modulus, we consider only the weight which must be *added to* the load already borne by the wire, in order to produce the observed elongation.

To determine the elongation in question, a reading of the micrometer must be taken with and without the weight. The difference in the readings gives, allowing for the pitch of the screw (see ¶ 52), the distance through which the wire has been stretched by the weight in question.

For a determination of Young's Modulus of Elasticity, a fine steel wire will answer. Care must be taken, however, not to bend the wire sharply over the edge of the vices or split plugs to which it is fastened. If the wire is 0.25 *mm.* in diameter, and free from kinks or bends, it may be made to bear safely a total load of 1, 2, or even 3 kilograms.

If *f* is the force exerted by the weight when re-

duced to dynes (see ¶ 166), e the resulting elongation of the wire in *cm.*, l the length in *cm.* of that portion of the wire in which the elongation takes place, and q its average cross-section in *sq. cm.*, Young's Modulus of Elasticity (E) is found in *C. G. S.* units (§ 8) by the formula

$$E = \frac{fl}{qe},$$

or by the method of reduction explained in ¶ 166.

EXPERIMENT LXVI.

BREAKING STRENGTH.

¶ 168. **Determination of the Breaking Strength of a Wire.** — A steel or spring-brass wire about $\frac{1}{4}$ *mm.* in diameter (No. 31, B. W. G.), free from kinks or sud-

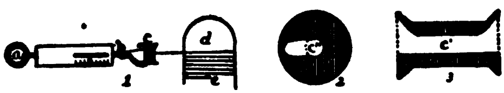

FIG. 180.

den bends, is to be attached at one end to the eye (b, Fig. 180) by which the hook (bc) is attached to the spring balance (abc). The other end is to be fastened to some fixed point, as, for instance, a nail (e) driven into a post (d). A bobbin, c, is to be cut out (as shown in c' and c'' of the cross-sections 2 and 3), so as to fit over the hook of the balance without danger of turning. A few turns of the wire are made about the bobbin; the rest is wound around

a post, *d*. The index of the balance is to be watched as a steadily increasing force is applied to the wire.[1] When the wire breaks, the maximum reading is recorded. The position of the break must now be ascertained. If it occurs at *b*, or between *b* and *c*, the result must be thrown out. If the wire breaks at *c* or at *d*, the accuracy of the result is doubtful; because a sharp bend in a wire where it passes round a corner may cause it to break under forces far less than its average breaking strength. If the break occurs between *c* and *d*, the break is probably a fair one. Enough wire will probably remain about the post for several repetitions of the experiment. The results should agree within five or ten per cent. Suspected results, much smaller than the average, may be discarded.

The cross-section of the wire must be found both by measurements with a micrometer gauge and by weighing a known length of the wire, let us say 1 metre, as accurately as possible. (See ¶ 166, formulæ I. and II.) The density of steel may be taken as 7.9, of brass 8.4 in this reduction. The student should compute by simple proportion the force necessary to break a wire one *sq. cm.* in cross-section; he may also calculate what length of the given wire would break under its own weight. Thus if 100 *cm.* of brass weighs 0.42 grams, its cross-section must be $0.42 \div 100 \div 8.4$, or .0005 *sq. cm.* If it takes 2.94 kilograms to break such a wire, a wire 1 *sq. cm.* in

[1] *The hand should be held in such a position as not to be injured by the hook when the spring recoils.*

cross-section would require 2.94 ÷ .0005 or 5,880 kilograms to break it. At 0.42 grams per metre, it would take 2.94 ÷ 0.42 or 7000 metres of the wire to break under its own weight.

Obviously the result of this calculation should be the same whether a large or a fine wire is used, provided that the quality be the same, because both the breaking strength and the weight of a wire increase in proportion to its cross-section.

EXPERIMENT LXVII.

SURFACE TENSION.

¶ 169. **Determination of the Surface Tension of a Liquid.** — I. A piece of fine iron wire is bent as in Fig. 181, so as to form a fork (*fbg*) with parallel prongs (*cf* and *eg*) about 2 *cm.* apart. The fork is then suspended from the hook of a balance (*a*) so as to dip into a beaker of water, as in the hydrostatic method (Exp. 9). The fork must be entirely covered by water when the balance beam is lowered see (¶ 19); but when the latter is raised, the prongs only must dip into the water.

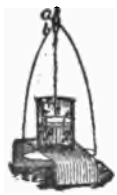

FIG. 181.

The weight of the fork is first balanced as accurately as possible; then the fork is lowered into the water, and suddenly raised out of it. A film of water will probably be found to fill the space between *fcdeg* and the surface of the water. This film will tend to pull the fork back into the water. To balance the

pull which it exerts, an additional weight of about 3 decigrams must be placed in the opposite scale-pan. This weight is to be adjusted, by a number of trials, as accurately as possible. As the film gradually evaporates, it becomes lighter and lighter; but as its weight is, in any case, so small that it may be neglected, the change of weight will probably have no visible effect. The student will notice that the tension of the film of water remains sensibly constant as it grows thinner and thinner, until it breaks. This is entirely unlike the tension of solid substances, which depends upon their cross-section. The tension which liquids exert depends simply upon the *breadth of the surface* which tends to contract, not on the cross-section of the solid contents included by that surface. For this reason, the phenomenon is called "surface tension."

In the case under consideration, the film has two surfaces, each let us say 2 *cm.* broad. The total breadth of surface is therefore 4 *cm.* The student is to calculate what force (in dynes) is exerted by a *single* surface 1 *cm.* broad.

The surface tension of liquids depends upon temperature; hence the temperature should be noted. It is greatly affected by impurities in the liquids. An invisible quantity of oil, for instance, produces variations of ten or twenty per cent. Great care must therefore be employed in obtaining the purest distilled water. Both the inside of the beaker and the lower part of the wire should be cleaned with caustic potash, and afterwards rinsed in several changes of

distilled water. The parts thus cleaned must not afterwards be touched by the finger.

II. A piece of thermometer tubing with a round bore about $\frac{1}{4}$ to $\frac{1}{2}$ *mm.* in diameter is carefully cleaned with caustic potash, which may be sucked through it with a medicine dropper (of course not by the mouth), then cleaned with distilled water. It is now dried by heat and filled with mercury. The contents are to be placed in a beaker, and weighed. If the quantity of mercury is too small to be weighed accurately, ten tubefuls may be weighed together (§ 39). The length of the tube is to be measured. The tube is now placed in a clean beaker containing pure distilled water (see I.). It should be at first inclined somewhat, so that the water which rises into it through "capillary attraction" may thoroughly wet its inside surface. It is next made vertical (see Fig. 182). The height of the column of water in the tube above the level in the beaker is then measured, both when it barely dips into the water, and when it dips so deep that the water rises nearly (but not quite) to the top of the tube. Other measurements should be taken similarly with the tube turned end for end. All results should agree closely, if the tube is of uniform calibre.

FIG. 182.

¶ 170. **Calculations relating to Capillary Attraction.** — If w is the weight in grams of the mercury which fills a tube, 13.6 the density of the mercury, and l

the length of the tube in *cm.*, the cross-section is (see ¶ 166, formula II.)

$$q = \frac{w}{13.6\,l}.$$

The radius of the tube is connected with the cross-section by the formula

$$q = \pi r^2;$$

hence, solving, we find

$$r = \sqrt{\frac{q}{\pi}} = 0.564\sqrt{q}, \text{ nearly.}$$

If h is the average height of the water in the tube above its level in the beaker, 1.00 the density of water, the volume of water raised is qh, or $\pi r^2 h$; the weight in grams is $1.00 \times qh$, or $1.00 \times \pi r^2 h$, and the weight in dynes (allowing g dynes to the gram) is qhg, or $\pi g r^2 h$. This weight is sustained by the tension of a film lining the inside of the tube. The breadth of this film is evidently equal to the circumference of the tube ($2\pi r$). If a film $2\pi r$ centimetres broad can sustain a force $\pi g r^2 h$ dynes, a film 1 *cm.* broad would evidently sustain $\pi g r^2 h \div 2\pi r$, or $\frac{1}{2} grh$ dynes. That is, the "surface tension" of water (S) is given by the formula

$$S = \tfrac{1}{2} grh = 490\, rh \textit{ dynes per centimetre (nearly)}.$$

Obviously, if S is constant, the product, $r \times h$, must be constant; that is, the height to which a liquid will rise in a tube is inversely as the radius of that tube.

EXPERIMENT LXVIII.

COEFFICIENT OF FRICTION.

¶ 171. **Determination of Coefficients of Friction.**— I. A piece of planed plank (*b*, Fig. 183) measuring let us say 5 × 20 × 40 *cm.*, is drawn horizontally by a spring balance, *a*, over a planed board *c*. The force necessary to maintain a uniform velocity after the plank is once started, is observed and noted. Then the plank is suspended from the spring balance and weighed. The ratio of the force required to draw a body to the force required to lift it is called a "co-

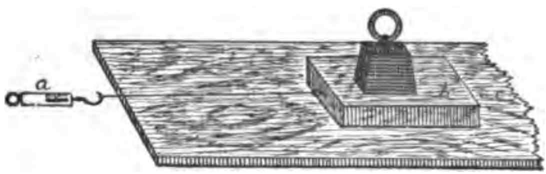

FIG. 183.

efficient of friction." The coefficient of friction in this case is that of wood on wood. If the force of traction varies in different parts of the board, the average should be calculated; and from this the average coefficient of friction may be found. It is instructive to repeat the experiment with the plank edgewise, so as to see whether the diminished area of the surfaces in contact is or is not compensated for by the increased intensity of pressure. For a fair comparison, the side and the edge of the plank

should of course be equally smooth, and both parallel to the grain of the wood.

The experiment may also be repeated with the plank flatwise, but with a heavy weight upon it as in the figure. The value of this weight should be found as in ¶ 159, and added to that of the board, in calculating the coefficient of friction in question.

The student will notice that it takes considerably more force to start a body than to drag it after it is once started. This is attributed to the cohesion of

FIG. 184.

particles which takes place at various points, particularly when two surfaces remain long in contact. The ratio of the force required to start a body when resting upon a horizontal surface to the force required to lift it is sometimes called the "coefficient of starting friction." This must not be confounded with the ordinary "coefficient of friction."

II. The board $A\ C$ (Fig. 184) already used in I. is inclined (by means of a nail, A, and a block D) so that the plank a, when once started, slides down it with uniform velocity. A measuring rod BC is placed at a point B, 1 metre from A, and the verti-

cal distance BC to the under side of the board is then measured. The "slope" of the under surface ($BC \div AC$) is thus found. The slope necessary to maintain a uniform velocity may not be the same from one end of the board to the other. If it is not the same, the average slope should be calculated.

If we resolve the weight of the block ac into two forces, one, ab, perpendicular to the board AC, the other, bc, parallel to it, then by definition (see I.) the coefficient of friction is $bc \div ab$; but, by similar triangles, this is equal to the ratio of BC to AB, which measures the "slope" of the board AC. The average slope which must be given to this board in order that the plank, when once started, may slide down it with uniform velocity, gives accordingly the "coefficient of friction" between the two surfaces in contact. The result should agree closely with that determined as in I.

¶ 172. **Fluid Friction.** — When a well-shaped boat moves through water with a velocity of v *cm. per sec.*, the opposing force (F) which it encounters is approximately equal to the square of this velocity multiplied by the area (a) of the surface wet by the water, measured in *sq. cm.*, and by a certain constant, f (about .003), which is called the *coefficient of friction of water*, that is: —

$$F = fav^2 \text{ dynes.}$$

Coefficients of fluid friction must not be confounded with coefficients of friction in the case of solids, which are calculated in an entirely different

way. The frictional resistance between two solid surfaces depends, as we have seen (¶ 171), upon the pressure between them, but not upon the relative velocity of the surfaces. On the other hand, the resistance offered by a fluid to the motion of a solid does not depend upon the pressure between the surfaces in contact, but does depend upon their relative velocity. The nature of the fluid, the shape and smoothness of the solid, modify the result; but the material of which the solid is composed is generally unimportant. The resistance offered by fluids to the motion of solids or the reverse depends upon disturbances which are wholly confined to the fluid. Every fluid has, therefore, its own coefficient of friction.

When a current of water flows through a *large*[1] tube of the length l and radius r (both in *cm.*), since the area of wetted surface is $2\pi rl$, the force opposing the flow is

$$F = 2\pi rlfv^2 \text{ (dynes).} \tag{1}$$

This force is supplied by the pressure (p) of the water (measured in dynes per *sq. cm.*) exerted upon an area equal to the cross-section (πr^2) of the tube; that is:—

$$F = \pi r^2 p. \tag{2}$$

Equating (1) and (2), we find,—

$$p = \frac{2lfv^2}{r} \quad (3), \quad \text{or } f = \frac{pr}{2lv^2} \tag{4}$$

[1] In capillary tubes, the force encountered is proportional directly to the velocity (see ¶ 250). In tubes from 1 to 5 *mm.* in diameter, for velocities between 10 and 100 *cm. per sec.*, no simple law can be given.

The velocity (v) can be estimated from the cross-section of the tube and from the volume of water which flows through it in a given length of time (¶ 147, 4), the pressure may be found by a pressure-gauge (see Exp. 69) at the point where the water enters the tube, provided that there is a free outlet at the other end, and that both ends of the tube are on the same level. If, as in Fig. 185, one end is higher than the other by an amount ac, equal let us say to h, then if g is the acceleration of gravity and 1.00 the density of water, the hydrostatic pressure is (see § 63)

$$p = 1.00\,gh, \text{ nearly.} \tag{5}$$

The length (l) of the tube may be directly measured. The capacity (c) may be found by measuring, or (as in ¶ 32), by weighing the quantity of water required to fill it. The cross-section (q) may then be calculated by the equation -

$$q = \frac{c}{l} \tag{6}$$

Hence the radius (r) is given by the formula —

$$r = \sqrt{\frac{q}{\pi}} = \sqrt{\frac{c}{\pi l}} \tag{7}$$

The coefficient of friction, f, may now be calculated by formula (4), since all the quantities are known.

The "resistance" of a tube to the flow of a given liquid may be defined as the pressure in *dynes per sq. cm.* required to maintain through that tube a flow

of 1 *cu. cm. per sec.* Thus if a rubber tube (*ab*, Fig. 185) 2 metres long and 3 *mm.* in diameter is used as a siphon to conduct water from a cistern, *a*, to a point *b*, it will be found that the outlet (*b*) must be about 10 *cm.* below the level (*a*) in the cistern in order that water may flow through *ab* at the rate of 1 *cu. cm. per sec.* The hydrostatic pressure corresponding to a difference of level of 10 *cm.* is nearly 10 grams per *sq. cm.*, that is, 9800 *dynes per sq. cm.* The "resistance" is therefore about 9800 units.

The resistance of a conduit may also be defined as

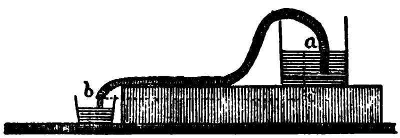

Fig. 185.

the power (in ergs per second) necessary to maintain a unit current (1 *cu. cm. per sec.*) through the conduit in question. This definition bears a strong resemblance to the definition of electrical resistance (§ 136). The fact that power is required to maintain a current through the tubes and valves of a water-motor, together with the friction between the solid parts of the motor, will be found to modify the "efficiency" of the machine. The next experiment relates to determinations of "efficiency."

EXPERIMENT LXIX.

EFFICIENCY.

¶ 173. **Nature of Efficiency.** — Let us suppose that a 20-kilogram weight is suspended by a tackle (Fig. 186) consisting of two double blocks, with four cords passing between them. Let us first suppose that the cords run with absolute freedom round the pulleys which the blocks contain. The force on each cord must evidently be 5 kilograms; and a force of 5 kilograms, applied by a spring balance to the free end of the cord, as in the figure, will just hold the weight in place. If the weight were started upward by any impulse, no matter how small, the force of 5 kilograms constantly applied to the free end of the cord would (in the absence of friction) continue to raise it with a uniform velocity, until the two blocks met together. If the two blocks were 1 metre apart in the beginning, we should have 20 kilograms raised by the tackle through a height of 1 metre. Each of the four cords would be shortened 1 metre in this process, hence there would be 4 metres of slack to be taken up at the free end of the cord. The spring balance must accordingly retreat 4 metres. The work spent upon the machine by a force of 5 kilograms re-

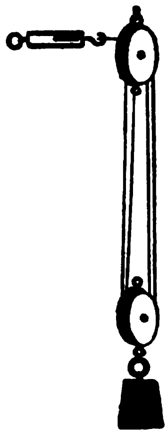

Fig. 186.

treating 4 metres (20 kilogram-metres), would be the same as that utilized by the machine in raising 20 kilograms 1 metre high (see § 14).

Let us now suppose that a slight downward impulse is given to the weight, so that it descends to its original position. The work spent by gravity upon the machine, being 20 kilogram-metres as before, is utilized in pulling the spring balance forward through a distance of 4 metres. In the absence of friction, the pull would be 5 kilograms as before. The amount of work utilized (20 kilogram-metres) would be equal, accordingly, to the amount spent upon the machine.

It is not necessary to consider the magnitude of the impulse by which the weight is started upward or downward; for if the weight moves with uniform velocity, it is capable of giving back this impulse, when it has been raised or lowered to any desired point (see § 121), in the act of stopping, when its energy of motion is lost. In the absence of all friction in the pulley-wheels, stiffness in the cords, and resistance in the air, a tackle devoid of weight would constitute a theoretically perfect machine, — that is, all the work spent upon it would be utilized by it. In practice, a considerable part of the work spent upon a machine is always transformed by friction into heat. That *proportion* of the work spent upon a machine which is utilized by it is called the "efficiency" of the machine.

Let us suppose that, instead of 5 kilograms, a force of 10 kilograms is required to raise a 20 kilogram

weight by means of the tackle represented in Fig. 186. Then since, in raising 20 kilograms 1 metre, 10 kilograms retreat 4 metres, the work spent is 40 kilogram-metres; but the work utilized is only 20 kilogram-metres. The "efficiency" of the tackle as a machine for raising weights is accordingly $\frac{20}{40}$ or 50%.

Again, let us suppose that a weight of 20 kilograms, descending one metre, exerts a force of only 2 kilograms on the spring balance, which advances 4 metres. Then the work spent by gravity is 20 kilogram-metres, but that utilized is only 8 kilogram-metres; hence the efficiency of the tackle as a machine for utilizing potential energy (§ 122) is $\frac{8}{20}$ or 40%.

Finally, let us consider the tackle as a machine for storing and utilizing energy. A force of 10 kilograms is required to raise the weight, and this force must retreat 4 metres to raise the weight 1 metre. 40 kilogram-metres of work are thus spent upon the machine. The free end of the cord is now attached to some resistance which it is desired to overcome. A force of 2 kilograms is thus applied through a distance of 4 metres. The work utilized by the machine is only eight kilogram-metres. Evidently the efficiency of the tackle as a machine for storing and utilizing energy is only $\frac{8}{40}$ or 20%.

When energy is stored in a machine, part of it is lost. When this energy is utilized, part of what is left is lost. When energy undergoes a series of transformations, a certain proportion is lost in each.

Obviously, in stating the efficiency of a machine, it is necessary to specify where or how the work is spent upon it, and where or how the work is utilized.

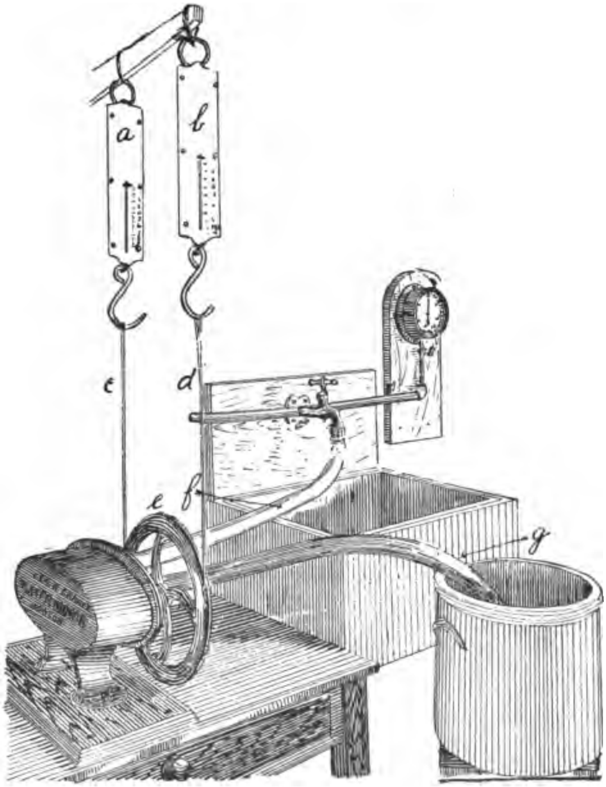

Fig. 187.

¶ 174. **Determination of the Efficiency of a Water-Motor.** — (1) To find the work *utilized by* a water-motor, the circumference of the driving-wheel (*e*, Fig. 187) is first measured, then two spring bal-

ances, *a* and *b*, are connected by a cord (*cd*) passing round the wheel. The motor is then started, and the tension of this cord increased until, through the friction which it exerts upon the wheel, the velocity of the latter is reduced to about one-half of its maximum. The speed of the wheel is then determined by counting the number of revolutions made in a given length of time. The reading of each spring balance is also found. If it varies, several observations must be made, and the mean calculated.

The difference between the two readings is equal to the force opposed by friction to the motion of the rim of the wheel, and must be reduced to dynes or megadynes. If the value of this force in dynes is F, if the number of revolutions in one second is n, and if c is the circumference of the wheel in centimetres, then in traversing the distance cn centimetres against the force F dynes, the work done must be cnF ergs. If we suppose that the force reduced to megadynes is equal to f, then cnf represents the work in megergs. Since cnf megergs of work are performed against friction in 1 second, and might be utilized for turning machinery (see ¶ 175), we infer that the work thus utilized would be cnf megergs per second. This measures, therefore, the power of the machine.

(2) To find the work spent in *driving* the motor, we must measure the quantity of water which passes through it in a given length of time. The water may be collected in a stone jar (*g*, Fig. 187), and weighed on a pair of rough platform-scales (Fig. 188). The

pressure of the water must also be found by means of a pressure-gauge connected with the supply pipe (see Fig. 187). The gauge should be as nearly as possible on a level with the outlet by which water escapes from the motor. The pressure must be reduced to dynes (or megadynes) per square centimetre. If v is the calculated volume in cubic centimetres of the water which flows through the motor in one second, and if P is the pressure of this water in dynes per square centimetre, then the work spent on the motor

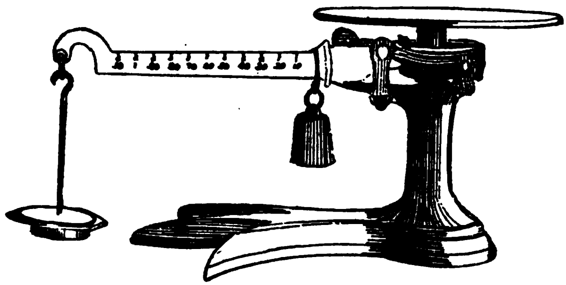

Fig. 188.

is vP ergs per second (see § 118). If p is the value of this pressure when reduced to megadynes per square centimetre,[1] then the work spent on the machine is vp megergs per second.

(3) For the accurate determination of efficiency, it is desirable to make *simultaneous* determinations of the power utilized by the motor, and of the power spent upon the motor. For this purpose, it is well for several students to work together. One may, for in-

[1] The ordinary atmospheric pressure (15 lbs. per sq. in.) is equal very nearly to 1 megadyne per square centimetre. See Table 50.

stance, record the readings of the spring balance, a, another those of b; a third those of the pressure-gauge; a fourth may attend to turning the stream of water into the stone jar at a given time, and cutting it off at a given time; and a fifth may count the number of revolutions made by the wheel of the motor in the interval in question. When the experiment is performed by a single person, the mean readings of the balances and pressure-gauge must be inferred from observations just before and just after the determinations of velocity.

To calculate the efficiency (e) of the motor, the work *utilized* in one second *by the machine* is to be divided by the work *spent* in one second *on the machine.* We have, accordingly, —

$$e = \frac{cnf}{vp}.$$

In repeating the experiment, the tension of the cord should be increased or diminished. The maximum *power* of a water-motor is usually realized when, by the resistance which it has to overcome, the speed of the motor is reduced to about half its maximum speed. To obtain the maximum *efficiency*, the speed of the motor must be still further reduced.

¶ 175. **The Transmission Dynamometer.** — To measure the power of a motor actually doing useful work, a transmission dynamometer must be employed. One of the simplest forms of this instrument is represented in Fig. 189. Instead of carrying two cords (c and d) from the driving-wheel (g) of the motor to two spring

balances (a and b) as in Fig. 187, these cords are made to pass around two pulleys (a and b, Fig. 189) to a second wheel (h), to which the motion is thus transmitted. The pulleys are suspended by two spring balances (A and B). The work done by the motor depends as before upon the difference in tension of the cords c and d; but if the pulleys run freely, the tension of e and f will be the same as that of c and d respectively; hence the forces A and B registered by the spring balances A and B (allowing for the weight of the pulleys) will be $2c$ and $2d$, respectively. It follows that $(c-d) = \frac{1}{2}(A-B)$. The difference between the readings (A and B) must therefore be halved in order to find the difference of tension between the cords[1] c and d.

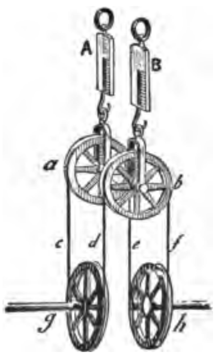

Fig. 189.

When the wheels move so fast that the revolutions cannot be counted, we may find the velocity of the cord, $cdef$, by measuring its length and counting the successive returns of a knot in the cord taking place in a given length of time. In other respects the work utilized is calculated as in ¶ 174, 1.

[1] In practice, if the cord c is approaching g the tension on c will be a little greater than on e; and the tension on d will be a little less than on f, hence the difference of tension between c and d will be greater than the difference between e and f. That is, the work done by g will be a little greater than that received by h. The average between these two quantities is measured by the dynamometer.

EXPERIMENT LXX.

MECHANICAL EQUIVALENTS.

¶ 176. **Different Methods for determining the Mechanical Equivalent of one Unit of Heat.** — (1) If a weight (*d*, Fig. 190) is suspended by a cord passing over a pulley (*a*) and round an axle (*c*), surrounded with water in a calorimeter, and made to descend slowly to a position *d′*, by applying a suitable resistance through a friction-brake, *b*, the work done by gravity in pulling the weight, let us say *w*, through the distance *l* (equal to *dd′*) will nearly all be converted by friction into heat within the calorimeter. Let us suppose that the total thermal capacity of the calorimeter and its contents is *c*, and that its rise in temperature is $t°$; then the quantity of heat developed is *ct*. If gravity exerts a force of *g* dynes on one gram, it will exert *wg* dynes on *w* grams; and a force of *wg* dynes acting through the distance *l*, must perform a quantity of work equal to *wgl* ergs (§ 14). If *wgl* ergs are equivalent to *ct* units of heat (§ 16), one unit of heat must be equivalent to $wgl \div ct$ ergs. To obtain exact results, allowances must be made for the friction of the pulley, *a*, for loss of heat by cooling, etc. By a device similar in principle to the one described above, Joule

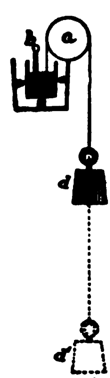

FIG. 190.

found that the mechanical equivalent of one unit of heat is about 41,660,000 ergs.

(2) Two heavy iron bars, A and B, suspended as shown in Fig. 191, may be released simultaneously by burning a cord (see ¶ 148) or by electrical means, so that when the bars meet endwise, a lead bullet (b) may be crushed between them. The work done by gravity in giving velocity to the bars is thus nearly all transformed into heat, through friction of the particles of lead against one another. Most of the heat will accordingly be found in the bullet. If the bullet is immediately lowered into a small calorimeter (c), the quantity of heat may be measured in the ordi-

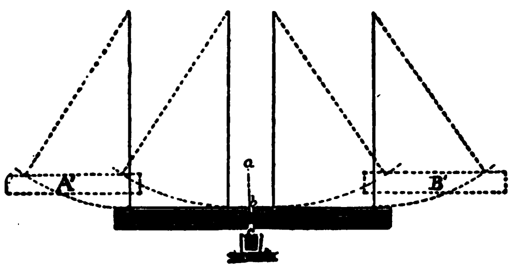

FIG. 191.

nary way (see ¶ 92). To obtain exact results, an allowance must be made for the energy of motion which remains in the bars after impact. If l is the *difference* between the original height of the bars and the height attained by them in their rebound, and w their combined weight, the work done by gravity is, as in (1), wgl. There is no way of allowing accurately for the energy taken up by the bars in the form of vibration, or for the energy of motion directly con-

verted within the bars into heat. It is said that the proportion of energy thus lost is small.[1]

(3) By measuring the temperature of a water-fall above and below the fall, it would be possible to estimate the mechanical equivalent of heat. Thus if the water is 0°.1 warmer at the foot of Niagara Falls than above the falls, where the height is 42.5 metres, we should infer that to cause a difference of 1°, a waterfall must be 425 metres high. Each gram of water falling 425 metres, or 42,500 *cm.* under a force of 980 dynes, nearly, must receive from gravity 980 × 42,500, or nearly 41,660,000 ergs, in the form of energy of motion. If the conversion of this energy into heat warms it 1°, then the mechanical equivalent of 1 unit of heat must be 41,660,000 ergs.

In practice, the difference of temperature between the top and bottom of a water-fall is generally too slight to be measured accurately with ordinary instruments. Unless, moreover, the volume of a waterfall is very great, evaporation and other causes may affect the result. A rough experiment illustrating this method of determining mechanical equivalents will be described in the next section.

¶ 177. **Determination of Specific Heats by Mechanical Equivalents.** — A kilogram of lead shot is placed in a pasteboard tube (*ac*, Fig. 192) about 5 *cm.* in diameter and 120 *cm.* long, closed by two corks, *a* and *c*.

1 For an experiment similar in principle, performed by Hirn, see Trowbridge's New Physics, Exp. 105. This modification of Hirn's method is due to Professor Guthrie. The geometrical principles connecting arcs and heights have been already considered in the case of a ballistic pendulum (see § 109).

The free space between the cork, *a*, and the level of the shot, *b*, is to be measured with a metre rod. The cork (*a*) must be removed for this purpose, and its thickness allowed for. A thermometer is now fitted through the cork (*a′*, Fig. 193) so that by inclining the tube the bulb may be completely surrounded by the shot. The temperature of the shot is to be taken; then the thermometer is removed and the hole closed by a wooden plug. The tube is now inverted 100 times in rapid succession. During each inversion the centre of the tube is held at a fixed height. The shot are kept at one end of the tube by centrifugal force until this end comes vertically over the other. Then the rotation should cease, so that the shot may fall through the distance *ab* almost like a solid mass. Care must be taken, however, not to heat the shot through agitation which would result from too suddenly arresting the motion of the tube. The cork, *c*, should be supported by a table or other solid object so as not to yield under the blow given to it by the shot. Under this condition only, the energy of motion of the shot will be converted into heat *within the mass of shot*. The temperature of the shot is again observed in the same manner as before. It should have risen 5 or 6 degrees.

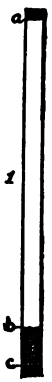

FIG. 192.

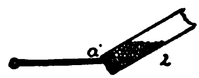

FIG. 193.

The experiment is now to be repeated with 1 kilogram of a substance in the form of shot, but of unknown specific heat; for instance, an alloy of zinc

and lead. If this substance takes up more space than the lead, the distance fallen through in each reversal of the tube will not be quite so great. In this case more than 100 reversals may be made. The total distance fallen through should be as nearly as possible the same. Thus, if the distance *ab* is 100 *cm.* in the case of the lead shot, and 98 *cm.* in the case of the alloy, the tube should be reversed 102 times in the latter case, instead of 100 times.

¶ 178. **Calculations relating to Mechanical Equivalents.** — If s is the specific heat of the lead shot, w its weight in grams, g the weight of 1 gram in dynes, d the distance in *cm.* fallen through in each reversal, n the number of reversals, and J the mechanical equivalent of 1 unit of heat, then the total work done by gravity is evidently $wg \times nd$ ergs; and the heat into which it is converted is (neglecting all corrections) wst units, which is equivalent to $Jwst$ ergs. We have, therefore, —

$$Jwst = wgnd;$$

whence

$$J = \frac{ndg}{st}.$$

It is interesting to compare the value of J calculated by this formula with that found by Joule (see ¶ 176, 1). On account of many large corrections which have not been considered, the result will probably be too great by some 20 or 30 per cent. The principal source of error usually lies in the cooling of the shot by contact with the sides of the

pasteboard tube. This can be avoided by cooling the shot before the experiment to a temperature about 6° below that of the tube. Before repeating the experiment, the tube must be allowed to return to its original temperature. The remaining errors have been found in the long run to balance one another with a probable resultant of about 10 per cent., which may be positive or negative according to the manner in which the manipulations are performed. Instead of computing the mechanical equivalent of heat, we may calculate the specific heat of the lead shot by the formula —

$$s = \frac{ndg}{Jt},$$

where J may be taken as 41,660,000; and if we distinguish by a prime ($'$) the qualities of an unknown substance, we find similarly, —

$$s' = \frac{n'd'g}{Jt'}.$$

Dividing, we find

$$\frac{s'}{s} = \frac{n'd't}{ndt'}, \text{ or } s' = \frac{sn'd't}{ndt'}.$$

In other words, the specific heats of two substances are to each other as the distances through which they must severally fall in order that each may be raised 1° in temperature. On account of the manner in which the two experiments are performed, the values of s and s' should be affected by constant errors in the same proportion, and hence the ratio between them will be affected only by accidental errors (§ 24). The

last formula is therefore less inaccurate than the preceding formulae. To obtain the most accurate results by the aid of mechanical equivalents, as has been described, special devices should be employed to limit the fall of the shot to a given distance. In the absence of due precautions in this respect, the results must be expected to compare unfavorably with those obtained by the ordinary methods (see Exps. 33 and 34). It is nevertheless considered desirable that a student should familiarize himself with a definite example of the conversion of work into heat.

MAGNETIC MEASUREMENTS.

EXPERIMENT LXXI.

MAGNETIC POLES.

¶ 179. **Determination of the Distance between the Poles of a Magnet.** — Compound magnets composed of thin strips of steel bolted together will be found convenient for several experiments in magnetism. Such a magnet, formed of pieces of clockspring, 10 or 15*cm.* long, and 1 or 2*cm.* broad, is represented in Fig. 194. In fitting the strips together it may be necessary to soften them by heat; but their temper must be restored (by again heating and *suddenly* cooling them) before they can be thoroughly magnetized. Each strip should be magnetized separately by stroking one end of it ten times from the centre outward with or upon the south pole of a powerful electromagnet. This end will become a north pole (§ 126). The other end is then to be magnetized similarly by the north pole of the electromagnet. The strips are afterward bound together with all the north poles turned carefully in the same direction.

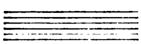

FIG. 194.

A piece of "ferroprussiate paper"[1] prepared for making "blue prints" is now to be stretched flat over a pane of window-glass, or over a stiff piece of pasteboard, with the sensitive surface uppermost. It is then to be placed over a powerful bar magnet constructed as has been described; and a few iron-filings are to be scattered over it. When the paper is jarred the iron-filings will arrange themselves as in Fig. 195. The sensitive surface is now to be ex-

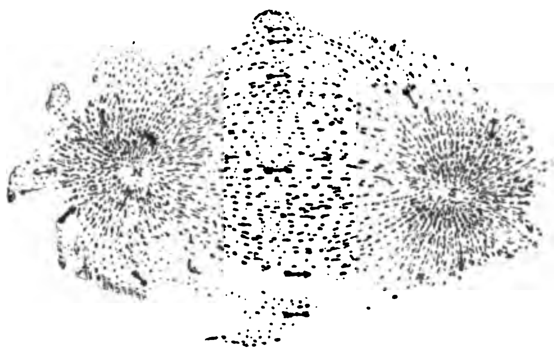

FIG. 195.

posed for about five minutes to direct sunlight, or to the light of the sky for a much longer period, until the surface not covered by the filings becomes quite

[1] To prepare ferroprussiate paper, take 1 gram citrate of iron and ammonia, 1 gram *red* prussiate of potash, pulverize together and dissolve in 10 grams of water. This quantity should cover 20 or 30 square decimetres of smooth (not porous) paper. It should be applied by lamplight, as rapidly and evenly as possible, with a small sponge, in strokes first lengthwise then crosswise, then dried in the dark. The student is cautioned that all "prussiates" are poisonous. Ferroprussiate paper, already prepared, may be bought of dealers in photographic apparatus.

blue. It is then to be placed in the shade, and the iron-filings removed.

The surface covered by the iron-filings should not have been affected by the light; hence the arrangement shown in Fig. 195 should be represented by a white tracing on a blue ground. To make the print permanent, it is necessary to soak it in water for about ten minutes, after which it may be dried in the sun. To avoid delay in waiting for the print to dry, the student is advised to defer this "fixing process" until the end of the experiment. In the meantime,

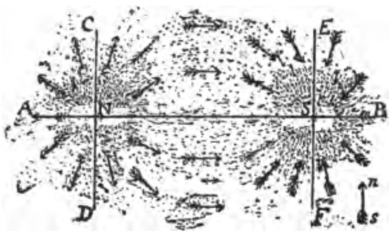

Fig. 196.

the print should be protected from excessive light either from the sun or from the sky. It may be illuminated freely by lamplight or gaslight.

The magnet is now to be placed *over* the print, directly above its former position. A small compass with a needle not more than 1*cm.* long, is to be put beside the magnet at different points in the print. The direction of the north pole of the compass-needle is to be indicated in each case by an arrow drawn in pencil upon the paper (see Fig. 196). The direction of the arrows should agree closely with the lines of

iron-filings, although the compass-needle is in a slightly different plane. The results of this experiment will be somewhat affected by the earth's magnetism. It is well, therefore, to note the direction (*sn*) in which the compass points when the magnet is removed to a distance.

A line *AB* is now drawn so as to bisect as nearly as possible the areas *N* and *S*, from which the "lines of force" (§ 127) seem to diverge. The line (*AB*) should agree with the general direction of the lines of force between *N* and *S*, whether indicated by the compass-needle or by the iron-filings. The areas *N* and *S* are again to be bisected by lines (*CD* and *EF*) perpendicular to *AB*. These lines should cut the edge of the areas (*N* and *S*) at a point where the lines of force are also perpendicular to *AB*.

The positions of the poles *N* and *S* are determined by the intersection of the first line (*AB*) with the perpendiculars (*CD* and *EF*.) The distance between the poles is to be measured. The experiment is to be repeated with at least two other magnets as nearly as possible like the first.

The student may be interested to make prints showing the arrangement of iron-filings due to two parallel magnets, both when their north poles are turned in the same direction and when turned in opposite directions.[1]

[1] See Experiment 40 in the Elementary Physical Experiments published by Harvard University.

EXPERIMENT LXXII.

MAGNETIC FORCES.

¶ 180. **Determination of the Strength of Magnetic Poles.** — One of the magnets (*ef*, Fig. 197), used in Experiment 71, is now to be placed horizontally in the pan-holder (*c*) of a balance (the pan being removed), and counterpoised by an observed weight

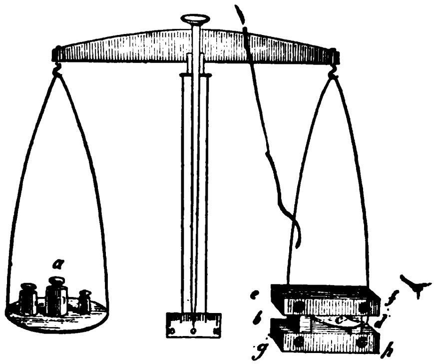

FIG. 197.

in the opposite pan (*a*). A second magnet (*gh*) is to be placed directly under the first, and parallel to it.

The north poles are at first to be turned in opposite directions, so that the magnets may attract each other. Small blocks (*b* and *d*) are now placed be-

tween them to keep them apart. The thickness of the blocks should be such that when the balance beam is raised upon its knife-edges, the index (b) may point to zero. The weight in the pan a is then gradually increased until the magnets are pulled apart. Care must be taken to find the *greatest* weight which the magnets can sustain; for if they be once separated a much smaller weight can hold them apart. In the final adjustment small weights (not over 1*cg*.) should be let fall into the scale-pan from a height not exceeding 1*cm*. The weight necessary to pull the magnets apart is to be noted.

The magnet gh is now to be turned end for end, so as to repel ef, and the weight in the pan a is gradually to be diminished until the magnet ef just touches the blocks (b and d). When a small weight is added to the pan a the beam will not turn suddenly as in previous observations; but, being in stable equilibrium, it may balance in any position. Care must therefore be taken to find the *smallest weight* which can cause a separation of the magnets, however slight.

The mean distance between the magnets, from centre to centre, is now to be determined by measuring the thickness of the magnets and the thickness of the blocks with a vernier gauge. In setting the gauge upon a magnet, if the jaws are of iron or steel the blocks of wood (b and d) should be interposed between the jaws and the surfaces of the magnet, since the strength of the magnet might otherwise be

perceptibly affected. The thickness of the blocks may then be found and allowed for. The experiment should be repeated with a third magnet, let us say *ij* in place of *gh*; then with *gh* in place of *ef*. In this way the forces of attraction and repulsion between each pair which can be formed out of the three magnets will be determined.

The student may be interested to prove that it makes no difference which of two magnets is the one suspended. This fact is an illustration of the general principle that action and reaction are equal and opposite. It will be noticed that the attraction between two magnets when close together, is much greater than their repulsion. This is due to the effects of induction (see § 129, footnote).

¶ 181. **Calculations relating to Magnetic Forces.** — If w be the weight in grams necessary to counterpoise a magnet; w_1 the weight of the counterpoise necessary to lift the magnet and at the same time to pull it away from the attraction of a parallel magnet at the distance d; and w_2 the weight similarly required when the two magnets repel each other; then if 1 gram $=g$ dynes, the force of repulsion which we call positive is $+(wg - w_2 g)$ dynes, and the force of attraction, which we call negative, is $-(w_1 g - wg)$ dynes. The numerical sum, or algebraic difference, Δ, between these forces is accordingly $(w_1 g - w_2 g)$ dynes. Substituting this value in the formula of § 129, we have, if any two of the magnets are equal

in respect to the strengths (s and s') of their poles,[1]

$$ss' = s^2 = \frac{\Delta\, d^2}{4}; \text{ or } s = \frac{d}{2}\sqrt{(w_1 - w_2)\, g}.$$

Thus if the attraction between two nearly equal magnets at a distance of 2 *cm.* is 600 dynes, and the repulsion 300 dynes, a force of 900 dynes (0.92 *g.*, nearly) will be required to offset the effect of reversing one of the magnets. the mean strength of their poles is, accordingly, about $\frac{2}{2}\sqrt{.92 \times 980}$, or 30 units each.

The results of this experiment are subject to errors which are sometimes (though rarely) almost as great as the quantities measured. They are nevertheless valuable in enabling us to form an *immediate estimate* of the strength of magnetic poles, which, though rough, may guide us in the less direct but more accurate methods which follow.

[1] If no two of the magnets are equal, we must form three equations from observations made with each pair of magnets; thus —

$$ss' = \frac{\Delta\, d^2}{4} \text{ (1)}; \; ss'' = \frac{\Delta' d'^2}{4} \text{ (2)}; \text{ and } s's'' = \frac{\Delta'' d''^2}{4} \text{ (3)}.$$

Multiplying (1) and (2) together and dividing by (3) we have —

$$s^2 = \frac{\Delta\ \Delta'}{4\ \Delta''} \times \frac{d^2\ d'^2}{d''^2}; \text{ or } s = \frac{dd'}{2p''}\ \frac{\sqrt{\Delta\Delta'}}{\Delta''}$$

EXPERIMENT LXXIII.

MAGNETIC MOMENTS.

¶ 182. **Determination of the Couple exerted by the Earth's Magnetism on a Suspended Magnet.** — A magnet (*gh*, Fig. 198) used in Experiment 72 is to be suspended horizontally by a wire *cf*. The coefficient of torsion of the wire has been found in Exp. 64. The wire is attached at *c* to a knitting-needle (*bd*) revolving on a graduated circle (*ae*) as in the torsion balance (Fig. 176, ¶ 165). The wire is, however, vertical, and the circle horizontal in this experiment. A short piece of wire should be attached vertically by wax to each end of the magnet to serve as a sight. The needle is first turned so that the north pole of the magnet points north, and its reading is taken. Then it is turned until the magnet points east, and the reading again taken. A distant object should now be sighted in the direction indicated by the sights. The needle is then turned so that the magnet points west. The same distant object should be in line with the sights. The reading of the needle is again observed. The experiment should be repeated with the other magnets employed in Experiment 72.

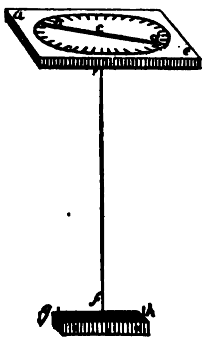

FIG. 198.

If the poles of the magnet are l centimetres apart, if they contain s units of magnetism each, and if the earth exerts on each unit of magnetism a force which has a horizontal component equal to H dynes, then the s units of magnetism in the north pole must be urged northward with a force of Hs dynes, and the south pole will be urged southward with an equal force. The two forces will constitute a couple (§ 113) C, with an arm equal to the distance l, between the poles; since the magnet is at right-angles to the forces in question. We have, therefore,

$$C = Hsl, \text{ or } H = \frac{C}{sl}.$$

This couple must be balanced by an equal and opposite couple due to torsion in the wire. It is obvious that in turning the magnet end for end it must be made to revolve through 180° so as to make an angle of 90° (on the average) with its original (north and south) direction. To produce torsion in the wire the needle must be turned through *more* than 180° in all, or more than 90° from its original setting.

Let us suppose that the needle has revolved through a total angle a, or an average angle of $\frac{1}{2}a$ from its original position; if the magnet had remained pointing to the north the twist in the wire would be $\frac{1}{2}a$; but the revolution of the magnet through 90° causes the wire to untwist through 90° at its lower end. The angle of torsion is therefore $\frac{1}{2}a - 90°$. It is now easy to calculate the couple exerted by the

earth. If it requires a couple of t dyne-centimetres to twist the wire through 1° (see Experiment 64) it must require $(\frac{1}{2}a - 90) \times t$ dyne-centimetres to twist it through the angle in question. Substituting this value for c in the formula above we have —

$$H = \frac{(\frac{1}{2}a - 90)\,t}{sl}$$

It is interesting to estimate the value of H by the rough values of s and l already determined in Experiments 71 and 72. If, for instance, the distance between the poles is 10 *cm.*, and the strength of each 30 units, and if the couple produced is 50 dyne-centimetres, then the earth must exert a force of $\frac{1}{6}$ of a dyne on each unit of magnetism when free to move only in a horizontal plane. This is what is meant by the statement that the "horizontal intensity" of the earth's magnetism is $\frac{1}{6}$ or 0.17, nearly. In practice large errors would be committed in estimating the horizontal intensity in this way, on account of the uncertainty of the factor s (see ¶ 181). A much more exact method will be considered in connection with Experiment 74.

The student should note that the couples acting on suspended magnets are proportional to the products of the distance between the poles and the strength of the poles, both of which have been already determined. These products (sl, $s'l'$, $s''l''$) are called the *magnetic moments* of the magnets to which they respectively belong.

EXPERIMENT LXXIV.

MAGNETIC DEFLECTIONS.

¶ 183. **Determination of Magnetic Deflections by means of a Magnetometer.** — A surveying-compass (Fig. 199) is placed in the middle of a wooden table, in the construction of which no iron has been employed even in the form of nails. All iron or steel objects are to be removed from the immediate neighborhood. The directions of the magnetic north, south, east, and west are to be determined by this compass, and marked by pencil lines upon the table. In all experiments in magnetism the magnetic points of the compass will be those referred to, unless otherwise stated. A magnet already tested in Experiment 71, considerably longer than the compass needle, is now placed at the east of the compass with its north pole toward the compass (see Fig. 200, 1). The distance of the magnet from the compass must

Fig. 199.

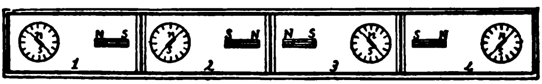

Fig. 200.

be noted. It should be small enough to cause a measurable deflection of the compass, let us say 5 or

10 degrees, but at least twice the length of the magnet.[1] The position of each end of the magnet is then marked in pencil on the table, and the deflection of the compass observed by the reading of two pointers, attached one to each end of the needle.

The magnet is now turned end for end (as in Fig. 200, 2) and the deflection again observed. The experiment is to be repeated with the magnet at an equal distance from the compass, but at the west of it, as in Fig. 200, 3 and 4. There will thus be 8 readings in all, from which the average deflection of the needle may be calculated. The mean distance of the centre of the magnet from the centre of the needle may be found quite accurately by measuring the distance between the outer and between the inner pencil marks on opposite sides of the needle, adding, and dividing by 4. The experiment is to be repeated with the other magnets employed in Experiment 71.

The results of this experiment are to be reduced as will be explained in ¶ 185.

¶ 184. **Theory of the Magnetometer.** — When a magnet is placed near a compass-needle, and at the east or west of it, as in Fig. 200, so that one of its poles is nearer than the other, the needle is deflected under the influence of the nearer pole. The lines of force due to a magnet at any point nearly in line with the two poles are (see Fig. 195) nearly parallel to the magnet; and hence in the case which we have

[1] For very accurate measurements the distance of the magnet from the compass should be at least 4 times the length of the magnet and 12 times the length of the needle.

supposed they are nearly east and west. That is, the magnet tends to make the compass-needle point east and west.

Let us suppose that the magnet is at the east of the compass, and that its south pole is (as in Fig. 200, 2) nearer than the north pole. Then the north pole of the compass-needle (*c*, Fig. 201) will be attracted by the south pole of the magnet more than it is repelled by the north pole. The resultant force will there-

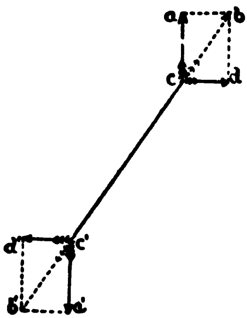

FIG. 201

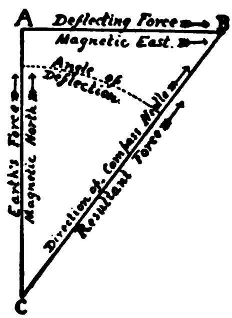

FIG. 202.

fore be an attraction toward the east, which we will represent by the line *cd* (Fig. 201). At the same time the earth pulls the north pole of the compass-needle northward, with a force represented let us say by the line *ca*. The resultant of these two pulls is a force *cb*, easily found by geometrical construction (§ 105).

On the other hand the south pole of the compass-needle (*c'*) will be repelled by the south pole of the

magnet more than it is attracted by the north pole. It will accordingly be urged westward with a force $c'd'$. At the same time it is drawn southward by the earth's magnetism with a force $c'a'$. The resultant force, $c'b'$, may be found as before. Assuming that the forces acting upon the south pole of the needle are equal and opposite to those acting upon the north pole, it follows that $c'b'$ must be equal and opposite to cb. If the needle cc' is free to turn, it will obviously take the direction of the two resultants.

The relation between the forces exerted by the earth and by the magnet upon the north pole of the compass-needle is shown in Fig. 202. The magnetic force is represented by AB; the earth's force by by CA; the resultant by CB. The angle BAC is called the angle of deflection. The tangent of this angle is by definition equal to AB ÷ CA; since AB and CA are at right-angles. Obviously, the magnitude of a deflecting force bears to that of a directive force at right-angles to it a ratio equal to the tangent of the angle of deflection produced.

It has been stated that when the two poles of a magnet are at unequal distances from a compass-needle, the nearer pole has the greater effect. Since the two poles are always equal and opposite, the action of a magnet as a whole evidently depends not only upon the strength of its poles, but also upon the difference of their distances from a given point. We must accordingly consider the length of a magnet, as well as the strength of its poles, in calculating the effect which it will produce. It is found, in fact, that the

forces produced by different magnets at a given distance are very nearly proportional to the "moments" of the magnets in question, that is (see ¶ 182), to the products of the strength of the poles and the distance between them. The moments of the magnets (sl, $s'l'$, etc.) employed in this experiment have been already determined (¶ 182). If a, a', etc., are the deflections produced, we should have —

$$\frac{sl}{tan\ a}=\frac{s'l'}{tan\ a'}, \text{ etc., nearly.}$$

The student should satisfy himself that this is the case before proceeding to the calculations of the next section.

A compass, having on each side of it a pair of revolving supports, capable of holding several magnets, successively at a given distance from the needle, affords one of the most direct and accurate methods of comparing magnetic moments together, and is properly called a magnetometer.

¶ 185. **Calculations relating to Magnetic Deflections.** — EXAMPLE. Let us suppose that in Fig. 200 the average distance between the centre of the magnet *NS* and the centre of the needle *ns* is 25 *cm.*, and that the distance between the poles of the magnet (¶ 179) is 10 *cm.* so that as in (2) the south pole is 20 *cm.* from the needle and the north pole 30 *cm.* from it. Assuming that each pole has a strength of 30 units (see ¶ 181) the attraction of the south pole for a unit of positive magnetism *at the centre of the needle* (see § 129) must be $30 \div (20)^2$ or $\frac{3}{40}$ dyne. The

opposite pole must exert a repulsion on the same unit of magnetism equal to $30 \div (30)^2$ or $\frac{1}{30}$ dyne. The resultant of these two forces is evidently $\frac{3}{40} - \frac{1}{30}$ or $\frac{1}{24}$ dyne acting in an easterly direction parallel to AB (Fig. 202). The earth's magnetism acts in a northerly direction parallel to CA (Fig. 202).

Now since $$\frac{AB}{CA} = tan\ CAB,$$

we have $$CA = \frac{ab}{tan\ CAB}$$

If, for example, $CAB = 14°$, the tangent of CAB is .249 (see Table 5) or $\frac{1}{4}$, nearly; then CA is evidently 4 times as great as AB; hence if $AB = \frac{1}{24}$ dyne per unit of magnetism, $CA = \frac{1}{6}$ dyne per unit of magnetism.

In practice an estimate of the earth's magnetism made in this way will be found to differ greatly from that made as in the last experiment, on account of a tendency to underestimate the strength of the magnetic poles in Experiment 71.

Let us suppose that this strength were estimated at 15 units instead of 30 units. Then in the calculation above we should have estimated the earth's field at $\frac{1}{12}$ dyne per unit of magnetism (instead of $\frac{1}{6}$). In ¶ 182, however, we should have estimated the earth's field at $\frac{1}{3}$ dyne per unit of magnetism. That is, our estimate in Experiment 73 would be too great, and that in Experiment 74 too small in proportion to the error originally made in estimating the strength of the poles. Now when one of two estimates is too

great, and the other too small in a given proportion, the geometric mean between them must be equal to the quantity which we seek. Hence to find the true value of the horizontal component of the earth's magnetism, we multiply together the estimate of Experiments 73 and 74, and extract the square root of the result. Thus $\sqrt{\frac{1}{3} \times \frac{1}{12}} = \frac{1}{6}$. The result is independent of the value provisionally adopted for the strength of the magnetic poles. If the two estimates agree closely the arithmetic mean may be substituted for the geometric mean (§ 57).

Knowing now the true value of H, we may recalculate the moment (M) of the magnet and the strength of the poles by formulæ derived from ¶ 182:

$$M = sl = \frac{C}{H}; \quad s = \frac{C}{HL}.$$

EXPERIMENT LXXV.

DISTRIBUTION OF MAGNETISM, I.

¶ 186. **Determination of the Distribution of Magnetism on a Rod by the Method of Vibrations.** — A steel rod (*aj*, Fig. 203) one metre long, and about 1 *cm.* in

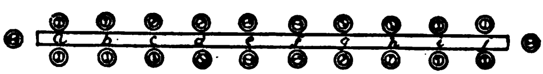

FIG. 203.

diameter, is marked with a file at ten points (*a* . . . *j*) 10 *cm.* apart, beginning with a point *a*, 5 *cm.* from one

end of the rod. It is then magnetized by stroking it from *e* to *a* 10 times with the south pole of a powerful electro-magnet, and by stroking it 10 times from *f* to *j* with the north pole of this magnet. A small piece of a sewing-needle (*f*, Fig. 204) about 1 *cm.* long, and highly magnetized is attached horizontally by sealing-wax to a bullet *e*, and suspended by a fine fibre (*cd*) of untwisted silk from a cord (*a*) in a test tube (*hg*).

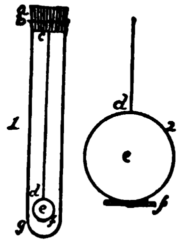

Fig. 204.

The torsion of the fibre (*cd*) should be so slight that the cork (*a*) may be twisted through 360°, without deflecting the needle (*f*) more than a few degrees from the magnetic north, toward which one end should point. The needle is then to be deflected by a magnet; and when the magnet is suddenly taken away the needle should make a series of vibrations in a horizontal plane. The weight of the bullet should be so proportioned to the magnetic strength of the needle that there may be about 10 vibrations completed in one minute. The exact time required for 10 vibrations of the needle is to be determined when it is vibrating in an arc not exceeding 30° or 40° (see Table 3, *g*). The north pole of the needle should be distinctly marked.

The test tube is now to be placed opposite the end of the rod, then held successively on each side of each of the ten points (*a*—*j*, Fig. 203). The direction indicated by the north pole in each position is to be represented by arrows (drawn as in Fig. 203) the

direction of which may be compared with that of the lines of force issuing close to the magnet in Fig. 196. In addition, the rate of vibrations of the needle is to be determined by counting the number of vibrations completed in 1 minute, or in whatever time may have been required for 10 vibrations under the influence of the earth's magnetism alone. In all cases the arc of vibration should be limited to 30° or 40° (see Table 3, *g*).

The number of vibrations made in the given time on one side of *a* is to be averaged with that made on the other side; and in the same way the average number of vibrations for each of the ten points is to be found. These numbers are then all to be squared (see Table 2). The results are to be plotted on co-ordinate paper (see § 59). Distances in centimetres are represented by a horizontal scale at the top of the figure, and the square of the number of oscillations is shown by the vertical scale at the left of the figure.

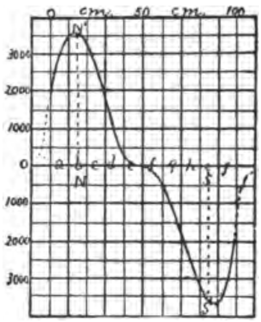

FIG. 205.

Thus, if opposite the point *b*, 15 *cm.* from the end of the magnet, the needle makes 60 vibrations per minute, we place a cross at the right of the square of 60 (3600) and under 15 *cm.* The vertical distances are measured upward if the north pole of the needle is repelled by the bar, and downward if it is attracted by it. In the same way other points may be found through which

a curve is to be drawn as in Fig. 205. Evidently, in this figure, N represents the "positive" or "north" end of the magnet.

This method of representing the distribution of magnetism depends upon the general principle that forces are proportional to the squares of the rates of oscillation which they produce (see § 110). The curve represents accordingly the strength of the magnet at different points as compared with the strength of the earth's magnetism. We should strictly allow for the effect of the earth on all the rates of oscillation; but as it is represented only by 100 units on the vertical scale, this effect would be hardly perceptible.[1]

The student should draw by the eye two vertical lines NN' and SS', dividing each area enclosed by the curve as nearly as possible into two equal parts. The distance between these lines indicates approximately the distance between the poles of the magnets. This latter may therefore be found by the scale at the top of the paper.

EXPERIMENT LXXVI.

DISTRIBUTION OF MAGNETISM, II.

¶ 187. **Magneto-Electric Induction.** We have seen that when iron-filings are brought into the neighbor-

[1] The effects of "induced magnetism" may introduce errors of 5 or 10 per cent in this experiment (see ¶ 207). The shape of the curve in Fig. 208 will not, however, be materially altered.

hood of a powerful magnet, they tend to arrange themselves along certain lines called "lines of force." These lines of force are not, like the meridians upon the surface of the globe, purely geometrical conceptions. According to Tyndall, the apparently empty space between the poles of a powerful electro-magnet "cuts like cheese." The most surprising fact connected with this phenomenon is that a knife with which such a magnetic field is cut becomes temporarily electrified. The point and the handle of the knife resemble, for the time being, the two poles of a voltaic cell, from which a current of electricity can be derived by making the proper connections. It is not necessary to use a knife; any piece of metal, a wire for instance, will do as well. All tendency to produce a current ceases when the knife or wire stops moving, or as soon as all the lines of force have been cut. The effect of a sudden motion upon a galvanometer may accordingly be almost instantaneous. In such cases it is measured by the "throw" of the needle (§ 109). It is found that the "throw" is proportional, other things being equal, to the intensity and extent of that part of the magnetic field which has been cut through, or, according to a system of representation universally adopted, it is proportional to the *number of lines of force* which have been cut.

If a loop of wire is placed around the middle of a long bar-magnet (Fig. 206) and suddenly made to slip off one end of the magnet, it will evidently cut nearly all the lines of force on that end of the magnet.

A delicate galvanometer connected with the ends of the loop will be affected. This affords a convenient method of comparing the strengths of different magnetic poles. In practice we employ a coil of wire instead of a simple loop; for when each turn cuts all the lines of force, the effect is found to be proportional to the number of turns which the wire makes about the magnet. It is not necessary to slide the coil completely off the magnet. A motion of a few centimetres may affect the galvanometer. When the motion is confined to one end of the magnet it will be found to deflect the needle in opposite ways according to which way the coil is moved. In other words the direction of the electrical current depends

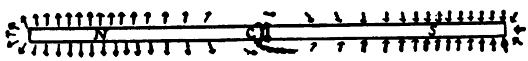

FIG. 206.

upon the direction of the motion. Let us suppose the direction of the motion to be always the same, that is, from left to right, or from the north toward the south end of the magnet. Then the galvanometer will be deflected one way when the motion of the coil takes place near one end of the magnet, and the other way when it takes place near the other end of the magnet. That is, the direction of the electrical current depends on the direction of the lines of force. Near the middle of the magnet a neutral point will generally be found. If the coil be moved from this neutral point toward either end of the magnet, it follows from the statements made

above that the direction of the current will always be the same. This direction is with the hands of a watch, as seen from the south pole of the magnet.

The throw of the needle is proportional, other things being equal, to the distance through which the coil is moved; hence it is important in comparing results that this distance should be always the same. If the coil is moved always through a given distance, the effect will be found to be greatest when the motion takes place near the ends of the magnet, where the lines of force are the thickest. In other words the magnitude of the electrical current depends upon the closeness of the lines of force. The effect is very nearly the same whether the coil moves *more* or *less swiftly*[1] through a given distance. In the first case we have a rapid motion, and hence a comparatively strong current lasting for a short time; in the second case we have a weaker current lasting for a proportionately long time. The forces exerted upon the galvanometer needle are proportional to the current; hence, by the fundamental law of motion (§ 106),

$$ft = mv,$$

since the product (ft) of the force and the time of its action is the same in both cases, the momentum given to the needle must be the same.

We shall make use of these facts to estimate the relative strength of the magnetism of a rod in differ-

[1] In order that this may be true, the duration of the motion must be several times less than the time occupied by one vibration of the galvanometer needle.

ent parts, and to distinguish positive from negative magnetism.

¶ 188. **Construction of an Astatic Galvanometer.**— A delicate galvanometer, such as has been already employed for the detection of currents created by a thermopile (Exp. 39), is represented in Fig. 207, and may be constructed as follows:—

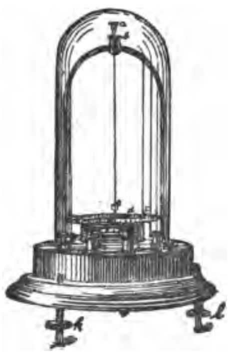

FIG. 207.

Two magnetized needles, *c* and *h* (Fig. 208), of nearly equal strength are connected by a vertical piece of wire, with their north poles *in opposite directions*, and suspended horizontally, by a fine thread (*bc*) of untwisted silk, from a screw *a*. This screw is held by a nut *b*, itself capable of rotation, so that the thread may be raised or twisted at pleasure. The two needles *c* and *h* should

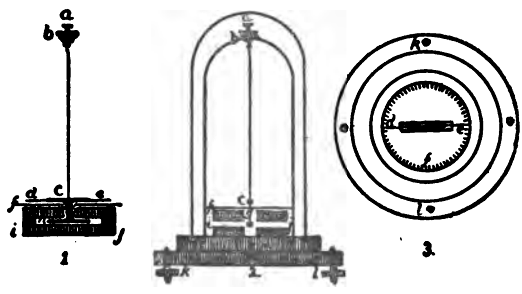

FIG. 208.

form a nearly "astatic" combination (*a* privative and ἵστημι, to stand); that is, one which, owing to the

equal and opposite forces exerted upon it by the earth, has no strong tendency to stand in any particular position.

The strength of either magnet may generally be increased by stroking one of the poles, as in ¶ 179, with the dissimilar pole of a powerful magnet, or diminished by touching similar poles together. A very light touch is usually sufficient to produce a perceptible change in a magnet. The delicacy of the instrument depends upon the delicacy of the balance which can be established between the two needles. It is generally possible to make the combination point permanently east and west. In practice, however, the needles are magnetized so that the time occupied by one oscillation is 5 or 10 times as great as that of either needle by itself. The needle is then sufficiently astatic for most purposes. It may be remarked that the rate of oscillation of an astatic needle is the best test of its adjustment (see ¶ 193, 4).

100 metres of insulated copper wire about $\frac{1}{2}$ *mm.* in diameter are now to be wound on the two rectangular bobbins *f* and *i* (Fig. 208, 1 and 2).[1] The bobbins are shaped so that the lower needle (*h*) may hang inside of them, and the upper needle (*c*) just above

[1] If it is desired to use the instrument later on (Exp. 86, II. and Exp. 95) as a differential galvanometer, the 100 metres of wire should be cut in two, and the two parts twisted together before winding them on the bobbins. The galvanometer will thus have four terminals instead of two. If two of the terminals are temporarily joined together, the other two may be connected with binding-posts in the ordinary manner.

them. Two indices of aluminum wire, *d* and *e* (Fig. 208, 1 and 3), are then attached to the upper needle, and a cardboard protractor (*f*) is set beneath them. The instrument is usually mounted on wooden supports, with levelling screws *k* and *l*, and covered with a glass shade to cut off currents of air. The galvanometer thus constructed should be sensitive to a few millionths of an ampère.

¶ 189. **Determination of the Distribution of Magnetism on a Rod by the Method of Induction.** — A coil (*b*, Fig. 209) consisting of about 100 turns of No. 20 insulated copper wire, wound on a brass bobbin, is fitted to a brass tube *ad* so as to slide freely between

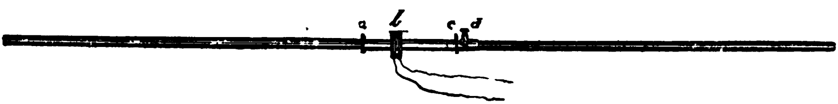

FIG. 209.

the stops *a* and *c*, through a distance of about 10 centimetres. The tube must be large enough to admit the long magnet employed in Experiment 75. It is first to be fastened near one end of this magnet by means of the clamp *d*, so that a point (*a*, Fig. 203) 5 *cm.* from the end of the magnet may come half-way between the stops *a* and *c* (Fig. 209).

The needle of a delicate galvanometer (Fig. 207), such as has been already employed for the detection of electrical currents (Exp. 39), is now to be loaded, if necessary, by att ching small bits of lead with sealing-wax to each end of the needle, so that its time of oscillation may be at least 10 seconds. The

instrument is to be set up with the plane of its coils approximately north and south. The nut *b* is then turned so that, by the torsion of the thread *bc*, the needle of the galvanometer is made to point to 0°. The terminals of the coil *b* (Fig. 209), are then to be connected with the terminals of the galvanometer.

The coil (*b*) is then suddenly made to slide from *a* to *c* (Fig. 209), and the throw of the galvanometer is noted. When the oscillation of the needle has ceased [1] the coil is made to slide back suddenly from *c* to *a*, and the throw of the galvanometer is again noted.

The experiment is to be repeated with the tube clamped so that other points (*b*, *c*, *d*, *e*, etc., Fig. 203) may come successively half-way between the stops *a* and *c* (Fig. 209).

In each case two throws of the galvanometer are to be observed. The direction of each throw is to be noted, and the average deflection calculated.

The positions of the centre of the tube with respect to the magnet are also to be noted. The results are to be plotted on co-ordinate paper as in Fig. 205,

[1] The student should learn to stop the vibrations of a magnetic needle. If a magnet is directed toward a needle as in Fig. 200, ¶ 183, a deflection in either direction may be produced. If the magnet be turned so as to tend to cause a deflection at every instant opposite to the motions of the needle, the latter will come very quickly to rest. To stop a wide oscillation, the magnet must be brought near the needle, but when the oscillation becomes feeble, the process should be continued from a greater distance. To affect an ordinary astatic needle, the magnet should be held not only at right-angles with it, but also considerably above or below it. A perfectly astatic needle should not be affected by a magnet in the same horizontal plane.

¶ 186, except that the vertical distances are to represent throws[1] of the galvanometer needle, instead of squares of the rates of oscillation. If the throw in a given case is in the same direction as at the north end of the magnet when the coil is stopped in a given direction, the distances are to be measured upward; otherwise downward. From the curves thus obtained the poles of the magnet are to be located as in ¶ 186, and the distance between them is to be estimated. The result should agree closely with that obtained in the last experiment.

EXPERIMENT LXXVII.

MAGNETIC DIP.

¶ 190. **The Earth's Magnetism.** — If fine iron-filings are sprinkled over a horizontal pane of glass, they will show a slight tendency to arrange themselves in lines parallel to the magnetic meridian, particularly if the glass be jarred. One might infer that the lines of force due to the earth's magnetism are horizontal. This is not, however, the case; the direction in which the lines are inclined is from north to south, according to the compass, but the lines make any angle with the horizon (§ 128); 70° or 80° for instance in the United States. We have already made use of

[1] If the throws exceed 80° the student should plot the *chords* of the angles in question (Table 8), instead of the angles themselves (see § 109).

the surveying-compass to find the magnetic meridian (¶ 183). The compass affords, however, little or no idea of the angle which the lines of force make with the horizon, because a compass-needle is suspended so as to move approximately in a horizontal plane.[1] To find the magnetic dip (§ 128), we may make use of an instrument known as the "dipping-needle." A simple form of this instrument consists of a knitting-needle *ad* (Fig. 210), with an axis *bc* soldered to it a right-angles and resting on two glass surfaces *b* and *c*, attached by sealing-wax to wooden supports (*be* and *cf*), and made horizontal by means of a spirit level.

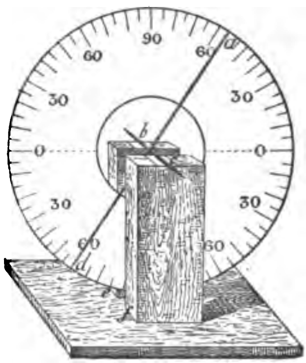

FIG. 210.

In practice the needle must be balanced by bending the axis *bc*, or by adding bits of sealing-wax or solder to it, so that it will stay, *when unmagnetized*, in any position, as *ad*. Then the needle is magnetized by stroking the end *a* ten times from the centre outward with the north pole of a powerful magnet, and by stroking the end *d* similarly with the south pole of the magnet. The needle will no longer balance in any position; but the north pole will, in north lati-

[1] The needles of surveying compasses intended for use in widely different latitudes are frequently provided with a small sliding weight by which variations in the magnetic dip and intensity may be counterpoised.

tudes, dip downward as in Fig. 210. To measure the angle of the dip, a cardboard protractor, cut out at the centre so as not to interfere with the axis of the needle *bc*, is attached vertically to one of the wooden supports (*be*), and turned round so as to be north and south according to the compass. The axis *bc* is made to point horizontally east and west, and to coincide as nearly as possible with the axis of the graduated circle. The mean reading of the two ends (*a* and *d*) of the needle should then give correctly the angle of the dip. Errors of parallax must of course be guarded against (§ 25). Various other sources of error may be eliminated by a series of experiments. In some of these the axis *bc* should be turned end for end, in some the whole instrument should be turned end for end, and in some the magnetism of the needle should be reversed by stroking the end *d* upon the north pole, and the end *a* upon the south pole of a magnet. By averaging the various results, the angle of the magnetic dip may be determined within a few degrees.

¶ 191. **The Earth Inductor.** — If a hollow square of wire *CDEF* is laid upon the floor with the side *CD* magnetically east and west, and rotated about *CD* as an axis into the position *ABCD*, it is evident that the wire *EF* must cut all the lines of force due to the earth's magnetism which pass through the areas *ABCD* and *CDEF*. The line *CD* will cut no lines of force, because it is stationary; and the wires *CE* and *DF* will cut none, because their motion is in a plane parallel to the lines in question. All

the lines cut will therefore be included in the area *ABEF*.

If the square is now held against the west wall of the room, in the position *C′D′E′F′*, and rotated as before about an axis (*C′D′*) perpendicular to the lines of force, into the position *A′B′C′D′*, the number of lines cut will be as before included in the area *A′B′E′F′*; and similarly if the square is rotated about an axis *C″D″*, in the north wall of the room

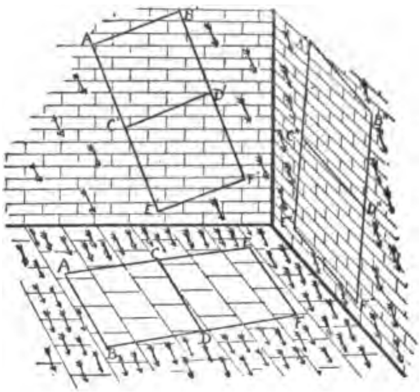

Fig 211.

perpendicular to the lines of force, the lines cut will all be included in the area *A″B″E″F″*. Now the areas (*ABEF*, *A′B′E′F′*, *A″B″E″F″*) are all equal, — each being twice the area included by the square. If, therefore, we connect the terminals of the square with a galvanometer, and observe the throws of the needle which take place when the square is suddenly turned over, we shall have a means of comparing the relative numbers of the lines

of force which pass through the square in its three different positions.

From these data we may infer the direction of the lines of magnetic force. If, for instance, the throw of the needle is much greater when the square is turned over on the north wall of the room than on the west wall, we may infer that more lines of force pass through the square in the former position; and that, accordingly, these lines are more northerly than westerly. If, again, the throw is much greater when

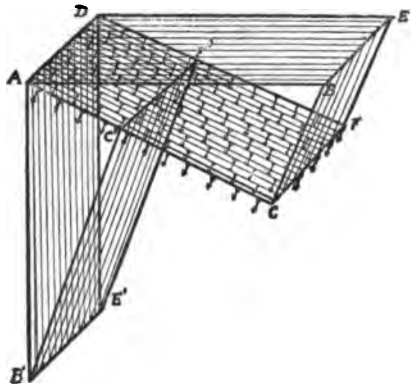

FIG. 212.

the square is turned over on the floor than on either wall, we may infer that the lines of force are more nearly vertical than horizontal. We will suppose, for simplicity, that the walls of the room face exactly north and west by the compass, so that no lines of force pass through the loop when held against the west wall of the room.

Let *ABED* and *AB′E′D* (Fig. 212) represent respectively the square in its horizontal and in its ver-

tical position, AD being magnetically east and west; let the plane $ADF'FCC'A$ be drawn perpendicular to the lines of force, and the planes $BEFC$ and $B'E'F'C'$ parallel to the lines. Then the areas $ADFC$ and $ADF'C'$ include respectively the lines which pass through the square in its two positions. Since the lines are equally spaced, their numbers are as the areas which include them. These areas are to each other as $AC:AC'$, or since by construction $BC = AC'$, they are to each other as $AC:BC$. This ratio $(AC:BC)$ is by definition the tangent of the angle ABC, which measures the magnetic dip.

Now if a' is the angle through which the needle is thrown when a loop of wire is turned over on the floor, and if a'' is the same for the north wall of a room, the impulses given to the needle are to each other as the chord of a' is to the chord of a'' (see § 109), or approximately as a' is to a''. It follows that the angle of the dip a is given by the formula —

$$tan\ a = \frac{chord\ a'}{chord\ a''}, = \frac{a'}{a''} \text{ nearly.}$$

The same proportion will be found to hold for a round loop of wire. In practice we employ a coil of wire, containing, let us say 100 turns, since the effect upon the galvanometer increases with the number of turns.

The student should note that a sliding motion given to such a coil either along the floor or along the wall causes no deflection of the galvanometer. This is because the lines of force are cut by the two

halves of the coil in opposite ways. It will be found to make no difference whether the coil is rotated about an axis passing through its centre, or on one side of it. We need to consider only the angle through which rotation has taken place. A coil capable of being thus rotated 180° about a horizontal and about a vertical axis constitutes what is called an "earth inductor," because of the currents of electricity which by the action of the earth's magnetism, may be "induced" in it.

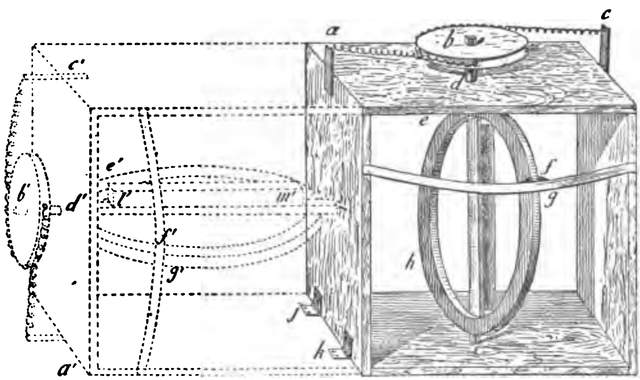

Fig. 213.

¶ 192. **Determination of the Magnetic Dip by means of an Earth Inductor.** — A convenient form of earth inductor is represented in Fig. 213.[1] It consists of a coil of wire *h*, mounted on a wooden axle *di*, with a head *b*, through which the coil may be set in rotation

[1] The instrument may be greatly simplified if it is intended only to be turned by hand. This generally requires the co-operation of two students, one to turn the earth inductor properly, the other to observe the throws of the galvanometer.

by the spring *cbd*. An auxiliary spring *ad* may also be employed to hasten the rotation through the first right-angle, and to slacken it in the second right-angle, so that the coil may be arrested by the catch *f*, when it has rotated through exactly 180°. By winding the spring *abd* round the head of the axle in the other direction, the coil may be made to return to its original position. The apparatus is permanently attached to the floor by means of two hinges *j* and *k*, the axes of which are east and west. If the coil is properly counterpoised, it will operate also when the whole instrument is tipped on its side, as represented by the dotted lines in Fig. 213.

Wedges are to be placed beneath the frame so that the axis of the coil may be exactly vertical in one position, and exactly horizontal in the other position. The catch *f* must be adjusted if necessary, so that the coil may be horizontal in the second position. If the hinges are properly placed the plane of the coil will be at right-angles to the magnetic meridian in both positions.

The axis of the coil is first to be made horizontal, and the terminals of the coil are to be connected (see ¶ 193, 11) with a galvanometer (Fig. 207, ¶ 188), placed at a considerable distance from the earth inductor so as to avoid jarring, and adjusted as in ¶ 189. The catch *f* is then to be lifted by pulling a string attached at *g*. The throw of the needle is to be noted. When the needle has come to rest (see ¶ 189, footnote) the coil is made to return suddenly to its original position by the same mechanism. The throw of

the needle is again observed, and the mean throw (a') calculated.

The experiment is to be repeated with the axis of the coil vertical. The mean throw (a'') is to be found. The angle of the dip (a) is then to be calculated by the formula (see ¶ 191),

$$\tan a = \frac{a'}{a''}, \text{ nearly.}$$

ELECTRICAL MEASUREMENTS.

CURRENT STRENGTH.

¶ 193. **General Precautions in the Measurement of Electric Currents.** — Nearly all measurements of electric currents involve the use of galvanometers depending upon the deflection of a magnetic needle. The same precautions must accordingly be observed in electrical as in magnetic measurements.

(1) DELICACY OF SUSPENSIONS. A needle weighing less than 10 grams may be safely suspended by a single fibre of the best cocoon silk. When several fibres are employed they should be fastened together with wax, but *not twisted together*. If great delicacy is desired, the finest possible thread should be employed.

When a needle is hung on a pivot, as in an ordinary compass, great care must be taken to preserve the sharpness of the steel point upon which it turns. A lever should be arranged so as to lift the needle from the pivot when the instrument is not in use; and when in use, care should be taken not to jar the compass. A slight jarring may be used as a last resort to relieve the friction between the needle and its pivot when the latter has been already dulled. It is preferable, when possible, to observe the turning-points of the needle while oscillating in a small arc,

and from these to infer its position of equilibrium (see ¶ 20).

(2) PRESERVATION OF MAGNETISM. The needle of a galvanometer should be carefully protected from strong magnetic forces, whether due to permanent magnets or to electric currents, since such forces are apt to affect the magnetism of the needle. This precaution is especially important in the case of "astatic" needles (¶ 188), since the slightest change in either of the two parts of which such needles are composed may completely destroy the balance between them, and thus seriously injure the delicacy of the combination.

Strong currents should never be sent through delicate galvanometers. The terminals of such galvanometers (*a* and *b*, Fig. 214) should be joined together with a wire or "shunt" (*c*), forming a cross-connection between the wires (*d* and *e*) which convey the current to and from the galvanometer. An electric current of unknown strength should be first tested by the galvanometer *with the shunt*. If the galvanometer shows little or no deflection, the shunt may be safely removed.

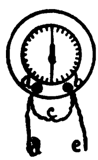

FIG. 214.

(3) MAGNETIC SURROUNDINGS. All iron, steel, or other magnetic substances should be removed, if possible, from the neighborhood in which magnetic measurements are to be performed. The positions of magnetic bodies which cannot be moved should be accurately noted. Especial care must be taken to guard against *changes* in the position of magnetic

bodies in a course of experiments.[1] The position of a galvanometer should be accurately located, since considerable variations, both in the direction and in the strength of the earth's magnetism, often occur in different parts of the same building, unless special care has been taken to avoid the use of iron in its construction. When there is no simpler way of describing the place of an instrument, its distances may be found from the floor and from two walls of the room.

(4) Rate of Oscillation. Any change in the strength of the magnetic forces acting upon a needle, in the magnetism of the needle itself, or in the freedom of its suspension will be found to affect its rate of oscillation. It is well, therefore, to determine this rate before and after every experiment in which such changes are likely to occur. This precaution is particularly important in the case of astatic needles and in the method of vibrations (Exp. 82).

(5) Excentricity. When a compass-needle is suspended at a point not exactly in the centre of the graduated circle by which its position is determined, errors due to "excentricity" may be introduced. Such errors are avoided by *reading both ends of the needle.*

(6) Zero-Reading. A galvanometer is always to be adjusted (except in the method of vibrations, Exp. 82) with the plane of its coil vertical, and parallel to the needle in its zero position, — that is, the position which the needle takes when no current is flowing

[1] Students should be cautioned against carrying small objects made of iron or steel about their person.

through the coil. In the case of a galvanometer provided with an ordinary compass-needle, the plane of the coil is accordingly to be made parallel to the magnetic meridian. In this position the reading of the needle should be zero. It is well to make sure (§ 32) that the zero-reading is not disturbed in the course of an experiment, either by dislocation of the galvanometer or by changes in the position of magnetic bodies in the vicinity (see 3).

(7) MUTUAL INDUCTION. To prevent the coils of one instrument from affecting the needle of another instrument, these instruments should be separated as widely as may be practicable. In certain

FIG. 215.

delicate experiments the effects of magnetism produced in one building are measured by electrical wires carried to an entirely separate building. Coils of wire are in general made horizontal if possible; magnets vertical; since in these positions minimum magnetic effects are usually produced on galvanometers in their vicinity.

(8) CONNECTING WIRES. The wires conveying an electric current to and from an instrument should be parallel and close together, so that the equal and opposite currents in these wires may neutralize each other as far as magnetic effects are concerned. A typical case is represented in Fig. 215, where by the parallel wires *bc*, *de*, and *af*, a battery *B* is connected

through a rheostat R with a galvanometer G (see Exp. 92). It will be found convenient in practice to twist the wires together. In rheostats the wires are wound double (see Fig. 240, Exp. 86) to avoid magnetic effects.

(9) Reversal of Currents (§ 44). Every instrument capable of being affected by magnetic influences from outside should be provided with means of reversing the current through it, without changing its direction in other parts of the circuit. Any such instrument is called a "commutator." A convenient form of "commutator" is represented in Fig. 216.[1]

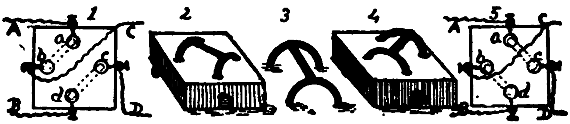

Fig. 216.

(10.) Waste of Power. The commutator may be made also to serve as a "key,"— that is, to cut off

[1] This commutator consists of a square block of mahogany or ebonite, with four holes *abcd* (Fig. 216) bored half-way through it. The screws of four binding-posts are driven horizontally into these holes, which are then filled with mercury. Two copper rods (Fig. 216, 3), bound together by a handle of mahogany or ebonite, are bent so as to reach respectively either from a to b and from c to d, or from a to c and from b to d (see Figs. 216, 2 and 4). The wires (A and B) from the positive and negative poles of a battery are connected with two *opposite* mercury cups, as a and d; the wires C and D, leading to the instrument in which the current is to be reversed, are connected with the other pair of opposite cups (as b and c). It will be seen that in one position of the commutator (Fig. 216, 1 and 2), the wire A is connected with C, while B is connected with D; in the other position (Fig. 216, 4 and 5) A is connected with D, while B is connected with C.

the current from the battery. This is done by simply removing the rods (Fig. 216, 3) from the mercury cups. In the absence of a commutator or key, one of the battery wires should be disconnected when the battery is not in use, not only to prevent unnecessary waste of power, but also to avoid serious errors which may result either from the deterioration of the battery or from heating the wires.

When a battery is not required for several days it is well to empty out the fluids which it contains, each into a separate vessel, in which it may be preserved for future use, if not already exhausted. The zincs and coppers or carbons should be placed in pure water; the porous cups left to soak in a solution of dilute sulphuric acid so as to be ready for immediate use; the clamps, being disconnected from the poles of the battery, should be carefully cleaned and dried.[1]

(11) ELECTRICAL CONNECTIONS. All electrical connections depending upon metallic contact should be carefully examined. The metallic surfaces should be scraped bright and bound together with considerable pressure. A good electrical connection between two copper wires may generally be made by twisting them together. A soldered joint is to be preferred if the connection must remain good for an indefinite length of time. A liberal supply of binding-posts, screw-cups, and couplings, will be found of value in electrical measurements.

[1] These remarks apply particularly to cells of the Daniell or Bunsen type (Figs. 234 and 235, Exp. 84). With a Leclanché cell (Fig. 236), these precautions are unnecessary.

The best temporary connection is undoubtedly made by dipping copper into mercury (see 9). The surface of the copper should first be amalgamated by dipping it into nitrate of mercury and rubbing it with a cloth.

(12) INSULATION. Care must be taken that electrical connections are not made when they are not wanted. The student should carefully examine the insulating material with which his wires are wound, particularly when the wires are to be twisted together. He should make sure that there is no current between any two of the binding-posts of a commutator or rheostat which can be detected by a galvanometer when the metallic connections are broken. The outside of battery cells should be dry for if they are not, electrical leakage is apt to take place. There is in fact more or less leakage in all experiments; but if the apparatus be perfectly dry this will probably not be enough to affect the accuracy of any of the measurements which follow.

EXPERIMENT LXXVIII.

CONSTANTS OF GALVANOMETERS.

¶ 194. **Construction of a Single-Ring Tangent Galvanometer.** — A form of galvanometer frequently employed, because of its simplicity of construction, is represented in Fig. 217. A horizontal cross section

is given also in Fig. 218. The instrument consists of a compass (*a*, Fig. 217, and *dgif*, Fig. 218) mounted on a wooden support in the middle of a coil of insulated wire. The compass-needle (*eh*) is made very short[1] so that the *whole* of it may be virtually at the centre of the coil. To assist in reading the deflections of the needle, two long light pointers (*f* and *g*)

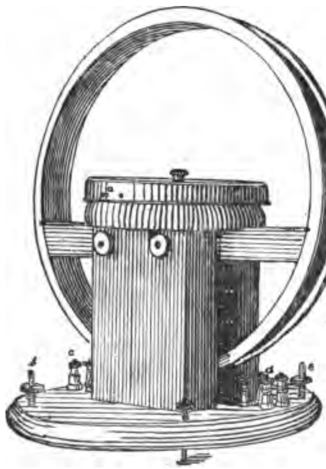

FIG. 217.

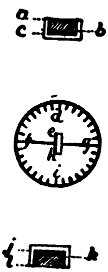

FIG. 218.

are attached to it at right angles. The wire is wound on a grooved brass ring in a single layer. The ends of the wire are carried to binding-posts (*c*, Fig. 217) at the base of the instrument as close together as possible. Levelling screws (*b* and *e*, Fig. 217) are usually added. In the construction of the instrument

[1] The length of the needle should not exceed $\frac{1}{12}$ the diameter of the coil. Kohlrausch, Physical Measurement, Art. 63.

neither iron nor steel must be used (¶ 214, 3) except in the magnet itself, and in the steel pivot upon which it turns. The compass should have a lever to lift the needle from the pivot when the instrument is not in use (¶ 214, 1).[1]

¶ 195. **Law of Tangents.** — When an electrical current of sufficient strength is sent through the coils of a galvanometer, lines of magnetic force due to the current may be recognized by the the aid of iron-filings scattered upon a horizontal piece of glass. We will suppose that the plane of the coil is parallel to the magnetic meridian (that is, vertical, and magnetically north and south ¶ 214, 6), and that the glass passes through the centre of the coil. Lines of force will then be formed in a direction which, if the current is sufficiently powerful, may differ imperceptibly from east and west near the centre of the coil.

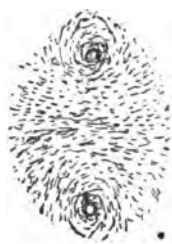
FIG. 219.

When a compass-needle is placed at the centre of the coil, it takes a direction, as might be expected, parallel to the lines of force passing through that point. If we suppose the current to be ascending on

[1] Single-ring galvanometers in the Jefferson Physical Laboratory have been constructed with 10 turns of No. 16 insulated copper wire, wound on a brass ring 36 *cm.* in diameter. The supports are made of wood. The needle is $2\frac{1}{2}$ *cm.* long. The pointers are of aluminum, and each about 5 *cm.* long. The circle is divided into degrees and half-degrees. The coil is arranged in sections of 1, 2, 3, and 4 turns, with connections so that any number of turns can be employed from 1 to 10. By sending the current through these sections in different directions the sections may be tested against one another.

the north side of the coil, and descending on the south side, the north pole of the needle will point nearly to the east. The electric current *tends* in fact to deflect the compass-needle due east and west, but the earth's magnetism combined with it always gives to the needle a more or less northerly direction.

The actual direction of the compass-needle is determined (see ¶ 184) by two forces: one, H, due to the horizontal component of the earth's magnetism acting in a northerly direction; the other, F, due in this case, not (as in ¶ 184) to a magnet, but to the magnetic effect of the electrical current acting in an easterly or westerly direction. The angle (a) of deflection is given accordingly, as in ¶ 184, by the formula,

$$\frac{F}{H} = \tan a. \qquad (1)$$

The units of current now in use have been defined (§ 132) with reference to the magnetic field which a current produces in a coil of wire. If L is the length of the wire, R its mean radius, and c the current in absolute units, we have

$$F = \frac{cL}{R^2}. \qquad (2)$$

Or if C is the current in ampères (§ 19), we have —

$$F = \frac{1}{10}\,\frac{CL}{R^2}. \qquad (3)$$

Substituting this value in (1) we have —

$$\frac{CL}{10\,R^2H} = \tan a. \qquad (4)$$

Let us suppose that two currents C' and C'' produce the deflections a' and a'' respectively; then

$$\frac{C L}{10\ R^2 H} = \tan a'; \qquad (5)$$

and

$$\frac{C'' L}{10\ R^2 H} = \tan a'. \qquad (6)$$

Dividing (5) by (6) we find —

$$C' : C'' :: \tan a' : \tan a''; \qquad (7)$$

that is, in a given galvanometer two currents are proportional to the tangents of the angles of deflection which they respectively produce. This is known as the *Law of Tangents.*

¶ 196. **Calibration of a Tangent Galvanometer.** — The single-ring galvanometer described in ¶ 194

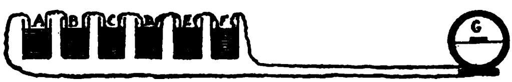

FIG. 220.

may approximate more or less closely to the conditions required of a perfect tangent galvanometer. To test the accuracy with which the "Law of Tangents" (¶ 195) is fulfilled, a battery of six small Daniell cells may be employed. The cells should be as nearly as possible of the same size and composition.

The plane of the galvanometer coil is to be made parallel to the magnet meridian (¶ 193, 6) so that the compass-needle points to 0° at both ends; then the two terminals are to be connected, with the poles of the battery arranged in series, as in Fig. 220, and in

Fig. 221, 1, so that the cells may all act together. The connecting wires should be well insulated (¶ 193, 12) and twisted together (¶ 193, 8). The deflection of the galvanometer is to be found by reading both ends of the needle (¶ 193, 5).

The connections of the poles of the first cell (A) are now to be interchanged (Fig. 221, 2) so that it acts against the other five. The deflection is to be found as before. Then the original connections of A are to be restored, but those of the second cell (B) reversed (as in 3), and the deflection again noted;

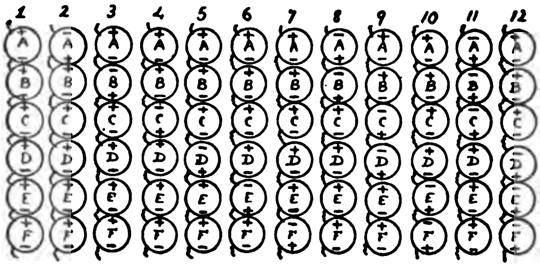

FIG. 221.

and so in turn each cell is to be opposed to the rest (as in 4, 5, 6, and 7). Then A and B are both to be reversed (as in 8), then C and D (as in 9), then E and F (as in 10). The student may be interested to test the equality of the cells by opposing A, B, and C against D, E, and F (as in 11, or as in 12). In repeating the measurements, the connections of the galvanometer should be interchanged (¶ 193, 9), and the measurements should be repeated in the inverse order, to eliminate variations in the strength of the

cells. The results are to be reduced as in ¶ 197, below.

¶ 197. **Reduction of Results of Calibrating a Tangent Galvanometer.** — In (1) we have six cells in series; in (2), (3), (4), (5), (6), and (7), we have in each case one cell opposed to five others or the equivalent of four cells. The *average* deflection gives, therefore, the effect of four cells of the same *average* strength as the six cells in (1). In (8), (9), and (10), we have in each case two cells opposed to four others, or the equivalent of two cells in all; the average deflection corresponds accordingly to two cells of the average strength.

In 11 and 12 there should be little or no deflection. Since the galvanometer is sensitive to the direction as well as to the magnitude of the current, the deflections in 11 and 12 should be equal and opposite.

The results are arranged in tabular form below:

1. No. of cells acting.	2. Average deflection.	3. Tangent of deflection.	4. Ratio of 3 to 1.
6	56°.5	1.511	.252
4	45°.3	1.011	.253
2	27°.1	.512	.256

We notice that the path of the electrical current is the same in all the arrangements, except that in some cases it passes through a given cell in one direction, in other cases in the opposite direction. It is stated that the electrical resistance of a cell is the same, regardless of the direction of the current.[1]

[1] Work is required to drive a current backward through a cell, whereas if a current passes through it in the ordinary direction, the cell is a source of power (see § 137). In calculating the *electrical re-*

The total electrical resistance is accordingly the same in each of the twelve arrangements shown in Fig. 221. It is also stated that the electro-motive force of a battery is proportional to the number of cells acting, hence by Ohm's law (§ 138) the ratio of the numbers in the third column to those in the second column should be nearly constant. If it is not, the galvanometer should be discarded for accurate purposes. The experiment should be repeated with a galvanometer in which the Law of Tangents is at least approximately fulfilled.

¶ 198. **Determination of the Constant of a Single-Ring Galvanometer.** — It is evident from formula 4, ¶ 195, that the deflection of a galvanometer depends

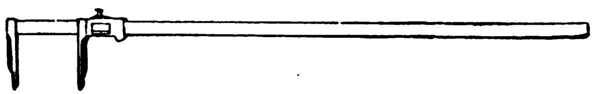

FIG. 222.

not only upon the electrical current, but also upon the length and radius of the coil of wire through which it flows. In order to measure currents with a galvanometer, it is therefore necessary to determine

sistance of a cell we do not consider the gain or loss of power due to chemical agency, but only the loss of power due to conversion into heat. The statement that the resistance of a cell is the same without regard to the direction of the current does not mean, therefore, that it is as easy to drive a current backward through it as to drive it forward, but that the cell would be *equally heated* in both cases. The truth of this statement has recently been called into question, but the method of calibration described above has been found practically to yield accurate results.

accurately the dimensions of the coil of wire. To find the diameter of a coil, we measure with a long vernier gauge (Fig. 222) the distance between the flanges of a bobbin (*al*, Fig. 223) upon which the coil is wound. Then we find the thickness of two blocks *ab* and *kl* which fill the space between the wires and the edges of the flanges. Subtracting *ab* and *kl* from *al* we have the outside diameter (*bk*) of the coil. We now measure the width of the bobbin and the width of the flanges. Subtracting the latter from the former, we have the width of the coil of wire. The whole number of turns of wire is now to be counted. Usually the groove is broad enough for one more turn of wire than that actually wound upon it, since this amount of space is necessary for turning the wire. The width of the groove is to be divided by the number of turns which would fill it, to find the average diameter (*bc*, or *jk*) of the wire. Subtracting this from the outside diameter (*bk*) we have the mean diameter (*bj*, or *ce*) of the coil. Dividing by 2 we have the mean radius of the coil.

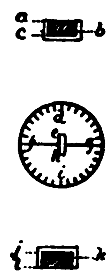

FIG. 223.

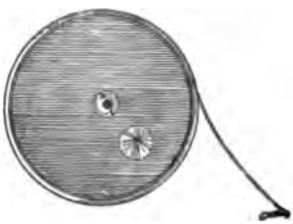

FIG. 224

Instead of measuring the diameter of the coil, we may find its circumference by passing a thin steel tape graduated in *mm.* around the outside of the coil. If *c* is the circumference, the outside diameter is

$c \div \pi$. From this the mean diameter and radius may be calculated as before. The results are to be still further reduced as in ¶ 199.

¶ 199. **Calculation of the Constant and Reduction Factor of a Tangent Galvanometer.** — The constant (K) of a coil of wire is equal to the ratio of its length to the square of its radius (§ 133). That is, in the notation of ¶ 195,

$$K = \frac{L}{R^2}. \qquad (1)$$

Substituting this value in formula 4, ¶ 195, we have

$$\frac{CK}{10\,H} = tan\ a, \qquad (2)$$

or solving for C,

$$C = 10\,\frac{H}{K}\,tan\ a. \qquad (3)$$

The constant, K, of a given galvanometer is therefore an important factor in the calculation of a current from the deflection which it produces in that galvanometer.

If n is the number of turns in the coil,[1] we have

$$L = 2\,\pi\,n\,R, \qquad (4)$$

which substituted in (1) gives

$$K = \frac{2\,\pi\,n\,R}{R^2} = \frac{2\,\pi\,n}{R}. \qquad (5)$$

[1] The student must remember that when a coil is made in two parts, so that half the current flows through each, the effect is the same as if the whole current flowed through one half. The total number of turns must therefore be halved in order to find the effective number n.

By this formula the constant of the tangent galvanometer is to be calculated. Thus for 6 turns of radius 18 *cm.* we have a constant $2 \times 3\frac{1}{7} \times 5 \div 18$, or 1.75, nearly. With such a galvanometer, assuming that the horizontal intensity of the earth's magnetism is 0.175, nearly, we should have from (3) —

$$C = 10 \times \frac{.175}{1.75} \tan a = \tan a \text{ (nearly)};$$

that is, the current in ampères would be numerically equal to the tangent of the angle of deflection produced.

In most galvanometers this is not the case. To find the current, we have to multiply the tangent of the angle of deflection by some factor, which may be greater or less than unity. This is called the *reduction factor* of the galvanometer.[1]

Denoting it by I, we have from (3) —

$$I = 10 \frac{H}{K}. \qquad (6)$$

It is important to find the reduction factor of a galvanometer which is to be used often, since it greatly shortens the reduction of results.

Substituting from (6) in (3) we have simply —

$$C = I \tan a. \qquad (7)$$

It may be observed that if $a = 45°$, so that *tan a*

[1] Some writers call the reduction factor "the constant" of a galvanometer. Since the reduction factor depends upon the earth's magnetism (see 6), it is evidently not constant. The effect of changes in the earth's magnetism in a short course of experiments may, however, generally be disregarded.

$= 1$, we have $C = I$. The reduction factor of a galvanometer is therefore numerically equal to the current which deflects it 45°; that is, the current which produces a field of force at the centre of the coil equal to the horizontal component of the earth's magnetism.

EXPERIMENT LXXIX.

COMPARISON OF GALVANOMETERS.

¶ 200. **Construction of a Double-Ring Tangent Galvanometer.** — A "double-ring" tangent galvanometer is represented in Fig. 225, also in horizontal section in

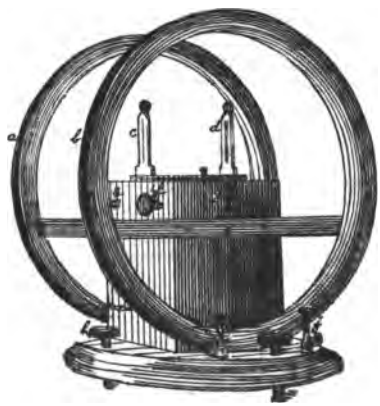

Fig. 225.

Fig. 226. It consists of two parallel coils of wire wound on brass or wooden rings *a* and *b*, with a surveying-

compass *cd* between them (see also Fig. 199, ¶ 183). In the case of a single-ring galvanometer, it has been stated that the length of the needle should not exceed $\frac{1}{12}$ the diameter of the coils. In the double-ring galvanometer, it may be $\frac{1}{4}$ of this diameter without introducing any serious error into the results (Kohlrausch, Art. 93). For measuring battery currents, each coil should contain about six turns of No. 12 insulated copper wire. It is recommended that the average diameter of the coils should be 32 *cm.* and the mean distance between them 16 *cm.*[1] The needle of the surveying-compass should be not more than 8 *cm.* long. When a current is made to divide in such an instrument into two parts, so that half flows through each coil, it is found that the tangent of the angle of deflection is approximately equal to the magnitude of the current in ampères.

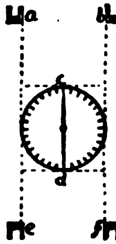

FIG. 226.

¶ 201. **Determination of the Reduction Factor of a Galvanometer by the Method of Comparison.** — The single-ring galvanometer (Fig. 217) is to be adjusted with its coil north and south (¶ 193, 6), as near as possible to the place (¶ 193, 3) where the horizontal intensity of the earth's magnetism was determined (¶ 183). The double ring galvanometer (Fig. 225) is to be similarly adjusted in some position conven-

[1] These dimensions have been calculated for places where the horizontal component of the earth's magnetism is .169 or .17 nearly. In places where this horizontal component is nearly .18 the dimensions should be 30 and 15 *cm.* respectively.

ient for future measurements. This position should be accurately noted. The two instruments (*A* and *C*, Fig. 227) are then to be connected in series with a constant battery (B) capable of yielding a current of one or two ampères. The deflection of each galvanometer is to be found by reading both ends of each needle (¶ 193, 5). The connections of *C* are then reversed (see ¶ 193, 9), and both deflections again noted. The connections of *A* are next reversed and new readings taken. Finally the connec-

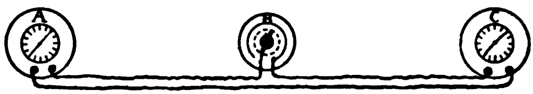

FIG. 227.

tions of *C* are again reversed, so as to be the same as at the start,—the needles being read as before.

The observations of the two galvanometers should be made at the same time, as nearly as possible. Let a be the average angle through which *A* is deflected; a' that through which *C* is deflected; then if the reduction factors (¶ 199) of *A* and *C* are I and I' respectively, the current C which traverses both galvanometers must be (see ¶ 199, formula 7) —

$$C = I \tan a = I' \tan a';$$

hence the reduction factor (I') of *C* may be found by the equation —

$$I' = I \frac{\tan a}{\tan a'}.$$

We notice that the reduction factors of two galvanometers are to each other *inversely* as the tangents.

of the angles of deflection produced by a given current.

The student should be cautioned *not* to connect the two galvanometers in multiple arc (§ 140); for in this case the current divides into two parts, which may or may not be equal. Not knowing the ratio between the two parts, we can draw no conclusion as to the relative sensitiveness of the two galvanometers.

When the instruments are connected as above *in series*, the same current (if there is no leakage) must traverse the coils of both.

EXPERIMENT LXXX.

THE DYNAMOMETER.

¶ 202. **Construction of a Dynamometer.** — A form of dynamometer useful for measuring battery currents is represented in Fig. 228. It consists of a wooden bobbin, *fgpn*, with two grooves, in each of which are wound 50 turns of No. 16 insulated copper wire. Small holes are bored through the bobbin at *f*, *g*, *n*, and *p*, so that it is possible to measure directly the inner and outer diameters of the coil. The average diameter is about 25 *cm*.

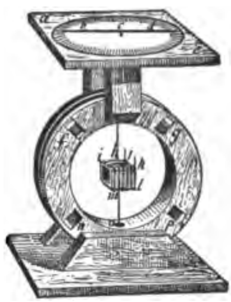

Fig. 228.

A small hollow wooden cube (*ijkl*), measuring 5 *cm.* each way, is now wound with $80\frac{1}{2}$ turns of No. 24 copper wire, the ends of which

are connected by No. 31 spring brass wires (*ch* and *mo*) to a fixed point beneath, *o*, and to the centre (*c*) of a knitting needle (*bd*), as in the torsion balance (see Fig. 176, ¶ 165). The length of the wire should be taken so that the coefficient of torsion of the wire *ch* may be some round number, let us say 10 dyne-centimetres, per degree (see ¶ 165). Thus if 100 *cm.* of the wire has been found (Exp. 64) to have a coefficient of torsion of 2 dyne-centimetres per degree, we may make *ch* just 20 *cm.* long, so that it may exert a couple of $\frac{100}{20} \times 2 = 10$ units per degree.

It will be observed that the constant of the large coil, having in all 100 turns, and a mean radius of 12·5 *cm.*, is (see ¶ 133) —

$$K = \frac{2 \times 3.1416 \times 100}{12.5} = 50, \text{ nearly,} \qquad (1)$$

while the magnetic area of the smaller coil is (see § 134) —

$$A = 80\tfrac{1}{2} \times 5 \times 5 = 2000, \text{ nearly.} \qquad (2)$$

The constant of the dynamometer is accordingly (§ 135) —

$$D = 50 \times 2{,}000 = 100{,}000 \text{ absolute units, nearly.} \quad (3)$$

In other words, a current of 1 *absolute unit* would create a couple of 100,000 units, tending to twist the wire. A current of 1 ampère (being $\frac{1}{10}$ of the absolute unit) will have $\frac{1}{10}$ the effect, not only in the cube (*ijkl*), but also in the large coil (*fgpn*). The couple produced, depending upon the product of these two effects (see §§ 133, 134), will be accord-

ingly less than D (in formula 3), in the proportion of 100 to 1. It follows that 1 ampère will exert in this instrument a couple of about 1000 dyne-centimetres; and that it will require a twist of 100° in the wire *ch* to balance it if, as has been supposed, 1° corresponds to 10 dyne-centimetres. Since the couple produced is proportional to the square of the current (§ 135), the current must be proportional to the square root of the angle of torsion which is required to balance this couple.

The proportions of the dynamometer have been chosen above so that the square root of the number of degrees indicated by the needle *bd* may give at once (approximately at least) the current in tenths of an ampère.

¶ 203. **Determination of the Constants of a Dynamometer.** — Before making use of a dynamometer to measure electrical currents, it is necessary to find (1) the constant of the large coil (*fgpn*, Fig. 228), (2) the magnetic area (§ 134) of the small coil (*ijkl*), and (3) the coefficient of torsion of the wire.

(1) The diameter of the large coil may be determined as in ¶ 198; but as the coils of the dynamometer contain several layers of wire, it is more accurate to measure directly the outside and inside diameters. For this purpose holes are made at *f*, *g*, *n*, and *p*, in the side of the bobbin. The number of turns, if unknown, may be estimated by counting the layers and the number of turns in each. From the whole number of turns and from the mean diameter of the coil, the constant (K) is to be calculated as in ¶ 199.

(2) To find the mean diameter of the square coil, the outside diameters *jk* and *kl* are to be measured by a Vernier gauge. The diameter of the wire is to be found by measuring the width of the 80 or more turns between *i* and *j*, then dividing by the number of turns. Subtracting this diameter from the outside diameters *jk* and *kl*, we have the mean diameter of the coil. Unless a wire passes through the middle of the cube in the direction *co*, it is obvious that there must be a whole number of turns plus one half turn on the cube *ijkl*. To avoid making a mistake, the turns should be counted on both sides of the cube. The magnetic area, *A*, of the square coil is then calculated as in § 134.

(3) The instrument is now to be laid upon its side, and a light balance-arm is to be attached to the cube (see Fig. 176, ¶ 165). The wire *ch* will probably have to be supported near *h* to prevent it from sagging under the weight of the cube. The wire should, however, rest freely upon the support, so as not to affect the torsion. The coefficient of torsion of the wire *ch* is then to be found as in ¶ 165.

¶ 204. **Determination of Reduction Factors by means of a Dynamometer.** — The Dynamometer is now to be set upright with the plane of the large ring north and south, and adjusted by twisting the needle *bd* so that the planes of the large and small coils are at right-angles. A fixed mark should be placed on the wall of the room so as to be in line with two sights *jk* on the small coil, when the coil is at right-angles to the large coil. The reading of the needle is to be ob-

served. The instrument is then to be connected (as in ¶ 201) in series with a single-ring tangent galvanometer, and with a battery of several Bunsen cells, capable of sending a current of about 1 ampère through the circuit. The needle *bd* is to be turned until the sights *j* and *k* on the small coil come in line with the same mark as before. The reading of the needle is to be again observed, and also that of the tangent galvanometer.

The current is now to be reversed in the large coil, but not in the small coil of the dynamometer; then reversed in the battery; then the original connections of the dynamometer are to be restored. In each case readings of the dynamometer and of the galvanometer are to be made.[1]

If t is the coefficient and a the angle of torsion of the wire, the couple is ta. If K is the constant of the large coil, A the magnetic area of the small coil, we have for the current c, by § 135 —

$$c = \sqrt{\frac{ta}{KA}}, \text{ in absolute units;}$$

or in ampères, $$C = 10\sqrt{\frac{ta}{KA}},$$

since an ampère is one tenth of an absolute unit.

From the current, C, and the mean deflection, d, which it produces in the tangent galvanometer, we

[1] The couple produced by a current may also be measured by turning the instrument on its side as in ¶ 208, 8, and directly counterpoising the current with weights placed in one pan of the balance.

may find the reduction factor of the latter by the formula —

$$I = \frac{C}{tan\ d}.$$

We may also find the horizontal component (H) of the earth's magnetism by the formula —

$$H = \frac{IK}{10},$$

derived from ¶ 199, 6, using the new value of I.

If the values of I and H found by means of the dynamometer differ from those previously determined (Exps. 74 and 78) by more than 5 or 10 %,[1] the student should repeat all the measurements upon which these values depend.

EXPERIMENT LXXXI.

ELECTRO-CHEMICAL METHOD.

¶ 205. **Determination of the Reduction Factor of a Galvanometer by the Electro-Chemical Method.** — The galvanometer is to be adjusted with the plane of its coil parallel to the magnetic needle (¶ 193, 5), and its exact position noted (¶ 193, 3). The terminals

[1] The use of a small square coil in a dynamometer is simply for convenience in the explanation of the instrument to students. For accurate measurements, a round coil is to be preferred. In any case there are certain corrections to be applied to the dynamometer on account of the size and shape of its coils (unless these be carefully proportioned) which if neglected may account for errors of 3 or 4 %.

of the galvanometer (*h* and *i*) are to be connected with the poles of a Daniell cell, *a* and *b* (Fig. 229, 2), through a commutator *defg* (see ¶ 193, 9). The ordinary copper (or positive) pole is replaced by a spiral of copper wire (*b*, Fig. 229, 1 and 2) with a coupling *c*, provided for convenience in weighing. The spiral should have been cleaned with nitric acid before the experiment. The solution of sulphate of copper with which it is surrounded should be saturated and free from all impurities, especially acid, ammoniacal, and oxidizing or reducing agents. The deflection of the galvanometer should be about 45,

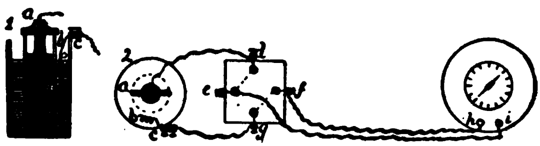

Fig. 229.

— more rather than less. If it is less than 30° the porous cup should be changed, or another cell substituted. When the spiral has been freshly coated with copper by the action of the battery, it should be disconnected from the coupling (*c*), dipped in three changes of fresh water, then in alcohol, and dried in a temperature not exceeding 100°, to avoid oxidation of the copper. Its weight is then to be found within a milligram, if possible, by a series of double weighings (Exp. 8).

The spiral is now to be replaced in the cell, and connected with the galvanometer as before. The time

when the connection is made must be accurately noted. The deflection of the galvanometer is to be recorded at intervals of one minute. Each end of the needle should be alternately observed (¶ 193, 5). At the end of 25½ minutes the commutator *defg* is to be suddenly turned (see ¶ 193, 9) so that the current through the galvanometer may be reversed. Observations of the galvanometer needle are to be continued, at intervals of one minute, for another 25 minutes. There will thus be 50 observations in all. At the end of 50 minutes and 50 seconds, exactly, the current is to be suddenly cut off. The copper spiral is to be cleansed in three changes of water, with care not to dislodge any of the fresh deposit, then dipped in alcohol, dried, and reweighed accurately as before. The results are to be reduced as in ¶ 230.

¶ 206. **Theory of the Electro-Chemical Method.** — It has been found that a current of 1 ampère deposits 1 gram of copper in the course of 50 minutes and about 50 seconds (the total duration of the experiment). The strength of the solution has little or no effect upon the result, always provided that *enough* copper is present in it (§§ 142, 143). The amount of copper deposited varies only with the strength and duration of the current.

If C is the strength of the current in ampères, t the time in seconds, and w the weight of copper deposited, we have accordingly —

$$w = \frac{Ct}{3050}, \text{ nearly,} \qquad (1)$$

and $$C = \frac{3050\ w}{t}, \text{ nearly.} \qquad (2)$$

If, as in the experiment, $t = 50$ minutes and 50 seconds, that is, 3050 seconds, we find simply —

$$C = w. \qquad (3)$$

That is, the average value of a current in ampères is numerically equal to the weight in grams of copper deposited by it in 3050 seconds.

Now from ¶ 199, 7, we have, at any point of time,

$$C = I \tan a, \qquad (4)$$

where a is the angle of deflection produced by the current in a tangent galvanometer, and I is the reduction factor of the galvanometer. Hence, averaging the different results from the 50 observations of the needle, we find, comparing (3) and (4) —

$$w = \text{average of } I \tan a. \qquad (4)$$

In practice, if the angles do not differ by more than 10 %, the same result (nearly) may be obtained much more easily by averaging the angles themselves, then finding the tangent of this average. That is, if A is the average angle of deflection —

$$w = I \tan A, \text{ nearly.} \qquad (6)$$

The reduction factor may now be calculated by the formula —

$$I = \frac{w}{\tan A}. \qquad (7)$$

Having found the constant, K, of the galvanometer (¶ 199, 1), we may calculate the horizontal com-

ponent (H) of the earth's magnetism, as in ¶ 204, by the formula (derived from ¶ 199, 6) —

$$H = \frac{IK}{10}. \tag{8}$$

If the value of H obtained by the electro-chemical method does not agree with previous determinations (Exps. 74, and 80), the last experiment (Exp. 81) should be repeated until at least 3 results, obtained either by the same or by different methods, agree within let us say 5 %. All previous measurements leading to a different result should now be repeated.[1]

EXPERIMENT LXXXII.

METHOD OF VIBRATIONS.

¶ 207. **Construction of a Vibration Galvanometer.** — A form of galvanometer easily constructed is represented in Fig. 230. It consists of a coil *cfg* (made by winding 14 turns of No. 18 insulated copper wire upon a hoop of wood, brass, or pasteboard, 10 *cm.* in diameter) with a short magnetized needle *e*, attached to a bullet *d* and suspended at the centre of the coil by a fine waxed fibre (*cd*) of untwisted silk (see ¶ 186). The strength of the magnet and the weight of the bullet should be proportioned so that the

[1] The student will do well to examine his *calculations* before repeating the measurements upon which they depend. A common error is a miscount or misconception of the number of turns of wire utilized in the coil of a galvanometer or dynamometer, particularly when the coils are connected in multiple arc. See footnote, ¶ 199.

needle may complete 10 vibrations in about 1 minute. A short test-tube may be employed to cut off currents of air (see Fig. 204, ¶ 186).

The ends of the coil may be carried to binding-posts, f and g. Connections at f and g may also be made by simply twisting the wires together (¶ 193, 11).

When an ordinary battery current is sent through the coil, the magnetic field of force created by the current will greatly increase the rate of vibration of the needle. We have seen (¶ 186 and § 110) that a field of magnetic force is proportional to the *square*

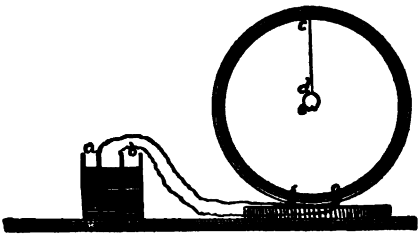

FIG. 230.

of the number of vibrations which it produces in a magnetic needle. In accordance with this law, the dimensions of the instrument have been chosen so that the square of the number of vibrations completed in 1 minute may represent approximately the strength of the current in thousandths of an ampère.

In calculating these proportions, it was assumed that the needle made exactly 10 vibrations per minute under the influence of the earth's magnetism, the strength of which was taken as 0.176 dynes per unit

of magnetism (see Exps. 72, 73, 74, 80, and 81). No allowance was made for the effects of magnetism induced in the needle, which (unless the needle be of the best steel and highly magnetized) may account for errors of 5 or 10 per cent with currents of 1 or 2 ampères. To obtain accurate results with a vibration galvanometer, it would be necessary both to calibrate it (see ¶ 196) and to compare it (as in Exp. 79) with a galvanometer of known reduction factor. When, however, as in this experiment, the instrument is to be used for rough work and for relative indications only, such tests need hardly be applied.

The influence of the earth's magnetism upon the vibration galvanometer must be allowed for, as will be explained in ¶ 209.

¶ 208. **Determination of the Relative Strength of Battery Currents by means of a Vibration Galvanometer.** — A vibration galvanometer (¶ 207) is to be set up with the plane of its coil vertical, but (contrary to the usual custom, ¶ 193, 5) at right-angles with the magnetic meridian. The time required for 10 vibrations of the needle (which should be about 1 minute) is now to be accurately determined. The needle may be set in vibration by bringing a magnet near it, then suddenly taking the magnet away. The arc of vibration should not exceed 30 or 40 degrees (see Table 3, *g*).

The terminals of the galvanometer, *f* and *g*, are now to be connected respectively with the poles, *a* and *b*, of a battery constructed as will be described below. The student must notice carefully whether the needle

points in the same direction as before, or whether the needle is reversed. In the latter case the connections of the galvanometer with the battery should be interchanged; that is, *f* should be connected with *b*, and *g* with *a*.

The number of vibrations made in 1 minute (or whatever time was required for 10 vibrations under the earth's magnetism) is now to be accurately determined. In no case should the arc of vibration exceed 30 or 40 degrees.

The battery to be employed in this experiment consists of a glass tumbler, half-filled with dilute sulphuric acid[1] (10 % by weight), a porous cup with an internal diameter not less than 5 *cm.*, containing a solution of sulphate of copper, and two strips, one of sheet zinc, the other of sheet copper, each 5 by 10 *cm.* Connecting wires should be soldered to both strips. The current from this battery is to be tested under the following conditions:

(1) When the zinc and copper strips are placed side by side in the sulphuric acid, but not touching each other.

(2) The same after the zinc has been amalgamated by rubbing it with mercury.

(3) (4) (5) The same after the current has been allowed to flow for five, ten, and fifteen minutes respectively.

(6) The same except that the bubbles gathered

[1] To avoid accidents in mixing sulphuric acid with water, the acid should be poured in a fine stream into the water, so that the heat generated may be quickly dissipated.

on the copper strip have been removed by a camel's-hair brush, without exposing the copper to the air.

(7) The same, except that the copper has been exposed for a few minutes to the air.

(8) The same except that the copper has been amalgamated by being rubbed with nitrate of mercury.[1]

(9) The zinc and copper strips are now to be carefully weighed; the zinc is to be replaced in the sulphuric acid, but the copper is to be immersed in the solution of sulphate of copper contained in the porous cup, and the latter is to be placed in the tumbler containing the acid.[2]

(10) (11) (12) The same after the current has been allowed to run for five, ten, and fifteen minutes respectively. The zinc and copper strips are now to be reweighed. The results are to be reduced as will be explained in the next section.

¶ 209. **Reduction of Results obtained with the Vibration Galvanometer.** — It has been stated that the square of the number of vibrations completed in one minute by a vibration galvanometer constructed as in ¶ 207, gives approximately the current to which these vibrations are due in thousandths of an ampère. To find, accordingly, the current in ampères, we square the number of vibrations produced in the given length of time, and divide by 1000.

[1] Copper may also be amalgamated by dipping it into nitric acid, then rubbing it with mercury by means of a cloth. Care must be taken not to let nitric acid come in contact with the hand.

[2] This combination constitutes a Daniell cell. See also Fig. 235, ¶ 211.

It must not, however, be forgotten that the earth's magnetism alone accounts for about 10 vibrations per minute. The earth's field is accordingly equivalent to that produced in the vibration galvanometer by $\frac{100}{1000}$ or 0.1 ampère. Care should have been taken in the experiment to have the earth's magnetism and the current acting always in the same direction. In this case all the results will be too great by 0.1 ampère. By subtracting this amount in each case, the effect of the earth's magnetism will be eliminated.

The strength of each current in ¶ 208, (1) to (12), should be calculated roughly in this way.

The student will notice that the visible action of the sulphuric acid on the zinc is arrested by amalgamating the zinc with mercury; that the action begins again when the zinc is connected with the copper strip, but that the bubbles of gas are then set free from the copper instead of from the zinc; that the amalgamation of the zinc does not impair the usefulness of the battery; that the current steadily decreases when both strips are in sulphuric acid, though it is temporarily increased by removing the bubbles from the copper, and by exposing the copper to the air; that amalgamation of the copper does not prevent the formation of bubbles upon it, nor improve in any way the action of the battery; that the formation of bubbles is arrested by placing the copper in the solution of sulphate of copper, and that in this case the battery furnishes a steady current; that the zinc plate loses in weight, but that the copper

plate gains in weight by a nearly equal amount,[1] owing to fresh copper deposited upon it. We have already made use (in Exp. 81) of the quantity of copper thus deposited to measure an electrical current.

EXPERIMENT LXXXIII.

THE AMMETER, I.

¶ 210. **Testing an Ammeter.**—The name "ammeter" (an abbreviation of ampère-meter) is given to any instrument indicating directly the strength of electrical currents in ampères. Ammeters are manufactured in various forms. Most of them depend upon the attraction which an electrical current, circulating in a coil of wire (*b*, Fig. 231), exerts upon a permanent magnet or upon a core of soft iron. In some instruments this electro-magnetic attraction is balanced by a spring, in others by gravity; in others again it is balanced by the attraction of a permanent magnet (*c*). Such instruments depend for their accuracy upon the constancy of the magnet, and even if correctly graduated at the start, are subject to errors which may be indefinitely great. Recently instruments have been

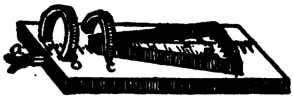

FIG. 231.

[1] If we assume that there is no wasteful action of the battery, the quantities of zinc dissolved and of copper deposited should be to each other as the atomic weights of zinc and copper, 64.9 and 63.1 respectively.

manufactured in which currents are measured by the attraction between two coils of wire traversed by the same electrical current. Such instruments are properly called electro-dynamometers (see Exp. 84). If carefully graduated, they may serve as standards for the determination of electrical currents.

Ammeters are usually intended to measure currents of at least 10 ampères, and being generally sen-

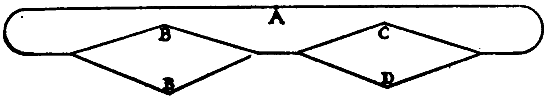

Fig. 232.

sitive only to about $\frac{1}{10}$ ampère, they cannot measure small currents very precisely. On the other hand, the tangent galvanometers described in ¶ 194 and ¶ 200 are intended to measure currents of a few ampères only. To compare an ammeter with such instruments, it must be connected with two or more of them in *multiple arc* (§ 140). A powerful battery

Fig. 233.

of three or four Bunsen cells is then included in the circuit. A diagram of connections is given in Fig. 232, where A represents the ammeter, BB the battery, C and D two galvanometers. To avoid the influence of the connecting wires upon the instrument (¶ 193, 8), the arrangement would practically be made as in

Fig. 233. The battery cells are represented in both diagrams (Figs. 232 and 233) as being connected in multiple arc (§ 140), since in this way they usually yield the greatest current through instruments of low resistance (§ 146).

If a, a', &c., are the deflections of the galvanometers; I, I', &c., their reduction factors, the currents through them are respectively $I \tan a$, $I' \tan a'$, &c. Hence the total current C is —

$$C = I \tan a + I' \tan a' + \&c.$$

The experiment should be repeated with batteries containing different numbers of cells, or the same number differently arranged, so as to produce currents of from 1 to 10 ampères.

The results should be tabulated in the ordinary manner, in three columns, containing respectively, (1) the current calculated from the galvanometer deflections; (2) the current indicated by the ammeter, and (3) the corresponding correction of the ammeter.

EXPERIMENT LXXXIV.

THE AMMETER, II.

¶ 211. **Determination of Battery Currents by means of an Ammeter.** — The electrical resistance (§ 136) of ammeters is usually so slight that it may be neglected. To measure the maximum current which a battery can produce, the screw-cups of the ammeter are to be connected by short thick copper wires with the pole-

cups of the battery in question. The wires should be parallel or twisted together, as in the last experiment (see Fig. 233), and scraped bright at both ends (¶ 193, 11). The indication of the instrument is to be noted.

With any instrument of the class known as ammeters, the student is to determine the maximum current which can be derived from various well-known forms of voltaic battery, as, for instance, the Bunsen

FIG. 234.

FIG. 235.

FIG. 236.

cell (Fig. 234), the Daniell cell (Fig. 235) and the Leclanché cell (Fig. 236). The observations may be continued in each case at intervals of five minutes for half an hour.[1] The material employed in each cell, and the dimensions of every part,[2] should be

[1] An old Leclanché cell may be employed for this experiment. It may serve subsequently for experiments with Wheatstone's Bridge, but for other purposes it will be rendered nearly useless.

[2] If a sufficient current cannot be obtained from a single cell of a given sort, two or more cells should be employed. The student should notice that with instruments like the ammeter having a very low resistance, it is more effective to arrange batteries in multiple arc than in series. See § 136, also Figs. 232 and 233, ¶ 210.

carefully noted. The corrections for various currents indicated by the ammeter have been found in the last experiment. The proper correction should be applied to each reading. The results are to be represented by a series of curves (Fig. 237) plotted on the same sheet of coordinate paper. A scale at the top of the paper indicates the time in minutes, and a scale at the left of the paper represents the current in ampères. Each curve should be marked with the name of the cell or battery to which it belongs.

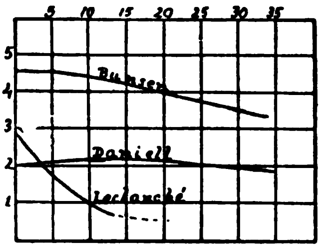

FIG. 237.

ELECTRICAL RESISTANCE.

EXPERIMENT LXXXV.

METHOD OF HEATING.

¶ 212. **Determination of Resistances by the Method of Heating.** — A short spiral (*a*, Fig. 238) of fine German silver wire, .01 *cm.* diameter (about No. 36) and 15 *cm.* long, is soldered to the two terminals *b* and *c* of two insulated copper wires, *d* and *e*, passing through a cork fitting the inner cup of a calorimeter (*B*, Fig. 239). The wires (*bd* and *ce*) should be so thick that their electrical resistance may be neglected in comparison with that of the spiral. The cork and wires are then inverted and placed in the calorimeter (*B*, Fig. 239) containing a sufficient quantity of distilled water to cover the spiral. The temperature of the water, which should be slightly below that of the room, is found by a series of observations (¶ 92, 10) made with a thermometer passing through the cork as in Fig. 239. The thermometer is provided with a stirrer (see ¶ 65, Fig. 50) so that a uniform temperature may be maintained.

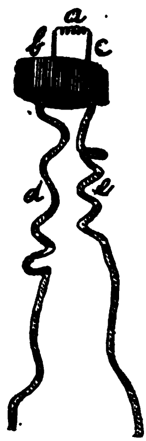

FIG. 238.

The instrument thus constructed (*B*, Fig. 239) is

to be connected in series with a Bunsen cell (A) and with a tangent galvanometer (C) adjusted in the same place and manner as in Exp. 83.

The time when the connection is made must be accurately noted. The tangent galvanometer is to be observed at intervals of one minute. Between the observations, the water in the calorimeter is to be stirred by twisting the stem of the thermometer. When the temperature reaches that of the room, the direction of the electrical current is to be suddenly reversed by interchanging the battery connections (see ¶ 193, 9). The observations of the galvanometer are

FIG. 239.

to be continued until the temperature of the water rises as high above that of the room as it was originally below it. Then the circuit is to be broken. The time when the current is interrupted must be accurately recorded. Several more observations of the temperature within the calorimeter are to be made at intervals of one minute, so that the resulting temperature may be accurately determined.

The weight of the calorimeter and of the water which it contains are finally to be found by weighing the calorimeter with and without the water.

¶ 213. **Calculation of Resistance by the Method of Heating.** — Let w be the weight of water, and W that of the calorimeter from which its thermal capacity

c is to be calculated,[1] and let t_1, and t_2 be the temperatures of the water at the moment when the circuit was first made and finally broken. These temperatures are to be inferred from the observations made before and after the experiment (see ¶ 93, 2). Since the average temperature of the water agrees with that of the room, no allowance need be made for cooling in the mean time (¶ 93, 3). The quantity of heat, H, generated by the electrical current is therefore —

$$H = (w + c) \times (t_2 - t_1).$$

Now let T be the time in seconds during which this heat was generated; then the average rate at which the heat was generated must have been $\frac{H}{T}$ units per second. Since 1 unit of heat per second corresponds to a power of 4.166 watts (§ 15), the power, P, spent by the electrical current, in watts, is —

$$P = 4.166 \frac{H}{T} = \frac{4.166 (w + c) \quad (t_2 - t_1)}{T}. \qquad \text{I.}$$

We now calculate the average current, C, in ampères, from the angles of deflection (a) averaged as in ¶ 206, and from the reduction factor of the galvanometer, I, already determined (Exps. 78–81) by the formula —

$$C = I \tan a. \qquad \text{II.}$$

We have finally, by Joule's Law (§ 136) for the resistance, R, of the conductor in ohms —

$$R = \frac{P}{C^2}. \qquad \text{III.}$$

[1] If the calorimeter is of brass, its thermal capacity is .094 W. nearly. To this should be added about 0.5 units for the thermal capacity of the thermometer and stirrer. See ¶ 90 (2).

If the experiment were varied so as to make the current just 1 ampère, then, since $C = I$, R would be equal to P. This is in accordance with the definition of resistance (§ 136). The student should bear in mind that the resistance of a conductor in ohms is nothing more or less than the power in watts required to maintain in that conductor a current of 1 ampère.

EXPERIMENT LXXXVI.

COMPARISON OF RESISTANCES.

¶ 214. **Construction of a Rheostat.** — A rheostat may be constructed as in Fig. 240. A series of brass blocks (IJ) is firmly attached to a plate of ebonite

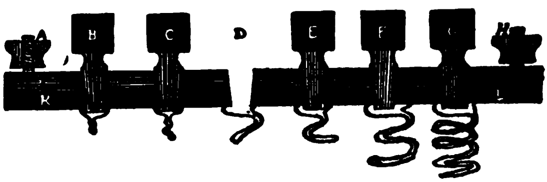

FIG. 240.

(KL), which is a non-conductor of electricity. The brass blocks are connected by coils of German-silver wire, which should be well insulated with silk. Each wire should be doubled in the middle (see Fig. 240), and the double wire should be coiled up or wound on a bobbin. The equal and opposite currents in any part of the coil thus neutralize each other as far as

external magnetic effects are concerned.[1] Brass plugs *B*, *C*, &c., are fitted into hollows between the blocks, so as to make good electrical connections. When all the plugs are in place, a current flowing through the blocks in series from the binding-post *A* to the binding-post *H*, should meet with a hardly appreciable resistance. If, however, one of the plugs (as *D*) is removed, the current is obliged to pass through one of the coils. It meets therefore, with a certain electrical resistance.

The resistance of the first coil in the series is usually 1 ohm (§ 20); that of the second is 2 ohms; the third and fourth are either 2 and 5 or 3 and 4 ohms. It is thus possible, by taking out one or more plugs at the same time, to introduce resistances from 1 to 10 ohms into the path of a current. The series of resistances may be extended by adding three new coils of 20, 20, and 50 ohms' resistance. With seven coils, we may thus obtain any resistance from 1 to 100 ohms. With three more coils of 200, 200, and 500 ohms resistance, we may extend the limit to 1000 ohms. With additional coils of 0.1, 0.2, 0.2, and 0.5 ohms, the resistance may be adjusted to a tenth of an ohm, &c. For convenience, extra coils of 1, 10, 100, and 1000 ohms are usually provided. The same results may be obtained by the series 1, 2, 3, 4, 10, 20, 30, &c. The line of resistances is usually bent, as in Fig. 241, so as to occupy as little space as possible. Connections with the two ends of the series are made

[1] The effects of "self induction" should also be to a great extent eliminated by this method of winding the coils.

by means of the binding-posts *a* and *d*. It is convenient for many purposes to include an entirely separate line of resistances, *befc*, in the arrangement. In the first part of this experiment the inner line will not be required. It should therefore be entirely disconnected from the outer line by the removal of the plugs which join the two lines together.

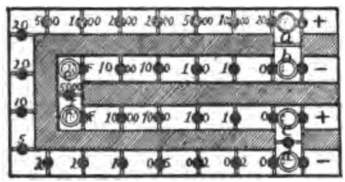

Fig. 241.

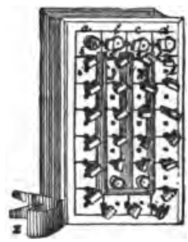

Fig. 242.

Both series of resistances are usually packed in a box (Fig. 242), variously called a "box of coils," a "resistance box," or simply a "rheostat."

¶ 215. **Determination of Resistances by the Method of Substitution.** — To find the electrical resistance of any conductor, as for instance the coil of the dynamometer employed in the last experiment, the coil

Fig. 243.

mometer employed in the last experiment, the coil (*C*, Fig. 243) is to be connected in series with a battery (*B*) and a tangent galvanometer (*G*). The deflection of the galvanometer is to be carefully observed. The dynamometer is now to be disconnected,

and in its place a rheostat, *R* (Fig. 244), is to be introduced into the circuit by means of the binding-posts *c* and *d*. The plugs connecting the inner and outer lines of resistance are to be removed, so that the current can circulate only through the outer line. The plugs along this line should all be driven lightly into place, and turned round in their sockets, so as to make good electrical connections. Enough plugs are now to be removed to reduce the deflection of the galvanometer to its former magnitude.

The resistance in ohms brought into play by the removal of each plug is indicated by the number op-

Fig. 244.

posite its socket (Fig. 241). If the first resistance tried is too small, that is, if it fails to reduce the current sufficiently, one about twice as great is tried; if the first resistance is too large, we try one about half as great. In fact we use with a set of resistances the same method of approximation as with a set of weights (¶ 2).

In the process of trying the several resistances, the current from the battery is liable to change. It is well, therefore, to replace the dynamometer in the circuit, and having observed the galvanometer, to substitute immediately the box of resistances (as previously adjusted) for the dynamometer. When two conductors

can be thus substituted one for the other in an electrical circuit without affecting the current, their electrical resistances are evidently equal according to the general principle of substitution (see § 43). We have only, therefore, to add together the resistances of those coils in the box through which the current flows, in order to find the resistance of the dynamometer.

To save time in making connections, the terminals of the coil *C* may be carried to the binding-posts *a* and *e* of the rheostat (Fig. 245). One of the battery wires is then carried to *d*, the other to the galvanometer *G*, and back to *f*. Plugs connecting *b* with *c*,

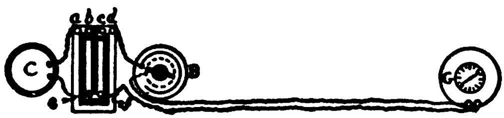

Fig. 245.

b with *e*, and *c* with *f*, are to be removed; the others are to remain. The binding-posts *e* and *f* are thus insulated from the rest of the instrument. The battery current then flows from *d* to *a* through the outer line of resistances, then from *a* to *e* through the coil *C*, then through *f* to the galvanometer *G* and back to the battery. If *b* and *c* be now connected by the insertion of a plug, the current will flow directly from *d* to *a*, and thus the rheostat resistance will be "cut out of the circuit." If the plug connecting *b* and *c* be removed and inserted between *b* and *e*, the current, after flowing through the outer line of resistances,

will make a short circuit from b to e, instead of passing through the coil C. The coil will therefore be "cut out of the circuit." By moving a single plug, accordingly, from one place to another, the rheostat may be substituted in the circuit for the dynamometer, and *vice versa*. The accuracy of the units indicated by the box of resistances may be provisionally taken for granted.

¶ 216. **Determination of Resistances by the Method of Interchange.**—A battery, B, (Fig. 246), is to be connected with a coil, C, of unknown resistance, and with a rheostat, R, of variable resistance in multiple arc (§ 140). The wires from the coil and from the rheo-

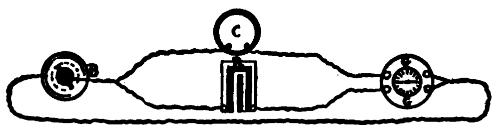

FIG. 246.

stat are to be carried back to the battery, each through one half of a differential galvanometer, $G\,G$. The resistance of the rheostat is to be adjusted if possible, by the removal of plugs, so that the deflection of the galvanometer may be reduced to zero. Since this occurs when the currents through the two halves of the galvanometer are equal, the total resistance in the two branches of the circuit containing C and R must be equal. Assuming therefore that the two halves of the galvanometer and the connecting wires have equal resistances, the resistance of the coil C must be equal to that of the rheostat R.

To make sure that the two halves of the galvanometer are exactly alike, the positions of the coil (C) and rheostat (R) should now be interchanged, and the resistance of the rheostat readjusted if necessary.

In the absence of a set of resistances by which the rheostat may be adjusted within, let us say, $\frac{1}{10}$ of an ohm, two adjustments must be made. In one, the resistance (R_1) of the rheostat will be too small, and the galvanometer will be deflected $x°$ in one direction. In the other adjustment the resistance (R_2) of the rheostat will be too great, and the galvanometer will be deflected $y°$ in the opposite direction.

The resistance (R) sought can evidently be found by the ordinary method of interpolation (§ 41, ¶ 26), that is —

$$R = R_1 + \frac{x}{x+y}(R_2 - R_1), \text{ nearly.}$$

In the absence of a differential galvanometer, the student should make by the method of substitution (¶ 215) as many determinations of resistance as time will allow. Other methods of comparison will be considered in experiments which follow.

EXPERIMENT LXXXVII.

WHEATSTONE'S BRIDGE.

¶ 217. **Determination of Electrical Resistances by a Wheatstone's Bridge.** — A form of Wheatstone's Bridge used by the British Association and ordinarily known

as the "B. A. Bridge," is represented, with slight modifications, in Fig. 247, which gives a view of the apparatus from above. Three strips of copper, *ab*, *ce*, and *fg*, are arranged in a line on a piece of wood, with small spaces between them. A fine German-silver or platinum wire *hj*, often called the "Bridge wire"[1] is stretched over a rail 1 metre long, graduated in *mm*. The wire is soldered at both ends to corners of the strips (*ab* and *fg*), which are turned up so as to be on a level with the wire. A cross-wire is attached to a slider (*i*, Fig. 248) so that it may be made to touch the wire *hj* at any point. Binding-posts are usually added at *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *i*. The latter serves to connect any conductor (as *Gi*) with the cross-wire, and thus to make an electrical connection between it and any point of the wire *ij*.

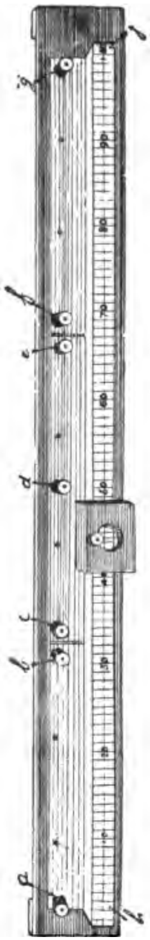

FIG. 247.

The terminals of a delicate galvanometer *G*, (see also ¶ 188, Fig. 207) are to be connected with the binding-posts *d* and *i*. The resistance coil *C*, tested in Exp. 85, is to connect *b* and *c*. Two binding-posts (*a* and *d*, Fig. 242)

[1] To avoid misconceptions arising from this name, it may be well to point out to the student at the start that the "Bridge wire" is *not* the "*Wheatstone's Bridge*" (§ 141).

of the rheostat used in Exp. 86 (*R*, Fig. 248) are to be connected by thick copper wires with *e* and *f* (Fig. 248). One of the plugs is to be removed from the rheostat, so as to give a resistance of 1 ohm. The poles of a battery (*B*) are then to be connected with the binding-posts, *a* and *g*.

The current from the battery is thus made to divide into two parts. One part flows from *a* to *d* through the coil *C*, then from *d* to *g* through the resistance *R* (or the reverse); the other part flows from *a* to *i*, through the resistance of the wire *hi*; then from *i* to *g* through the resistance of the wire

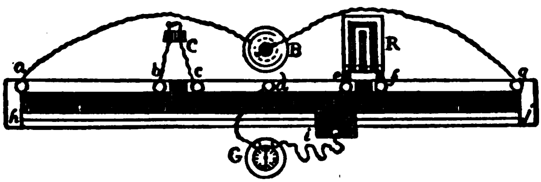

FIG. 248.

ij (or the reverse). The resistance of all other conductors may be neglected. The galvanometer circuit forms a cross-connection or "Wheatstone's Bridge" (§ 141) between the points *d* and *i* of the parallel circuits *adg* and *aig*. The points *a*, *d*, *g*, and *i* correspond accordingly to *A*, *B*, *C*, and *D* in Fig. 18, § 141. The slider *i* is to be moved from one end of *hj* to the other until a point *i* is found having the same potential as *d* (§ 141), so that the galvanometer shows no deflection. The distances *hi* and *ij* are to be carefully measured. The poles of the battery are next to be interchanged and the experiment repeated.

The average of the distances hi and ij is to be found. Assuming that the wire is uniform, the resistance of these portions A and B will be to each other as their lengths, hi and ij. That is —

$$\frac{A}{B} = \frac{hi}{ij}.$$

The resistance C is now calculated from the resistance R in the box of coils (1 ohm in this case) by the formula (§ 141) —

$$C = R \times \frac{hi}{ij}. \qquad \text{I.}$$

The experiment is to be repeated with the places of C and R interchanged. In this case the formula will become —

$$C = R \times \frac{ij}{hi}. \qquad \text{II.}$$

By removing from the box of coils different plugs, other measurements of the resistance C may be made. The student should satisfy himself that with various values of R, the same value of C is always obtained. The most accurate value is usually that which is found when R is nearly equal to C.

If the value of C thus determined differs by more than 10 % from that found in the last experiment, the latter should be repeated. By this means gross errors in the box of coils may be found out. It should be remembered that the British Association Unit which is copied in many boxes of coils is only about 987 thousandths of a true ohm.

EXPERIMENT LXXXVIII.

SPECIFIC RESISTANCE.

¶ 218. **Specific Resistance.** — The specific electrical resistance of a given material may be defined as the resistance of a conductor made of that material, 1 *cm.* long and 1 *sq. cm.* in cross-section. In the practical units of the volt-ohm-ampère series, the specific resistance, S, is equal accordingly to the electromotive force in volts (see § 138) required to maintain a current of 1 ampère between two opposite faces of a centimetre cube cut out of a given substance; or again, it is equal to the *power in watts* (see § 137) required to do the same thing. The power required to maintain a current of 1 ampère through L centimetre-cubes of the substance, arranged in series, so that the same current traverses each, is obviously LS watts. If we place Q rows of centimetre-cubes side by side, each row containing L of the cubes, it is obvious that to maintain a current of 1 ampère in each row will require LS watts; hence the total power required for all the rows will be QLS watts.

Since each row is traversed by a current of 1 ampère, the compound conductor, consisting of Q rows, must carry a current of Q ampères.

The resistance of this conductor may now be calculated by Joule's Law ($P = C^2 R$, see § 136);

for substituting QLS for P, and Q for C, we have —

$$R = \frac{P}{C^2} = \frac{QLS}{Q^2} = \frac{LS}{Q}. \qquad \text{I.}$$

We notice that in the formula L represents the length and Q the cross-section of the compound conductor. The resistance of any conductor is accordingly proportional to its length, and inversely as its cross-section. To find it, we multiply the specific resistance by the length and divide the product by the cross-section. Obviously, specific resistances of different materials are important factors in calculations relating to electrical resistance.

To calculate specific resistance (S), we must first find the actual resistance (R) of a conductor of known length (L) and cross-section (Q); we then have, from I., —

$$S = \frac{RQ}{L}. \qquad \text{II.}$$

It will be found convenient to express the result in terms of microhms (§ 2) instead of ohms. This is done by moving the decimal point six places to the right (*i. e.*, multiplying by 1,000,000).

¶ 219. **Determination of Specific Resistance.** — A fine German-silver wire (not insulated), about 1 metre long, is soldered (near *a* and *b*, Fig. 249) to two copper strips. These strips are to be so thick that their electrical resistance may be neglected. They are to be scraped bright (¶ 193, 11), and connected with the binding-posts *b* and *c* of a Wheatstone's bridge

apparatus, in place of the coil used in the last experiment (see Fig. 248, ¶ 217). To prevent the wire from crossing itself at any point, it may be looped round a glass jar *a* (Fig. 249). The resistance (R) of the wire is to be found as in the last experiment.

The wire is now to be straightened, and the distance *between* the copper strips accurately determined. This gives the length (L) of the conductor spoken

FIG. 249.

of in the last section. The diameter (d) of the wire is to be measured at let us say ten different points with a micrometer gauge (¶ 50, II.), and the results averaged. The cross-section (Q) of the wire is then calculated by the ordinary formula —

$$Q = \tfrac{1}{4} \pi d^2.$$

The specific resistance of the German silver of which the wire is composed is finally to be calculated by formula II. of the last section.

The experiment may be repeated with wires of different lengths, diameters and materials.

EXPERIMENT LXXXIX.

THOMSON'S METHOD.

¶ 220. **Determination of the Resistance of a Galvanometer by Thomson's Method.** — The terminals of a galvanometer, *G* (Fig. 250), and of a rheostat, *R*, are to be connected with a Wheatstone's Bridge apparatus in the same manner as any other resistances would be connected, when it is desired to compare them

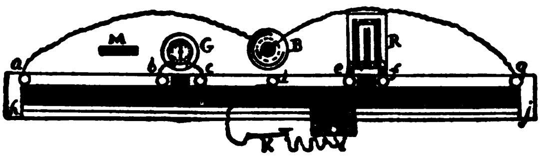

FIG. 250.

together (see Exp. 87). A battery, *B*, is also to be connected in the same manner. Instead, however, of putting a second galvanometer in the circuit *di*, to tell when the current in that circuit is reduced to zero, a simple key, *K*, is placed there.

The galvanometer needle will probably be strongly deflected by the current passing through the instrument. It must be brought back nearly to zero by a powerful magnet, M, properly placed. If the battery is too strong for the magnet, a weaker battery may be substituted, or the same result may be obtained by connecting the poles of the battery with a cross-wire or shunt of sufficiently low resistance. The key is

now to be closed. If the effect is to increase the deflection of the needle, the slider (i) is to be moved toward that end of the "Bridge wire" (hj) nearest the galvanometer. If the effect is to diminish the deflection, the slider is to be moved toward the rheostat. Finally a point (i) is found where the closing of the key has no effect upon the galvanometer. The resistance of the latter is then calculated as in the last experiment.

The experiment is to be repeated with a rheostat resistance as nearly as possible equal to that of the galvanometer. The current should be reversed, and the resistances interchanged as in Experiment 87.

The resistance of the galvanometer is to be calculated by one of the formulæ of ¶ 217.

¶ 221. **Explanation of Thomson's Method.** — Thomson's method of measuring the resistance of a galvanometer depends upon the fact that when the circuit di (Fig. 250) is closed through K, more or less current will ordinarily pass from i to d, or the reverse.

The electrical potential (§ 139) of the point d will therefore be affected, just as the pressure at a given point in a water pipe would be affected by connecting that point with one in another pipe where the pressure was different. Since the current from a to d depends (according to Ohm's Law, § 138) upon the difference of potential between those points, it is evident that if a retains the same potential as before, any change in the potential at d must affect the current. The deflection of the galvanometer is accordingly increased or diminished. The object of nearly neutral-

izing the deflection is that any change in it may be made perceptible; for if the needle were already deflected for instance 89°, since 90° is the maximum possible deflection, it would be hard to detect an increase in the current. We have seen that the electrical potential at *d* is changed when it is connected with a point *c* at a different potential; obviously if *d* and *i* are at the *same potential,* there will be no change in the potential of *d*, and hence no change in the deflection of the galvanometer. The student should note that we may find a point *i*, having the same potential as a point *d*, either (1) by observing the deflection of a galvanometer in the circuit *di* (see Exp. 87), or (2) by observing the *change* in the deflection of a galvanometer in any other branch of the compound circuit.

The chief difficulty in this experiment lies in the arrangement of a permanent magnet so as to neutralize the deflection of a galvanometer needle without destroying temporarily the sensitiveness of the instrument. The advantage of this method, aside from its theoretical interest, is chiefly in cases where it is impossible to obtain a second galvanometer sufficiently sensitive to measure the resistance of the first.

EXPERIMENT XC.

MANCE'S METHOD.

¶ 222. **Determination of the Internal Resistance of a Battery by Mance's Method.** — A rheostat (*R*, Fig. 251) and a galvanometer (*G*) are to be connected with a Wheatstone's Bridge apparatus as in Experiment 87; and a battery cell (*B*) is to be put in place of the unknown resistance (*C*, Fig. 248). Instead, however, of placing a second battery in the circuit *ag*, a simple key (*K*) is put there.

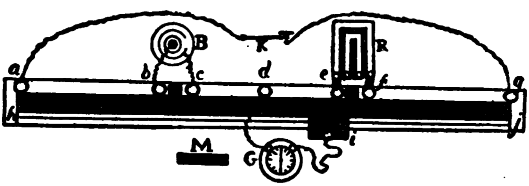

FIG. 251.

The needle of the galvanometer will probably be strongly deflected by the current passing from *d* to *i*, or the reverse. As in the last experiment, this deflection must be nearly reduced to zero, by bringing a powerful magnet (*M*) near the galvanometer. A shunt may be introduced if necessary between the terminals of the galvanometer (see ¶ 193, 2). The key is now to be closed. If the deflection of the galvanometer is increased, the slide (*i*) is to be moved toward the battery. If the deflection is diminished, it should be moved toward the rheostat. The change in the

position of the slider will probably throw the galvanometer and magnet out of adjustment. The position of the magnet must therefore be changed. After a series of trials the slider may be placed at a point i, where no *sudden* effect is produced upon the galvanometer by closing the key.

If the galvanometer is affected one way when the key is first closed, then the other way, the first effect is the one by which the adjustment of the slider is to be made.

The experiment is to be repeated with a resistance in the rheostat as nearly as possible equal to that of the battery; but the methods of reversal and interchange employed in Exp. 87 will hardly be justified by the accuracy of the experiment. The resistance of the battery is to be calculated by one of the formulæ of ¶ 217.

¶ 223. **Explanation of Mance's Method.** — The effect in Mance's method of the battery current upon the galvanometer has generally to be diminished by shunting the galvanometer. The opposite difficulty however, sometimes arises. When it is desired to measure the resistance of a battery composed of two nearly equal cells, opposed to one another, the current from these cells may be insufficient to affect the galvanometer. In this case an auxiliary battery must be introduced into the circuit *akg*. We will first suppose that such an auxiliary battery is employed. If the two cells of which the resistance is to be measured *exactly* neutralize each other, the case differs from that of an ordinary Wheatstone's Bridge

only in the nature of the resistance which is to be measured. The theory is therefore the same.

If, however, one of the two cells is stronger than the other, an allowance must be made for the current which flows from the battery (B) through the galvanometer, whether the auxiliary battery is connected or not. This is done by neutralizing the deflection of the galvanometer due to the battery B.

The fundamental principle upon which Mance's method depends is that two batteries in any system of conductors, however complicated, produce each the same effect as if the other were not present. The current in any part of the circuit is in fact the algebraic sum of the two currents which the batteries would separately produce. We have seen that a battery in the circuit *akg* affects a galvanometer in the circuit *di*, unless the resistances *ai* and *ij* are proportional to *ad* and *dg* respectively. If a current already exists in the galvanometer a *change* in that current must be produced by a battery in the circuit *akg*, unless the proportion above is fulfilled.

Let us now suppose that the battery in the circuit *akg* is just strong enough to neutralize the current from the battery B, which would naturally flow through the circuit *akg*. Then the effect of introducing this battery into the circuit may be simply to arrest the current in *akg*. The same effect is produced by breaking the circuit by means of the key K. Evidently the act of opening or closing the key in a circuit is equivalent to connecting or disconnecting a battery of considerable strength.

When the circuit is made the resistance between the poles of the battery is much less than when the circuit is broken. The result is an increased current from the battery, and in a very short time a change in its electromotive force. The observations should, therefore, be taken the moment that the circuit is closed. The galvanometer needle sometimes first jumps in one direction, then slowly changes to the other direction. The slow movement in the needle may be explained as the result of a gradual change in the electromotive force of the battery. The first effect indicates which of the resistances is too great or too small.

The chief advantage of Mance's method is that it enables us to measure the resistance of batteries at a given instant while furnishing a current. Concordant results must not be expected between Mance's and other methods. It is now thought that there is something not yet understood in the nature of battery resistances which causes these resistances to appear to be greater or less according to the manner in which they are determined.

EXPERIMENT XCI.

USE OF A SHUNT.

¶ 223. **Determination of the Resistance of a Galvanometer by means of a Shunt.** — I. Two tangent galvanometers (*ab* and *gh*, Fig. 252) already employed

in Exp. 79, are to be set up in the same places as in that experiment, and connected in series with a battery (B) capable of causing deflections of from 50° to 60°. The connecting wires *bcdeg* and *afh* are to be made bare at a point between the two galvanometers and at a point (e) between the galvanometer (gh) and the battery. The wires are to be clamped at these points by the binding-posts of a rheostat (R). All the plugs are now to be put into their places. The galvanometer *gh* will then be short circuited through the rheostat (R). The deflection of the galvanometer should accordingly fall to 0°. If it does not, the plugs in the rheostat should be turned

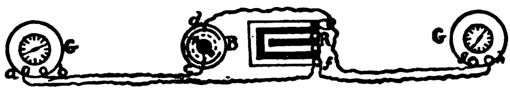

FIG. 252.

round in their sockets with light pressure until at least a minimum deflection is obtained.[1]

When plugs are removed from the box of coils, a part only of the current will flow through the rheostat. The galvanometer (gh) will then be deflected. Plugs are to be removed from the box until the deflection of the galvanometer (gh) reaches about 30° or a little more than half the deflection of *ab*. The resistance of the rheostat is to be noted, and the deflections of the two galvanometers are to be simultaneously determined as in Exp. 82. This method

[1] The plugs should be carefully cleaned if necessary by rubbing them with paper.

is applicable to galvanometers of low resistance. The results are to be reduced by ¶ 224, I., formula (5).

II. Instead of the galvanometer *ab*, a second rheostat resistance may be introduced into the circuit *edcbaf*. The value of this resistance is to be noted. The deflections of the galvanometer *gh* must be observed (as in I.) with and without the shunt *ef*. The resistance of the shunt must also be noted.

This method requires a constant battery (see Exp. 84), with an internal resistance which is either known (see Exps. 92 and 93) or so small that it may be neglected in comparison with the resistance in the circuit *edcbaf*. The method is used in practice only in the case of high-resistance galvanometers. On account of the extreme sensitiveness of such instruments, the current from an ordinary voltaic cell must be reduced by the use of a very large resistance in the circuit *edcbaf*. In comparison with this resistance, that of the voltaic cell may usually be neglected. The resistance of the shunt should be such that when connections are made through it, the deflection of the galvanometer may be about half as great as when these connections are broken. The results are to be reduced by ¶ 224, II., formula (12).

¶ 224. **Calculations of Resistance depending upon the Use of a Shunt.** — I. If I and i are the reduction factors of the two galvanometers, A and a their deflections, then since the whole current, C, passes through the first galvanometer (*ab*, Fig. 252), it must be given by the equation (see formula 7, ¶ 199) —

$$C = I \tan A. \qquad (1)$$

Only a portion (c) of this current passes through the second galvanometer (gh); this portion is —

$$c = i \tan a. \qquad (2)$$

The remainder (c') of the current flows through the rheostat. Evidently —

$$c' = C - c = I \tan A - i \tan a. \qquad (3)$$

Now the current (c) through the galvanometer (gh) must be to that (c') through the shunt inversely as the resistances (let us say G and S) in question (§ 140). That is —

$$c : c' :: S : G. \qquad (4)$$

The resistance of the galvanometer (G) may therefore be found by the formula —

$$G = \frac{c'\,S}{c} = S\,\frac{I \tan A - i \tan a}{i \tan a}. \qquad (5)$$

It should be remembered that the resistance of the galvanometer (gh, Fig. 252), calculated by this formula, includes that of the wires, eg and fh, connecting it with the rheostat. The result is rendered inaccurate by any bad connection within the rheostat. A minimum deflection of 1° in the galvanometer (gh), produced with all the plugs in place in the rheostat (R), indicates an under estimate of both the galvanometer and rheostat resistances not far from 1 or 2 %.

II. If E is the electromotive force of the battery (B, Fig. 252), R the resistance in the circuit $edcbaf$ (including strictly the internal resistance of the battery), and if G is the resistance of the galvanometer,

the current, C, produced (when the connection between e and f is broken) must be (see § 138) —

$$C = \frac{E}{R + G}. \tag{1}$$

If now a connection is made between e and f through a shunt of the resistance S, so that the current flows partly through G and partly through S, the resistance (r) of this multiple circuit will be (solving the equation in § 140) —

$$r = \frac{GS}{G + S}. \tag{2}$$

The current C' now becomes —

$$C' = \frac{E}{R + r}, \tag{3}$$

or, substituting the value of r and reducing, —

$$C' = \frac{E\,(G + S)}{RG + RS + GS}. \tag{4}$$

The portion (c) of this current which flows through the galvanometer is to the whole current (C') as S is to $G + S$ (§ 140); that is —

$$c = C' \frac{S}{G + S}. \tag{5}$$

Substituting the value of C' from (4) we have —

$$c = \frac{E\,(G + S)}{RG + RS + GS} \times \frac{S}{G + S} \text{ or}$$

$$c = \frac{ES}{RG + RS + GS}; \tag{6}$$

hence $$E = \frac{cRG + cRS + cGS}{S}. \tag{7}$$

But from (1) $$E = CR + CG;$$

hence $$\frac{cRG + cRS + cGS}{S} = CR + CG, \tag{8}$$

$$cRG + cRS + cGS = CRS + CGS, \tag{9}$$

$$cRG + cGS - CGS = CRS - cRS, \tag{10}$$

and $$G\,(cR + cS - CS) = RS\,(C - c), \tag{11}$$

whence, finally, $$G = \frac{RS\,(C - c)}{cR + cS - CS}. \tag{12}$$

In the use of this formula it is necessary to know only the relative values of the currents C and c. With nearly all instruments, when the deflections are small, the currents are proportional to these deflections. We may accordingly substitute the deflections produced in such cases for the currents which they represent.

EXPERIMENT XCII.

OHM'S METHOD.

¶ 225. **Determination of the Resistance of a Battery by Ohm's Method.** — A tangent galvanometer (G, Fig. 253) and a rheostat (R) are to be connected in series by the wires bc, de, and af, with a Daniell cell (B) capable of deflecting the galvanometer needle 50° or 60° when all the plugs of the rheostat are in their

places. The deflection of the galvanometer is to be accurately observed. The 1-ohm plug is now to be removed from the rheostat, and the deflection again noted. The resistance of the rheostat is then gradually increased until the deflection of the galvanometer is reduced to less than half of its original magnitude. In each case, the deflection is to be carefully observed, and the resistance noted.

The connections at *b* and *f* being now interchanged (¶ 193, 9) so that the direction of the current through the galvanometer is reversed, the experiment is to be repeated. If any differences are observed in the deflections corresponding to a given resistance,

FIG. 253.

the mean angle of deflection is to be calculated in each case.

If a_1 and a_2 are the mean angles of deflection in any two cases, R_1 and R_2 the corresponding rheostat resistances, C_1 and C_2 the currents through the galvanometer, I the reduction factor of the galvanometer (Exps. 78, 80, 81), B the resistance of the battery, galvanometer, and connecting wires, then we have (see ¶ 199, 7) —

$$C_1 = I \tan a_1 \quad (1); \quad C_2 = I \tan a_2. \qquad (2)$$

Now by Ohm's law (§ 138) these currents are inversely as the corresponding resistances, that is —

$$C_1 : C_2 :: R_2 + B : R_1 + B, \qquad (3)$$

hence we find —

$$\frac{R_2 + B}{R_1 + B} = \frac{C_1}{C_2}, \qquad (4)$$

$$R_1\, C_1 + B C_1 = R_2\, C_2 + B C_2, \qquad (5)$$

$$B C_1 - B C_2 = R_2\, C_2 - R_1\, C_1, \qquad (6)$$

$$B\,(C_1 - C_2) = R_2\, C_2 - R_1\, C_1, \qquad (7)$$

$$B = \frac{R_2\, C_2 - R_1\, C_1}{C_1 - C_2}, \qquad (8)$$

and finally, substituting the value of C_1 and C_2, and cancelling I, we have —

$$B = \frac{R_2 \tan a_2 - R_1 \tan a_1}{\tan a_1 - \tan a_2}. \qquad (9)$$

The student may thus calculate several values of B. The best value for R_1 is 0; that is, we obtain the most accurate results by utilizing the observation of the galvanometer when all the plugs are in place. Evidently if $R_1 = 0$, the value of B becomes simply

$$B = \frac{R_2 \tan a_2}{\tan a_1 - \tan a_2}. \qquad (10)$$

The best value for R_2 is one nearly equal to B. The simplest way to find this value is to calculate the value of B from any two of the observations. It must be remembered that the battery resistance thus calculated includes that of the galvanometer and connecting wires. Having found the resistance

of the galvanometer, &c. from the last experiment, we may find by subtraction the *internal* resistance of the battery. The results with a tolerably constant battery should agree with those obtained by Mance's Method (Exp. 90) within 5 or 10 %.

The calculation of the electromotive force of a battery from the results of Ohm's Method will be considered in ¶ 230. It may be remarked that if this electromotive force is not constant, formula (3) is not justified. In this case the succeeding formulæ which depend upon (3) may give false or even absurd results.

EXPERIMENT XCIII.

BEETZ' METHOD.

¶ 226. **Explanation of Beetz' Method.** — In Beetz' method two batteries, B' and B'' (Fig. 254) are placed in the same circuit $(abcda)$ but so as to be op-

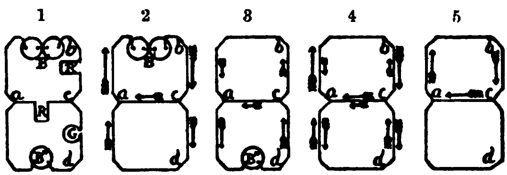

Fig. 254.

posed to each other; and the circuit is divided into two lobes, like a figure 8, by means of a wire ac, acting as a shunt to both batteries. A known resistance R' is placed between b and c; another known resistance (R) is introduced between a and c; a delicate

galvanometer (G) is placed between c and d. We will suppose that the two positive poles of the batteries are connected at c.

Let us now consider what effect the battery B' would produce if B'' were not acting. The current descending in the branch bc would divide into two parts (Fig. 254, 2); one flowing directly from c to a, the other indirectly from c to a through d. These two parts would unite at a, and thence return to the battery.

Let us next consider what effect B'' would produce if B' were not acting. The current ascending in dc (Fig. 254, 3) would divide into two parts; one flowing directly from c to a, the other indirectly from c to a through b. Both parts uniting at a would return to the battery.

When both batteries act together, each may be considered to produce the same effect as if the other were not acting. The result is represented in Fig. 254, 4. We notice that in the diagrams the *portion* of the current from B' which flows through d is as great as the *whole* current from B''. To produce this effect it is evident that the battery B' must be stronger than B''. It is also evident that two equal and opposite currents through d must neutralize each other; hence the result of combining two batteries as in Fig. 254 may be such as is represented in Fig. 254, 5; namely, a current entirely confined to the circuit bc, containing the stronger battery, *no current whatever flowing through the weaker battery.*

In practice we employ a battery, B', *more than suffi-*

cient to reverse B''; then we weaken the current which it sends through the circuit d, either by increasing the resistance R', so that the whole current from B' is reduced, or by diminishing the resistance R, so that a greater portion of the current may flow directly from c to a, without passing through the battery B''. The use of the galvanometer, G, is simply to tell when an exact balance has been established between the two opposing currents through d (see Fig. 254, 4). No current is then indicated by the galvanometer.

It is possible to calculate by Ohm's Law (§ 138) and by the principle of divided circuits (§ 140) the magnitude of each of the currents represented in Fig. 254, 4, and thus to find under what conditions the currents through d are equal and opposite. The expressions become, however, more or less complicated. The final solution, which is simple, may be obtained much more easily by the method which follows.

¶ 227. **Principle of Electromotive Forces in Equilibrium.** — Let E' be the electromotive force, and B' the resistance of the first battery; let E'' be the electromotive force of the second battery (B''), and let C be the current through the rheostat R. Then if, according to the diagram (Fig. 254, 5) the current through B'' has been reduced to zero, the current C, having no choice of circuits must flow through B' and R' as well as through R. The result is the same as if the circuit through B'' did not exist. We have accordingly an electromotive force E', causing a cur-

rent C through a total resistance $R + B' + R'$. Hence, by Ohm's Law (§ 138), —

$$E' = C\,(R + B' + R'). \qquad (1)$$

The power of the battery is spent in heating the several resistances R, B', and R'. We need to consider only the power (P) spent in heating the resistance R. We have (see § 136) —

$$P = C^2\,R. \qquad (2)$$

The ratio of this power (P) to the current (C) determines that part (E) of the whole electromotive force (E') which is required to maintain the current (C) through the resistance (R) in question: Since in passing through the resistance R the loss of potential is E, we have (see §§ 137, 138, and 139) —

$$E = \frac{P}{C} = \frac{C^2\,R}{C} = CR. \qquad (3)$$

The power spent by the battery B'' upon a small current C'' flowing through it in the ordinary direction (from a to c) will be $C''\,E''$ (§ 137); but the power required to take electricity from a point a to a point c, where the electrical potential is higher than at a by the amount E, is $C''\,E$. Evidently such a current through the battery can exist only on condition that E'' is greater than E.

On the other hand, a current C'' flowing from c to a would represent an expenditure of power equal to $C''\,E$. The power required to drive the current backward through the battery B'' is, however, $C''\,E''$.

Evidently a reversed current can exist only if E is greater than E''. It follows that if E and E'' are equal, the current through B'' will be reduced to zero. It is evident, conversely, that if the galvanometer in the diagram (Fig. 254, 1) shows no deflection, E and E'' must be equal; that is (from 3), —

$$E'' = CR; \tag{4}$$

from which we find —

$$C = \frac{E''}{R}, \tag{5}$$

a formula by which we may calculate the current from a battery (B') which, flowing through a known resistance, R, neutralizes a known electromotive force, E''.

¶ 228. **Calculation of Battery Resistances in Beetz' Method.** — For the determination of the resistance of a battery by Beetz' method, two experiments are necessary. Let r_1 and r_1' be the values of R and R' (¶ 226) in the first experiment, and let r_2 and r_2' be the corresponding values in the second experiment. Then from ¶ 227 we have, dividing (1) by (4), —

$$\frac{E'}{E''} = \frac{B + r_1 + r_1'}{r_1}, \tag{1}$$

and

$$\frac{E'}{E''} = \frac{B + r_2 + r_2'}{r_2}. \tag{2}$$

Assuming that the proportion between E' and E'' is the same in both experiments, we have, equating (1) and (2), —

$$\frac{B + r_1 + r_1'}{r_1} = \frac{B + r_2 + r_2'}{r_2} \tag{3}$$

$$Br_2 + r_1 r_2 + r_1' r_2 = Br_1 + r_1 r_2 + r_1 r_2' \tag{4}$$

$$Br_2 - Br_1 = r_1 r_2' - r_1' r_2 \tag{5}$$

$$B = \frac{r_1 r_2' - r_1' r_2}{r_2 - r_1}. \tag{6}$$

The same result may be obtained from formula (8), ¶ 225, namely, —

$$B = \frac{R_2 C_2 - R_1 C_1}{C_1 - C_2}, \tag{7}$$

by substituting for the total external resistances R_1 and R_2 their values, $r_1 + r_1'$ and $r_2 + r_2'$ respectively, and also substituting for the two corresponding currents C_1 and C_2 their values (from ¶ 227, formula 5) $\frac{E''}{r_1}$ and $\frac{E''}{r_2}$ respectively. The factor E'' is cancelled in the reduction.

Beetz' method differs from Ohm's method chiefly in the manner in which we estimate the relative strength of two currents. In Ohm's method the ratio between the currents is determined by the angles of deflection produced in a tangent galvanometer. In Beetz' method, it is determined by the resistance between the poles of a constant battery, enabling the current to neutralize the effect of that battery. Beetz' method is essentially a null method (§ 42).

Beetz' method may be used not only to measure the resistance of a battery (see 6), but also, when that resistance has been found, to determine the rela-

tive magnitude[1] of two electromotive forces (see 1 and 2, also ¶ 230, 8).

When the electromotive force of a battery is known, it furnishes us with the means of measuring currents with great precision (see formula 5, ¶ 227).

¶ 229. **Determination of Battery Resistances by Beetz' Method.** — The copper or positive pole (P, Fig. 255) of a battery (B), consisting of two Daniell cells in series, is to be connected by a wire ($PKK'P'$) with the positive pole (P') of a weaker battery (B'). The circuit is to be completed between the negative poles (N' and N) of the batteries through a delicate galvanometer (G) provided with a shunt (S) to pre-

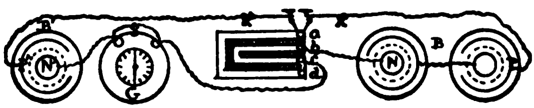

Fig. 255.

vent it from being injured by the battery currents (¶ 193, 2) and through the *inner* line of resistances, bc, of a box of coils. The inner and outer lines, bc and da, are to be connected with a plug between c and d, but separated at a and b throughout the experiment. The wire $PKK'P'$ is to be made bare at a and connected at that point with the binding-post of

[1] If a tangent galvanometer be introduced into the circuit of the stronger battery (B'), for instance between a and b (Fig. 254), so that the current C becomes known, we may calculate also the absolute values of the electromotive forces by formulæ (1) and (4) of ¶ 227. This important modification of Beetz' method is due to Poggendorff. See ¶ 230, 3, and Exp. 99.

the outer line of resistances. Keys (K and K') are to be placed one on each side of a. When all the plugs are in place, and the keys closed, the circuit of the battery (B) is completed through the lines of resistance bc and da, the course of the current being $PKadcbN$. The circuit of B' is also completed through the outer line da, thus: $P'K'adcGN'$. The student should note the direction in which the galvanometer is deflected.

When the connection between a and d is broken by removing the "infinity plug,"[1] both of the circuits named above are interrupted. If the keys K and K' are closed, the batteries will be opposed to one another. Neither battery can furnish a current unless it is strong enough to force it backward against the other battery. If the battery B is stronger than B', the current will follow the course $PKaK'P'N'GcbN$. Since the current in B' is reversed, the galvanometer will be deflected in the opposite direction. The student should make sure that this is the case. If it is not, there is probably some error in the connections, which must be corrected.

The infinity plug is now to be returned to its place, and other plugs removed between a and d.

It will be seen that when the resistance of the

[1] Two of the brass blocks in each chain of resistances should have no metallic connection between them, except that furnished by the plug. When the plug is removed there should be no perceptible current from one block to the other. In other words, the resistance between the blocks should be practically infinite. The plug in question is called accordingly the "infinity plug." It is usually marked ∞ or INF.

outer line *ad*, common to the two battery circuits, is very small, the galvanometer is deflected one way; when the resistance is very large the galvanometer is deflected the other way. The next step is to find, by gradually increasing the resistance, at what point the change in the deflection takes place.

To avoid using up the batteries (¶ 193, 10), the keys K and K' should be left open, except at the moment when it is desired to test the deflection of the galvanometer. The key K in the circuit of the stronger battery is always to be closed first, then the other key, K', immediately after it. As soon as the direction of the deflection has been recognized, the keys are opened in the inverse order.[1]

If the galvanometer is deflected in the same way as when all the plugs are in place, the resistance of the outer line (*ad*) is to be increased; if it is deflected as when the connection in *ad* is broken, the resistance is to be diminished. The sensitiveness of the galvanometer may be increased if necessary by removing the shunt (S) but the student must not forget to replace the shunt before proceeding to the second part of the experiment. The resistance of the outer line (*ad*) causing the deflection of the galvanometer to disappear is to be recorded. If no such resistance can be found, the two nearest resistances should be noted, and the deflections (one in one direction, the other in the other direction) caused by each should be observed. From these results the desired

[1] A "double key" or other mechanical contrivance for closing two circuits one after the other will be found useful in this experiment.

resistance is to be calculated as in ¶ 216, by interpolation (§ 41).

So far the resistance in the inner line *bc* has been zero. This resistance is now to be increased by removing the 10-ohm plug. If the keys be closed, the galvanometer will be deflected. To reduce the deflection to zero, it will be necessary to increase the resistance of the outer line (*ad*). The resistances of both parts of the rheostat (*bc* and *ad*), causing equilibrium in the galvanometer are to be noted.

The battery resistance is to be calculated by formula 6, ¶ 228; remembering that the values of *ad* correspond to the resistances r_1 and r_2, common to the two circuits, while the values of *bc* correspond to the resistances r_1' and r_2', in the circuit of the stronger battery.

ELECTROMOTIVE FORCE.

¶ 230. **Different Methods for the Determination of Electromotive Forces.**

I. ABSOLUTE METHODS. Electromotive force (see § 137) is defined as the ratio of the power spent by any source of electricity to the current which it produces. We must distinguish between methods (1–4) in which the power thus expended is absolutely measured and those (5–12) in which comparative results only are obtained.

(1) METHOD OF HEATING. The power spent by an electric current may be measured in the same way as electrical resistance (Exp. 85), by passing a current from a battery through a coil of wire surrounded with water, and calculating from the rise of temperature of the water how much energy has been spent by the current in a given length of time.[1] If the strength of the current be known, the loss of potential may be found by the general formula (§ 137) —

$$E = \frac{P}{C}.$$

Thus if a current of 2 ampères is found to heat the equivalent of 100 grams of water 15° in 1000 seconds, so that it generates 1½ units of heat in one second,

[1] See Glazebrook and Shaw, Practical Physics, § 74.

since 1 unit of heat per second is equivalent to 4.166 watts (§ 15), $1\frac{1}{2}$ units per second would be equivalent to 6.249 watts, or 6.249 ÷ 2 = 3.124 watts per ampère. We know, therefore, that the difference in potential (§ 139) between the two ends of the coil of wire must be 3·124 volts. It will not do, however, to assume that this is equal to the electromotive force of the battery; for we have left out of account the heat generated by the electrical current in the connecting wires and in the interior of the battery. Unless the electrical resistance of the battery be unusually small in comparison with that of the coil, a considerable portion of the electrical energy will be thus wasted.

At the same time that the method of heating can not in practice be employed to determine *directly* the electromotive force of a battery, it must be remembered that all determinations of electromotive force which involve a measurement of current and resistance may depend *indirectly* upon the method of heating, since this is one of the fundamental methods by which resistances are measured (Exp. 85).

(2) OHM'S METHOD. Having once determined a standard of resistance by the Method of Heating (Exp. 85), we have seen how by various methods of comparison (Exp. 86–93) the resistance of any part of an electrical circuit may be found. In Ohm's method, we find the current (C) in a simple circuit, and calculate the resistance (R) of this circuit by adding together the resistances of its separate parts.

Then, by Ohm's Law, we have for the electromotive force (E) the general equation (§ 138) —

$$E = CR.$$

Substituting in this formula the value of R, which in the absence of any resistance except that of the battery, galvanometer, and connecting wires, is given by formula 10, ¶ 225, namely —

$$B = \frac{R_2 \tan a_2}{\tan a_1 - \tan a_2},$$

and substituting also the corresponding value of C, namely, $I \tan a_1$, we have —

$$E = \frac{IR_2 \tan a_1 \tan a_2}{\tan a_1 - \tan a_2}.$$

The student may show that the same formula is obtained if we multiply the total resistance ($B + R_2$) in the second part of the experiment by the current ($C_2 = I \tan a_2$) which flows through it. The agreement of the two results must not be taken as an indication that the electromotive force is the same in both parts of the experiment, but as the necessary consequence of the formulæ of ¶ 225, in framing which we have *assumed* that the electromotive force of the battery is constant.

(3) POGGENDORFF'S METHOD. It has already been shown in Beetz' method (Exp. 93) that the current from a battery may be neutralized by meeting a counter current caused by division of a current from a more powerful battery into two parts. This is

the principle of Poggendorff's absolute method (see Exp. 99), which differs from Beetz' method simply in the fact that a tangent galvanometer is introduced into the circuit of the more powerful battery (B', Fig. 254) as a means of measuring the current (see note, ¶ 228). Given the current, C, and the resistance, R, the electromotive force (E) is calculated by the ordinary formula (§ 138) —

$$E = CR.$$

(4.) ELECTROSTATIC METHODS. The electromotive force of a powerful battery may be measured by the repulsion between two pith-balls charged by the battery under certain conditions (see ¶ 258). Electrostatic forces are also measured in absolute electrometers of various kinds (see ¶ 270). It should, however, be remembered that results obtained by such instruments are strictly in the electrostatic system. Since the relation between the electrostatic and the ordinary (electromagnetic) systems are not known with any great degree of accuracy, the use of electrometers, as far as the latter system is concerned, is practically confined to the comparison of electromotive forces (see ¶ 230, 11, also ¶ 270).

II. COMPARISON OF ELECTROMOTIVE FORCES. The absolute measurement of electromotive force is, like the absolute measurement of resistance upon which it depends, a more or less difficult problem. The *comparison* of two electromotive forces may, however, be made with a considerable degree of precision.

(5) THE VOLT-METER. Two electromotive forces may be compared by the currents separately produced by them through equal resistances. When the resistance of a battery is unknown, it is evident that this method cannot in general be applied; for the battery resistance may be a considerable part of the resistance of a circuit. In practice, few batteries have a resistance of more than 10 ohms; in fact 1 ohm would be much nearer the average battery resistance. Hence if a galvanometer has a resistance of several thousand ohms, the battery resistance may usually be disregarded. This is the principle on which volt-meters are constructed (Exps. 96 and 97).

(6) WIEDEMANN'S METHOD. In Wiedemann's Method (Exp. 94), two batteries are joined in series with a tangent galvanometer of low resistance. Whether the batteries act in the same or in opposite ways, the total resistance in the circuit is the same (see note ¶ 197). It follows, therefore, from Ohm's law (§ 138), that the current is proportional in one case to the sum, in the other case to the difference of the electromotive forces E and e; hence the sum $(E + e)$ is to the difference $(E - e)$ as the currents C and c produced, that is —

$$E + e : E - e :: C : c.$$

(7) METHOD OF OPPOSITION. Let us now suppose that N cells of the electromotive force E being opposed to N' cells of the electromotive force E' reduce the current to zero, then obviously the electromotive force $NE = N'E'$; or, $E' : E :: N : N'$.

This is a fundamental method of comparing electromotive forces, the usefulness of which is limited only by the difficulty of obtaining enough cells of each kind to make an exact balance. We note that, in this method, we compare the electromotive forces of two batteries when at rest, and not (as in previous methods) when in action. The method of opposition is essentially a "null method" (§ 42) for the comparison of electromotive forces.

(8) BEETZ' METHOD. When, as in Experiment 93, a battery current is neutralized by *part* of the current from a more powerful battery, we cannot find the electromotive force of either battery absolutely, unless, as in (3), the whole current from the stronger battery is measured, as well as the resistance which it traverses between the poles of the weaker battery. We may, however, find the relative electromotive forces from formulæ 1 and 2, ¶ 228. Hence if the electromotive force of one battery is known, that of the other may be determined. It may be remarked that by this method we compare the electromotive force of one battery *when at rest* with that of another *when in action.*[1]

(9) CLARK'S POTENTIOMETER. Again, if a current (C) flowing through a resistance R neutralizes one battery (as in Exp. 93), while the *same current* flowing through a resistance r neutralizes another

[1] By substituting one battery, B, for another, B' (Fig. 254), as the active source in Beetz' Method (Exp. 93) we may compare the two successively with a third electromotive force, B''. This gives us a null method by which we may compare the electromotive forces of two batteries (B and B') *when in action.*

battery (in the same manner), the electromotive forces of these batteries, being CR and Cr respectively, are to each other as R is to r. The proportion between them may therefore be found, independently of any measurement of electrical current. This is the principle of Clark's Potentiometer (Exp. 98), and is undoubtedly the best method of comparing the electromotive forces of two constant batteries *when not in action.*

(10) USE OF CONDENSERS. The relative strength of two batteries may be found by charging a condenser (see ¶ 257) first by one battery, then by the other. The quantity of electricity stored in the condenser is found to be proportional to the electromotive forces in question. It is estimated by discharging the condenser through a ballistic galvanometer, and observing, as in Experiments 76 and 77, the throw of the needle.

(11.) USE OF ELECTROMETERS. The electromotive force of a battery may be determined by connecting the poles with an electrometer (¶ 270); but in order to interpret the indications of the instrument, it must first be calibrated by a series of electromotive forces of known strength. The chief advantage of the use of an electrometer over that of a volt-meter is in the case of inconstant electromotive forces, especially those which disappear as soon as a current begins. The use of a condenser has the same advantage, and is frequently preferable on account of the liability of electrometers to be out of order. Neither instrument is suitable for an elementary class of students.

(12) USE OF AN ELECTRIC SPARK. Electromotive forces may be estimated roughly by the distance which an electric spark can be made to jump (see Table 36). This method is particularly suited for experiments with a Ruhmkorff coil, or other instrument in which large differences of potential exist for an instant only.

EXPERIMENT XCIV.

WIEDEMANN'S METHOD.

¶ 231. **Determination of Electromotive Forces by Wiedemann's Method.** — (1) Two Daniell cells, A and B, one of which (A) has been used in Ohm's method (Exp. 92), are to be connected in series with a tangent galvanometer (C, Fig. 256, 1). The connections are to be such that the cells act together. The deflection of the galvanometer is to be observed. (2) Then the connections of B are to be reversed (Fig. 256, 2), and the deflection again noted. (3) The galvanometer connections are then to be interchanged, and the deflection observed (Fig. 256, 3). (4) Finally the connections of B are to be interchanged, so that the two cells may act together as at first (Fig. 256, 4), and the deflection of the galvanometer determined.

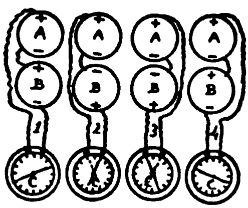

FIG. 256.

Let E be the electromotive force of the stronger cell, and e that of the weaker cell; let A be the average deflection caused by the joint action of the two cells, and C the corresponding current; let a be the average deflection, and c the current produced by the two cells when in opposition; then by formula 7, ¶ 199—

$$C = I \tan A, \tag{1}$$

$$c = I \tan a. \tag{2}$$

Now by Ohm's law (§ 138), as has been explained in ¶ 230, 6, we have—

$$\frac{E+e}{E-e} = \frac{C}{c}, \tag{3}$$

or

$$Ec + ec = EC - eC, \tag{4}$$

whence

$$eC + ec = EC - Ec, \tag{5}$$

or

$$e\,(C+c) = E\,(C-c); \tag{6}$$

from which we find—

$$e = E\,\frac{C-c}{C+c}. \tag{7}$$

Substituting the values of C and c from (1) and (2) and cancelling the factor I, we have—

$$e = E\,\frac{\tan A - \tan a}{\tan A + \tan a}, \tag{8}$$

or

$$E = e\,\frac{\tan A + \tan a}{\tan A - \tan a}. \tag{9}$$

It should be noted that if the reversal of the cell B does not affect the direction of the current,—that is,

if the deflections in Fig. 256, 2 and 3, are in the same direction as in 1 and 4 respectively, — the electromotive force of the cell *B*, being less than that of *A*, is to be calculated by formula 8; but if the reversal of *B* causes a reversal of the current, the electromotive force of *B* is greater than that of *A*, and is hence to be calculated by formula 9. The electromotive force of *A*, already computed, may be found from the results of Ohm's method by the formulæ of ¶ 230, 2. The electromotive force of the two cells combined is now to be calculated by adding *E* and *e* together.

II. The experiment is to be repeated with the battery composed of the two cells just employed and a Bunsen cell. The cells are first to be set up in series with the Bunsen cell and the galvanometer, then both of the Daniell cells are to be reversed.

The deflections are to be observed and the electromotive force of the Bunsen cell is to be calculated.

EXPERIMENT XCV.

THE THERMO-ELECTRIC JUNCTION.

¶ 232. **Determination of the Electromotive Force of a Thermo-electric Junction** — An iron wire (*ab*, Fig. 257) and a German-silver wire (*ac*), insulated by surrounding them with India-rubber tubes, are soldered together at *a*; and the junction (*a*) is enclosed in a steam heater. The other ends, *b* and *c*, are soldered to insulated copper wires, *bd* and *ce*. The junctions

b and *c* are placed in a beaker and covered with melting ice. A thermo-element is thus formed with an electromotive force of about 3 thousandths of a volt. The object of this experiment is to measure the electromotive force in question.

I. The terminals of the thermo-element (*d* and *e*) are to be connected with two pole-cups of a differential galvanometer (*dg*) so that the current from the thermo-element circulates in one half of the coil of the galvanometer.

The other half of the galvanometer is to be connected through a rheostat (*hi*) with the poles (*j* and

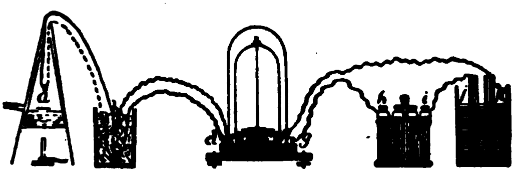

Fig. 257.

k) of a voltaic cell of known electromotive force (¶ 230, 2). There should be at first, let us say, 1000 ohms' resistance in the rheostat. The connections are to be made so that the current from the Daniell cell may produce upon the needle an effect opposite to that due to the thermo-element. The resistance of the rheostat is now to be increased or diminished until the two currents exactly neutralize each other. The rheostat resistance (R_1) is then noted.

An additional resistance (r) of known amount, about equal to that of the galvanometer (see Exp. 89), is now to be introduced between *b* and *d*, or be-

tween c and e, and the resistance of the rheostat (hi) again adjusted so as to produce equilibrium. The new value of the resistance (R_2) is also to be noted.

II. If a differential galvanometer cannot be obtained, the thermo-electric junction is first to be connected with the galvanometer, and the deflection (D) noted; then the resistance (r) is to be introduced, and the deflection (d) again noted. The Daniell cell is then to be connected with the galvanometer through a resistance (R_1), such that the deflection of the needle is the same as D. Then the rheostat resistance is increased to a value R_2 which produces a deflection equal to d. The results of I. and II. are to be reduced by formula (10), ¶ 233.

¶ 233. **Calculation of the Electromotive Force of a Thermo-electric Junction.** — If in the thermo-electric circuit (*abdeca*, Fig. 257), e is the electromotive force, and b the electrical resistance of the thermo-element, g the resistance of the galvanometer, or that part of it which is included in the circuit in question, c_1 the current in the first part of the experiment, c_2 the current in the second part of the experiment, and r the resistance added; if, furthermore, in the voltaic circuit (*fghijkf*, Fig. 257), E is the electromotive force, B the battery resistance, G the galvanometer resistance, R_1 and R_2 the two rheostat resistances, and C_1 and C_2 the corresponding currents, we have (§ 138), since the currents c_1 and C_1 are equal, —

$$c_1 = \frac{e}{b+g} = C_1 = \frac{E}{B+G+R_1}; \quad (1)$$

and since the currents c_2 and C_2 are equal —

$$c_2 = \frac{e}{b+g+r} = C_2 = \frac{E}{B+G+R_2}. \quad (2)$$

From (1) and from (2) we find —

$$e = E\frac{b+g}{B+G+R_1}, \quad (3)$$

and

$$e = E\frac{b+g+r}{B+G+R_2}. \quad (4)$$

By either of these formulæ (3 or 4) we may calculate the value of e from the observed values of r, R_1, and R_2, if b, g, B, G, and E, are known (Exps. 87–92). The student should bear in mind that the resistance of each part of the galvanometer in this experiment is about twice that of the two parts in multiple arc (§ 140), and half that of the two parts in series. A result independent of the battery and galvanometer resistances may be obtained by combining the observations obtained in the first and second parts of the experiment. Dividing (2) by (1) we have —

$$\frac{b+g}{b+g+r} = \frac{B+G+R_1}{B+G+R_2}, \quad (5)$$

whence $(b+g)B + (b+g)G + (b+g)R_2$

$$= (b+g)B + (b+g)G + (b+g)R_1 + r(B+G+R_1), \quad (6)$$

that is, —

$$(b+g)R_2 - (b+g)R_1 = r(B+G+R_1), \quad (7)$$

or

$$(b+g)(R_2 - R_1) = r(B+G+R_1); \quad (8)$$

from which we find —

$$b + g = \frac{r\,(B + G + R_1)}{R_2 - R_1}. \qquad (9)$$

Substituting this value in (3) and cancelling $(B + G + R_1)$, we have finally —

$$e = E\frac{r}{R_2 - R_1}. \qquad (10)$$

EXPERIMENT XCVI.

THE VOLT-METER, I.

¶ 234. **Calibration of a Volt-Meter.** — The name volt-meter is given to any instrument capable of indicating directly the value of an electromotive force in volts. One of the forms ordinarily employed (Fig. 258) is similar in external appearance to the ammeter shown in Fig. 231, ¶ 210. There is, however, an essential distinction between these instruments.

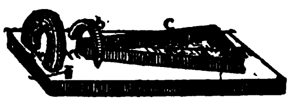

FIG 258.

In the ammeter, the coil a is made so as to have the smallest possible electrical resistance, in order that this resistance may be neglected. In the volt-meter, the finest possible wire is employed in this coil, so that the current which flows through it may be neglected. The simplest way to calibrate a volt-meter is to connect it with a battery containing different numbers of voltaic cells in series (see Fig. 220, ¶ 196). Having found the electromotive force of each cell (see

¶ 230), we may calculate that of the whole battery by adding these electromotive forces together. The difference between this calculated value and the observed reading of the volt-meter gives the correction of the volt-meter for the reading in question. A delicate galvanometer (G, Fig. 259) connected in series with a rheostat (R) is a convenient substitute for a volt-meter in the measurements relating to the electromotive force of batteries. The resistance in the galvanometer circuit should be so great that we may entirely neglect the current which flows through the instrument in comparison with the other currents used in this experiment. To test such a combination, it is to be connected with a battery of known electromotive force, as for instance, the Daniell cell employed in Experiment 92. If a common astatic galvanometer is employed (Fig. 207, ¶ 188), the resistance of the rheostat should be such as to give a deflection of about 45°. This resistance should be noted, and should remain unchanged through all the experiments with the instrument of which it now constitutes an essential part.

An ordinary astatic galvanometer does not obey the law of tangents (¶ 195) closely enough even for rough determinations. It is necessary, accordingly, to test the reading of the instrument with a series of electromotive forces bearing known ratios to one another.

A simple device by which this object may be attained consists of a uniform straight wire, traversed by a current from a constant battery. The "bridge-

wire" of the Wheatstone's apparatus (*hj*, Fig. 259) may be employed. A battery (*B*) of two Bunsen cells in series will probably be required to give the necessary current. The poles should be connected with the ends of the wire by means of screw cups (*b* and *f*) provided for that purpose.

Contact is now to be made between this wire and the terminals of the volt-meter (*GR*) at points 10 *cm.* apart. This may be done by the aid of two sliders, similar to the one used in Experiment 87. Pressure must be exerted upon the sliders to insure a good electrical contact (¶ 193, 11). The deflection

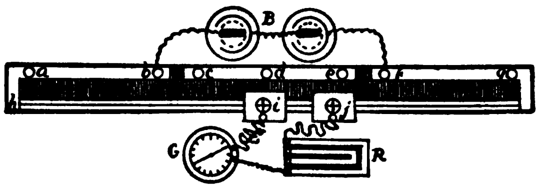

FIG. 259.

of the galvanometer is to be noted. The experiment is to be repeated with contact at two other points the same distance apart, but in a different part of the wire.[1]

The sliders are now to be interchanged and the deflections determined as before.

The direction of each deflection, whether between north and east or between north and west should be noted.

[1] A record of the reading of each slider corresponding to a given deflection should be preserved, since it may be useful in comparing the resistances of different parts of the wire.

The experiment is now to be repeated with contacts at two points 20 *cm.* apart, then 30 *cm.*, 40 *cm.*, &c., up to 80 or 100 *cm.* (the length of the wire). The observations should be repeated in the inverse order to eliminate variations in the strength of the battery.

The average deflections, corresponding respectively to 10, 20, . . . 80, or 100 *cm.*, are now to be calculated, and the results are to be plotted on co-ordinate paper as is Fig. 260. The distance between the sliders is here represented by a scale at the top of the figure, and the deflections by a scale at the left. The deflection produced by the Daniell cell is also to be plotted, and the number of centimetres corresponding to this deflection found (see § 59). If the electromotive force of the Daniell cell is E volts (¶ 230), and if D is the distance between the sliders which produces an equal current, the distance d corresponding to 1 volt is —

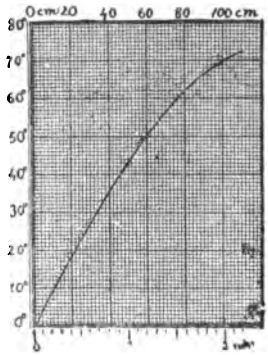

FIG. 260.

$$d = \frac{D}{E}.$$

This distance is to be indicated on the diagram and is to be divided into tenths or smaller parts. The division may be extended across the base of the figure. The theory and uses to be made of the diagram will be explained in the next experiment.

EXPERIMENT XCVII.

THE VOLT-METER, II.

¶ 235. **Determination of Electromotive Forces by means of a Volt-meter.** — A volt-meter, calibrated as in ¶ 234, is to be connected with various cells or batteries, one at a time. The deflection caused by each is to be noted. The electromotive force of each is then to be found (see § 59) by means of the curve already plotted (Fig. 260, ¶ 234). A point a is first located in the scale of degrees corresponding to the deflection in question. Then a point b is found on the curve at the right of a, and below b a point c is found in the scale of *electromotive force* into which the base of the figure has been divided.

The student is to determine rapidly in this way the electromotive forces of all the cells which he has employed.

The principle upon which this method depends is that the difference of potential between two points on a wire of *uniform resistance* is proportional to the distance between those points represented by the scale at the top of Fig. 260. For if R is the resistance of 1 *cm.* of the wire, the resistance of d centimetres will be Rd. Hence from the general formula of § 139 —

$$e = cr = cRd, \tag{1}$$

$$\frac{e'}{e''} = \frac{crd'}{cRd''} = \frac{d'}{d''}. \tag{2}$$

If the scale at the bottom of Fig. 260 is constructed so as to give one electromotive force correctly, all electromotive forces should be correctly represented.

EXPERIMENT XCVIII.

CLARK'S POTENTIOMETER.

¶ 236. **Comparison of Electromotive Forces by means of Clark's Potentiometer.** — The positive or carbon pole of a battery (*B*, Fig. 261), consisting of two

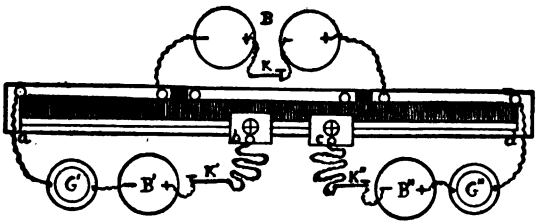

FIG. 261.

Bunsen cells in series, is to be connected with one end, *d*, of a Wheatstone's Bridge wire. The negative or zinc pole is to be connected with the other end (*a*) of the wire. A key, *K*, is to be included in the circuit. The negative (or zinc) pole of a Daniell cell (*B'*) is to be connected with *a*. The positive (or copper) pole is to be joined through a key, *K'*, and a delicate galvanometer, *G'*, to a slider (*b*), by which an electrical connection may be made at any point of the wire. The positive or carbon pole of a Lechanché cell is to be connected similarly with *d*, while the negative

(or zinc) pole is to be connected through a key, K'', and a galvanometer, G'', with a second slider at c.

The key K' is first pressed for an instant, and the direction of the deflection noted. Then K and K' are both pressed, the connection being completed first in K then in K'.

If the deflection is in the same direction as before, the distance ab is to be increased; if it is in the opposite direction the distance is to be diminished. The experiment is now repeated until a point b is found such that in pressing both K and K', no deflection is observed. In this case the point b has the same potential as the positive pole of the battery B'.

In the same way a second slider is to be placed at a point c, where the potential is the same as that of the negative pole of the Lechanché cell.

The key K being now closed, the keys K' and K'' are to be pressed simultaneously. If the adjustments have been accurately made, neither galvanometer will be deflected. If this is not the case, the adjustments must be repeated.

By the principle explained in ¶ 235, if the wire ad is of uniform resistance, so that the resistances of ab and cd are proportional to their lengths, the difference of potential between a and b must be to that between c and d as ab is to cd. We have, therefore, —

$$\frac{E''}{E'} = \frac{cd}{ab}, \text{ or } E'' = E' \frac{cd}{ab},$$

where E' and E'' represent the electromotive forces, respectively, of the batteries B' and B''. By this

formula, knowing the electromotive force of the Daniell cell (¶ 230), we may calculate that of the Lechanché cell. In repeating the experiment, the places of the Daniell and Lechanché elements should be interchanged. If the two sliders should interfere with each other, either 1 or 3 Bunsen cells should be used (in *B*) instead of 2. The experiment may also be repeated with other batteries. Clark's Potentiometer is especially adapted to the determination of the electromotive forces of *inconstant* elements.

EXPERIMENT XCIX.

POGGENDORFF'S METHOD.

¶ 237. **Determination of Electromotive Forces by Poggendorff's Absolute Method.** — The zinc pole *d* (Fig. 262) of a Bunsen battery is to be connected with one

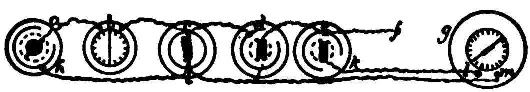

FIG. 262.

terminal (*c*) of the resistance-coil used in the Method of Heating (Exp. 85.) The zinc pole (*a*) of a Daniell cell is to be connected with the same terminal through a delicate galvanometer, *b*. The copper pole (*h*) of the Daniell cell is to be connected with the terminal (*i*) of the rheostat, and the carbon pole (*k*) of the Bunsen cell is to be connected through a tangent galvanometer (*glm*) with the same terminal (*i*).

A portion (*de*) of a German-silver wire (*def*) having in all a resistance about equal to that of the resistance-coil (*ci*), let us say 1 ohm, is to be included in the circuit of the Bunsen battery.

The wire *def* is to be disconnected for a moment, and the direction of the galvanometer deflection noted. Then the extreme end (*f*) of the wire (*def*) is to be bound in the clamp *e*. If the deflection is in the same direction as before, a longer wire must be employed, and if the two Bunsen cells are still unable to reverse the Daniell cell,[1] other cells must be added to the first, either in series or in multiple arc (§ 140).

We will suppose that a battery (*de*) and a wire (*def*) have been found such that when the wire is clamped at *f*, the current in the Daniell cell is reversed; but when clamped at *d*, the current flows in its natural direction.

The wire (*def*) is next to be clamped at a point (*e*), found by trial, so that the current in the Daniell circuit may be reduced to zero. The galvanometer (*b*) will then show no deflection.

In practice, we clamp the wire at a point (*e*) so that the Daniell cell is barely reversed, and wait for a condition of equilibrium to come about through the gradual weakening of the Bunsen cell. At the moment when the astatic galvanometer (*b*) points to 0° the reading of the tangent galvanometer (*g*) is to be taken.

[1] The student may be reminded that unless similar poles meet at *c* and at *i*, it will be impossible in any case to produce a reversal of the current.

The experiment is to be repeated with the connections of the galvanometers reversed one at a time, as in Experiment 79.

If a is the mean angle of deflection of the tangent galvanometer and I its reduction factor, the current C is (see ¶ 199, 7) —

$$C = I \tan a \text{ ampères.} \qquad (1)$$

If R is the resistance of the coil (ci) in ohms (Exp. 85) we have a difference of potential (e) between its terminals c and d (see § 139) equal to —

$$e = CR = RI \tan a \text{ volts.} \qquad (2)$$

This is equal to the electromotive force of the *Daniell cell* (see ¶ 130, 3).

For a simplified diagram of Poggendorff's Method, see Fig. 254, 1, ¶ 226. The only change to be made in this diagram is the introduction of a tangent galvanometer in the upper circuit (abc).

EXPERIMENT C.

ELECTRICAL EFFICIENCY.

¶ 238. **Determination of the Efficiency of an Electric Motor.** — A small electric motor, such for instance as is represented in Fig. 263, is to be connected through an ammeter (Fig. 231, ¶ 210) or through a tangent galvanometer (A, Fig. 264), with a voltaic battery (BB) containing at least twice as many cells as are

required to keep the motor (M) in motion. Thus if the motor can be started with 2, but not with 1 Bunsen cell, a battery of 4 Bunsen cells should be em-

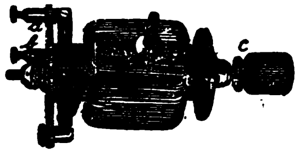

Fig. 263.

ployed. The poles of the battery are to be connected through a volt-meter or its equivalent (see Exp. 96) consisting of an astatic galvanometer (G) and a rheostat (R). The work done by the motor (M) is to be determined as in Experiment 70, by observing the readings of a pair of spring balances (SS') con-

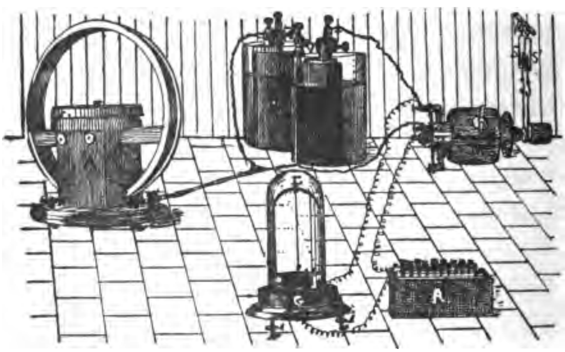

Fig. 264.

nected by a cord passing round the pulley of the motor. Ordinary letter-balances will probably answer for this experiment. The tension of the cord should be such as to reduce the speed of the motor

to about one half its maximum; but different experiments should be made with different tensions. The number of revolutions made by the wheel of the motor in a given length of time may be determined by an instrument called a "revolution counter" especially devised for this purpose. This consists of a shaft *ab* (Fig. 265) which can be easily connected with the axle of the motor, and a toothed wheel (*c*) with teeth fitting into a thread cut on the shaft at *b*, The revolutions of the shaft are indicated on a dial (*d*) by a pointer (*e*) attached to a wheel (*c*). The circumference of the pulley is to be measured.

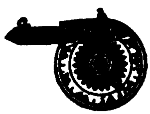

FIG. 265.

Instead of a revolution counter, we may make a band of thread 60 *cm.* long, passing from the pulley of the motor over a second pulley-wheel. Every time that the knot in this band passes a given point shows that the pulley-wheel has advanced 60 *cm.* The velocity of the circumference of the pulley-wheel can be found by this method by counting the number of times that the knot passes a given point in 1 minute. If the band is just 60 *cm.* long, this number represents the velocity in *cm. per sec.* without any reduction.

The power in ergs per sec. utilized by the motor is to be calculated from these data as in ¶ 174, 1, and reduced to watts (§ 15) by dividing by 10,000,000; that is, by pointing off 7 places of decimals. The power in watts spent upon the motor is found by multiplying together the current in ampères indicated by the ammeter (or its equivalent) and the electro-

motive force in volts indicated by the volt-meter, or its equivalent (see § 137).

The efficiency of the motor is to be found by dividing the power utilized by the power spent (see ¶ 174, 3).

II. Instead of an electric motor, we may employ a small dynamo-machine, driven by a water-motor. The work spent by the water is to be calculated as in Experiment 69. The work utilized is to be found as above by multiplying together the current in ampères and the electromotive force in volts. The former is to be measured by an ammeter in the main circuit of the dynamo-machine; the latter by a volt-meter connected with the poles of the dynamo-machine. The experiment should be repeated with greater or less resistance interposed in the main circuit.

The student can hardly fail to notice the similarity of the method by which we calculate the work of an electrical current to that used in the case of a current of water (§ 118). The same general method is employed in all measurements of electrical efficiency.

EXPERIMENTS FOR ADVANCED STUDENTS.

The principal methods by which physical quantities are measured have been considered in the course of the 100 experiments which have been described. Various modifications of these methods have already been alluded to. On account, however, of either the practical or the theoretical difficulties involved, and the expense of the necessary apparatus, measurements of certain physical quantities have been hitherto entirely omitted. This course would, however, be incomplete without an outline, at least, of the methods by which some of these quantities may be determined. Most of the experiments about to be mentioned are suitable only for advanced students. For this reason it has been been thought unnecessary to describe them in detail, or to include in the text proofs of the formulæ involved, except when these proofs are necessary to an understanding of the methods employed. The Proofs of other formulæ will be considered separately in Parts III. and IV.

¶ 239. **The Piezometer.** — To measure the compressibility of a liquid, we place it in a glass bulb (*C*, Fig. 266) with a narrow neck or stem (*D*) containing a small mercury index. The bulb is to be placed in a stout glass cylinder filled with water. A consider-

able hydrostatic pressure is then generated by means of the thumb-screw, *A*, and measured by a small air manometer, *E* (see ¶ 77). The contraction of the liquid in the stem is observed. Since the bulb is at the same pressure inside and out, there is no tendency to stretch or to crush it. An allowance must, however, be made for the compression of the sides of the bulb. It can be shown geometrically that the capacity of a bulb decreases, when thus subjected to a uniform pressure, in the same proportion as the volume of a solid would decrease under the same circumstances. The ratio of the pressure in dynes per square centimetre to the decrease in volume of 1 cubic centimetre is called the "Coefficient of Resilience of Volume."[1] It is usually calculated from "Young's Modulus" (Y), determined as in Experiment 65, or as in ¶ 248, I., and from the "Simple Rigidity" (S) of a solid. The simple rigidity may be found from the coefficient of torsion, T, (*i. e.*, the couple necessary to twist a wire 1°, see Exp. 64), and from the length, l, and radius, r, of the wire, by the formula —

FIG. 266.

$$S = \frac{360\ Tl}{\pi^2\ r^4}.$$

It may also be found as in ¶ 248, II. Denoting by M the "coefficient of resilience of the solid," or

[1] Everett, Units and Physical Constants, Arts. 63–65.

"modulus of volume elasticity," as it is sometimes called, we find —

$$M = \frac{SY}{9S - 3Y}.$$

A mean value of M for glass may be taken as 400, 000,000,000 dynes per square centimetre. The quantities S, M, and Y are (in the case of glass and many other substances) related to each other in about the proportion of the numbers 6, 10, and 15 respectively.

If C is the capacity in *cu. cm.* of the bulb (Exp. 11), and P the pressure to which it is subjected, measured in *dynes per sq. cm.*, the contraction of the interior volume of the bulb (V) in *cu. cm.* is —

$$V = \frac{CP}{M}.$$

If V' is the apparent contraction in *cu. cm.* of the liquid, its real contraction is $V + V'$, and the Coefficient of Resilience of volume (M') of the liquid is —

$$M' = \frac{PC}{V + V'}.$$

By making the bulb in two parts, a solid may be introduced into it and surrounded with liquid. The Coefficient of Resilience of the solid may be deduced from its effect on the apparent contraction of the liquid in question.

¶ 240. **Use of a Weight Thermometer.** — If a bulb similar to that employed in ¶ 239, be filled with mercury at an observed temperature t_1, then warmed to the temperature t_2, a certain quantity of mercury will

be driven out of it. Let the weight of this mercury be w, and let the whole original weight of the mercury be W_1, both weights being reduced to vacuo (§ 67), then the weight, W_2, remaining in the bulb is $W_1 - w$. If v_1 and v_2 are the specific volumes of mercury at the temperatures t_1 and t_2 (see Table 23, *A* and *B*), then the capacities of the bulb (c_1 and c_2) at these temperatures must be —

$$c_1 = W_1\, v_1 \text{ and } c_2 = W_2\, v_2.$$

It may be shown by geometry that when a vessel is expanded uniformly by heat, its capacity is increased in the same proportion as the volume of a solid would increase under the same circumstances. The cubical expansion, e, of glass is accordingly (see ¶ 63) —

$$e = \frac{c_2 - c_1}{c_1\,(t_2 - t_1)};$$

hence the linear coefficient, ϵ, is (see § 83) —

$$\epsilon = \tfrac{1}{3}\,\frac{c_2 - c_1}{c_1\,(t_2 - t_1)}.$$

This is considered to be one of the most accurate methods of obtaining the coefficient of expansion of various kinds of glass.

By collecting and weighing the mercury which is driven out of a bulb or *weight thermometer*, we may estimate the relative rise of temperature in different cases. The instrument is useful in determining precisely the maximum rise of temperature within an enclosure which has to be kept closed at the time when the temperature is taken.

The weight thermometer has also been employed to measure the cubical expansion of solids *enclosed in the bulb.* If c_1 is the capacity of the bulb at the temperature t_1, and if W_1 is the weight of mercury required to fill the space between the solid and the bulb, the volume of the solid V_1 is evidently $c_1 - W_1 v_1$. If when heated to the temperature t_2, at which the capacity of the bulb is c_2, w grams of mercury are driven out, so that W_2 (or $W_1 - w$) grams remain, then the volume V_2 of the solid is $c_2 - W_2 v_2$; hence we may find the cubical coefficient of expansion (e) by substituting these values of V_1 and V_2 in the ordinary formula (see ¶ 63) —

$$e = \frac{V_2 - V_1}{V_1 (t_2 - t_1)}.$$

¶ 241. **Conduction of Heat.** — (I.) The conductivity of various insulating materials may be found approximately by filling the space between the inner and outer cups of a calorimeter (¶ 85) with these materials, and finding the rate at which heat is lost. If A is the mean area of the surfaces between which conduction takes place, L the distance between them, t the difference of temperature, and T the time in which Q units of heat pass from one surface to the other, the specific conductivity (c) of the material is —

$$c = \frac{QL}{tTA}.$$

II. A metallic rod (AD, Fig. 267) is surrounded, one end by steam, the other by melting ice. The

central portion is covered with insulating material. Two thermometers, B and C, are inserted in holes in the rod, partly filled with mercury. If L is the length of the rod between B and C, A the area of its cross-section, t the difference of temperature between the points (B and C), and w the weight of ice melted in the time T, after a steady flow of heat has been established, *less the quantity melted in the same time when the rod is replaced by insulating material*, then since the latent heat of liquefaction of water is 79, the specific conductivity (c) of the rod is given by the formula—

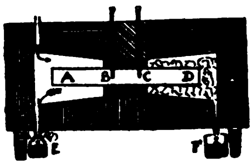

FIG. 267.

$$c = \frac{79\,wL}{tTA}.$$

The specific conductivity of a given material represents the quantity of heat which would flow in one second from one side of a unit cube made of that material to the opposite side of the cube when the difference of temperature between the two sides is 1°.

The results of this experiment will be slightly modified by the manner in which heat flows through the insulating material which surrounds it. To avoid errors from this source, the distance between the thermometers should be as small as, or smaller than the diameter of the rod. This method should be applied only to metals or to substances which are good conductors of heat.

¶ 242. **Latitude.** — The latitude of a place is usually determined by an observation of the "altitude" of the sun at "apparent noon;" that is, the time when it attains its greatest "altitude," or angular distance from the horizon. The true altitude (a) of the sun is defined as the angle which a line drawn from the centre of the earth to the centre of the sun makes with a plane passing through the centre of the earth and parallel to the horizon of the place in question. The declination (d) of the sun is defined as the angle which the same line makes with the earth's equator. The sun's declination may be found in nautical almanacs calculated in advance for every day of the year. The difference between local and Greenwich time, and the hourly change in declination must generally be allowed for. The latitude (l) of a place is by definition equal to the complement of the angle between the horizontal and equatorial planes. We have, accordingly, —

$$l = 90° - a \pm d. \qquad \text{I.}$$

If the sun is (as in summer) above the equator, the sign of d is to be taken as positive; if the sun is below the equator, d is to be called negative.

I. In nautical observations, the apparent altitude of the sun is determined by means of a sextant (see Exp. 44). The lower "limb" (or edge) of the sun is made to coincide with the sea-horizon. The observed altitude (A) must be corrected as follows: —

(1) FOR SEMI-DIAMETER. The apparent semi-diameter (s) of the sun (not far from 16′), given exactly in the nautical almanac for every day in the

year, is to be *added* to the observed altitude of the lower limb of the sun, since the altitude of the sun's centre is wanted.

(2) DIP OF THE SEA-HORIZON. A line drawn from the eye of the observer to the sea-horizon makes a certain angle with a true horizontal plane. This is called the "dip of the sea horizon." It may be calculated by the formula —

$$h = \sqrt{m} \times 1\tfrac{3}{4}' \text{ (nearly)},$$

where m is the height in metres of the eye above the sea-level. The dip (h) must be subtracted from the observed altitude.

(3) FOR REFRACTION. Atmospheric refraction tends to make heavenly bodies appear higher than they really are. The correction (r) is accordingly to be subtracted from the observed altitude. It is given by the equation —

$$r \doteq cotan\ A \times 1' \text{ (nearly)}.$$

(4) FOR PARALLAX. The apparent altitude of a body as seen from the earth's surface is obviously less than if it could be observed at the earth's centre. In the case of the stars, on account of their enormous distance, the difference is imperceptible. The correction for parallax (p) is given in general by the equation —

$$p = P\ cos\ A \text{ (nearly)},$$

where P is the "horizontal parallax" of the body in question; that is, its correction for parallax when

seen on the horizon. In observations of the sun with an ordinary sextant, since P is less than 9″, all corrections for parallax may usually be neglected. It is only in the case of the moon, where P is in the neighborhood of 1°, that the correction for parallax becomes important.

The true altitude (a) of a heavenly body is found in general from the observed altitude (A) by applying the corrections for semi-diameter (s), dip of the horizon (h), refraction (r), and parallax (p) as follows:

$$a = A + s - h - r + p. \qquad \text{II.}$$

II. Observations of latitude taken on land are usually made with an "artificial horizon." This may consist of a plate-glass mirror (made horizontal by two spirit-levels and levelling-screws) or simply a dish of mercury (B, Fig. 268) The lower limb of the sun is made to coincide with its own reflection in the horizontal surface. The observed angle (D) between the direct and reflected rays reaching the sextant (A, Fig. 268) is measured, and *halved*, to find the apparent altitude of the sun. The result is corrected as above for semi-diameter, refraction, and (if sufficiently accurate) for parallax.[1] We have —

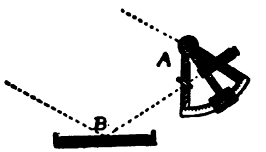

FIG. 268.

$$a = \tfrac{1}{2} D + s - r + p. \qquad \text{III.}$$

[1] The correction for "dip" is obviously to be omitted in the case of an "artificial horizon," since the plane of the reflecting surface should be perfectly horizontal.

The latitude is finally calculated by formula I. above.

¶ 243. **Longitude.** — The longitude of a place may be determined by a sextant observation of the altitude of the sun (see ¶ 242) an hour or two after sunrise or before sunset. For the reduction of the results, in general, the student is referred to works on navigation. A simple (though not very accurate) method of finding the longitude of a place is to measure the altitude of the sun at an observed time t' (about an hour before noon), then to determine exactly the time t'' (about an hour after noon) when the sun descends to the *same altitude*. Obviously the time of "apparent noon," t, (neglecting the change in the sun's declination), is half-way between t' and t'', that is —

$$t = \tfrac{1}{2}\,(t' + t''), \text{ nearly.} \qquad \text{I.}$$

If e is the "equation of time" (given in the nautical almanacs for every day of the year), the time (T) of "mean noon" is (by definition) given by the formula —

$$T = t \pm e. \qquad \text{II.}$$

The sign of the quantity e is positive if the sun is fast, but negative if the sun is slow.

It is assumed that the chronometer employed in this experiment has been set so as to indicate correctly the time of a given meridian, as for instance that of Greenwich, from which it is desired to measure longitude. If it does not indicate this time correctly, an allowance must be made for the error of

the chronometer. At sea, several chronometers are frequently carried. In certain cases a chronometer may have to be set by a lunar observation. For the reduction of such results (which is exceedingly complicated), the student is referred to works on navigation. On land, the standard time of a given meridian is usually obtainable by means of the electric telegraph.

It may be remarked that the longitude of a place is given by formula II. in hours, minutes and seconds.

¶ 244. **Indices of Refraction.** — I. If A is the angle of a prism (Exp. 45), and D the angle of minimum deviation (Exp. 46) of a ray of light of a given wave-length, the index of refraction (μ) of the material of which the prism is composed is (for light of that wave-length) —

$$\mu = \frac{\sin \frac{1}{2} (A + D)}{\sin \frac{1}{2} A}.$$

Certain "doubly refracting" substances have two indices of refraction instead of one. To determine them we employ a prism cut so as to produce the maximum separation of the two rays into which a single ray of monochromatic light can be decomposed by the given prism angle. The minimum deviation of *each* ray is then measured, and the two indices of refraction are calculated separately by the ordinary formula.

II. If R is a mean radius of curvature of the two surfaces of a double convex lens (Exp. 21), and F its principal focal length (Exps. 41–43), the index of

refraction of the material of which the lens is made may be found by the formula —

$$\mu = 1 + \tfrac{1}{2} \frac{R}{F}.$$

If the same lens (B, Fig. 269) be enclosed between two flat glass plates (A and C), and the space be filled with a liquid, with the index of refraction μ', then if F' is the principal focal length of the combination, we have —

$$\mu' = \mu - \tfrac{1}{2} \frac{R}{F'}.$$

If R_1 and R_2 are the two radii of curvature of the two sides of the lens, the mean radius of curvature should strictly be calculated by the formula —

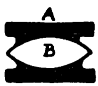

FIG. 269.

$$R = \frac{2 R_1 R_2}{R_1 + R_2}.$$

¶ 245. **Polarization.** — The vibrations which constitute ordinary light are, according to modern theories (§§ 92, 93), at right-angles with the direction in which the light is propagated. In a vertical beam of light, for instance, the vibrations are supposed to be confined to a horizontal plane. The vibrations appear in general to be distributed uniformly in every possible direction perpendicular to the path of the ray. Certain substances and certain optical combinations have, however, the property of stopping all the vibrations — or rather all their *components* (§ 105) — except those in a certain direction, as for instance

north and south. The light transmitted is then said to be polarized.

In many optical instruments, light passes successively through two such combinations. The first is called the "polarizer" (*e*, Fig. 270), the second is called the "analyzer" (*a*). If the polarizer and analyzer are placed so that the direction of the vibrations transmitted is the same in both cases, the light which has passed through one will also pass freely

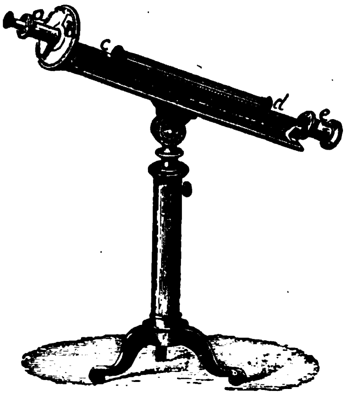

FIG. 270.

through the other; but since the polarizer transmits only vibrations in a given direction, if the analyzer is placed so as to stop all vibrations in this direction, a beam of light which has passed through the polarizer will be completely cut off by the analyzer. The position of the analyzer when this occurs is indicated by a pointer attached to it. The reading of the pointer with respect to the graduated circle *b* determines the zero-reading of the instrument.

Certain substances have the property of changing the direction of the vibrations in a beam of polarized light which they transmit. Thus in passing *upward* through a solution of cane sugar, north and south vibrations are gradually changed into a northeast and southwest direction.[1]

When a substance producing "rotation of the plane of polarization" is placed between the polarizer and the analyzer in its zero-position, the analyzer will no longer cut off all the light transmitted by the polarizer. To produce perfect darkness, the analyzer must obviously be turned through an angle equal to that through which the plane of polarization has revolved. The instrument shown in Fig. 270 affords, accordingly, a means of measuring the rotation of the plane of polarization.

To test the strength of a solution of sugar with this instrument, we pour the solution into a tube *cd* with glass ends, and interpose the tube in the path of the beam *ea* of polarized light. The analyzer is then turned to the right from its zero-position, until the light which it transmits is reduced to a minimum.

[1] When light is polarized by reflection, it is said to be polarized in a plane perpendicular to the reflecting surface, and containing both the incident and the reflected rays. According to Fresnel's theory the vibrations in a beam of polarized light take place at right-angles with the "plane of polarization." The action of a solution of sugar upon a beam of polarized light *approaching the eye* is to rotate the plane of polarization (and hence also the direction of the vibration) *with the hands of a watch*. The student should note that this is called a right-handed rotation in optics; but that it is opposite to the motion of an ordinary right-handed screw, which when turned to the right moves *away from* the eye.

Let a be the angle in degrees through which it is turned when ***sodium light*** is employed, and let d be the depth of the sugar solution, equal to the distance between the glass ends of the tube cd; then experiments show that the strength of the solution (s) in grams per *cu. cm.* is given by the equation (Kohlrausch, § 46), —

$$s = 1.5 \frac{a}{d} \text{ (nearly).}$$

The rotation varies considerably with lights of different colors (see Table 31 D). For this reason, when ordinary white light is employed perfect darkness can never be attained.

There are various optical effects (besides the darkness produced by an analyzer) which depend upon the plane in which light is polarized. Many of these have been applied to the determination of angles of rotation of the plane of polarization. The method described above has been chosen because of its simplicity.

¶ 246. **Color.** A piece of colored paper (c, Fig. 271) may be mounted in front of a white screen (d) and illuminated by a candle (a) through a piece of ruby glass (b), all other light being cut off. The distances ac and ad must be adjusted so that c and d appear equally bright when viewed from a point near b. The "relative luminosity" of the surface c is then equal to $(ac)^2 \div (ad)^2$ as far as reflected red rays are concerned.

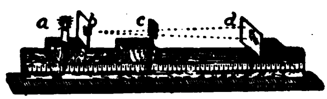

FIG. 271.

A transparent gelatine plate stained with an emerald green mixture of common green and yellow inks is now substituted for the ruby glass (*b*), and the relative luminosity is again determined. Finally, a gelatine plate stained with a violet mixture (Hofmann's violet containing a trace of soluble Prussian blue) is employed.

The three relative luminosities of the surface *c*, obtained as above by means of red, green, and violet rays, completely determine the color of the surface in question (see ¶ 115).

¶ 247. **Velocity of Light.** — The velocity of light was determined by Fitzeau in 1849.[1] A beam of light made intermittent by passing between the teeth of a revolving wheel, was sent to a distant mirror, then reflected back to the eye through the same wheel. When the wheel (which had 720 teeth) made 12.6 revolutions per second, the flashes of light, in traversing a total distance of 17,326 metres, were retarded so as to strike a tooth instead of the space between two teeth; hence the light was cut off. When the speed of the wheel was doubled, so that 18,144 teeth passed a given point in one second, the light reappeared; when trebled it disappeared, &c. It was inferred from this experiment that a beam of light required $\frac{1}{18144}$ of a second to traverse 17,326 metres; whence the velocity of light would be about $18{,}144 \times 17{,}326$ metres per second, or nearly thirty thousand million *cm. per sec.*

[1] See Deschanel's Natural Philosophy, § 686; Ganot's Physics, § 507.

Foucault has measured the time required by light to traverse short distances (a few metres only) by the use of a revolving mirror.[1] A beam of light (AB, Fig. 272) striking the mirror (B) was reflected to a fixed concave mirror (CC) *with its centre of curvature in the axis of the revolving mirror* (B), then back on its course to the revolving mirror (B), and thence to the eye. The beam strikes the eye only for a very short time during each revolution of the mirror, but on account of the rapidity of rotation a continuous effect is produced. When the speed of rotation reaches several hundred revolutions per second, the mirror turns through a perceptible angle while the light is passing from B to C or to C' and back again.

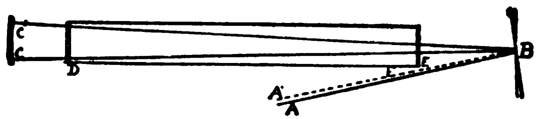

FIG. 272.

Hence the return path BA' differs slightly from the original path AB.

With a distance BC equal to about 4 metres, and with from 600 to 800 revolutions per second, divergences of about 40″ or 50″ were observed. The velocity of light was found to be 29·8 (or nearly 30) thousand million *cm. per sec.*

By passing the beam of light through a tube of water (DE, Fig. 272) it was found that the velocity of light in water is about ¾ that in air.

[1] Deschanel's Natural Philosophy, § 687; Ganot's Physics, § 506.

¶ 248. **Velocity of Sound in Wires.** — I. If a wire stretched between two vises be stroked horizontally near one end by a piece of resined cloth, a musical note may result from the *longitudinal vibrations* into which the wire is thrown. The pitch of the note is to be determined by a "pitch pipe" (Fig. 273) or any instrument serving a similar purpose. The number of vibrations corresponding to the note may be found by reference to Table 43. If l is the length of the wire between the vises, and n the number of vibrations per second, the velocity of sound (v) is —

FIG. 273.

$$v = 2nl.$$

II. If a strip of resined cloth be drawn slowly *round* the wire (like a belt round a pulley) a musical note may result from *torsional vibrations* set up in the wire. The velocity of these torsional vibations may be found by the same formula as above. The note due to longitudinal vibrations is usually about a "sixth" (¶ 134) above that due to torsional vibrations. Hence the two velocities of sound are to each other as 5 to 3, nearly.

If d is the density of the wire, Y Young's Modulus of Elasticity (¶ 166) and S the simple rigidity of the wire (¶ 239) v_1 and v_2 the velocities of longitudinal and torsional vibrations, we find —

$$Y = v_1^2 d. \qquad \text{I.}$$

$$S = v_2^2 d. \qquad \text{II.}$$

¶ 249. **Reversible Pendulum.** — A reversible pendulum (Fig. 274) may be made of cast iron,[1] so that although the two knife-edges A and B are at very unequal distances from the centre of gravity (C) the time of oscillation on both knife-edges is nearly the same. The position of C must be found approximately (Exp. 62), and the distances AC and BC measured. The distance AB must be accurately determined (by measuring DE, DA, and BE with a vernier gauge, and subtracting DA and BE from DE). If t' is the time of oscillation on the knife-edge A, and t'' that on B (see Exp. 58), the time t of oscillation of a simple pendulum of the length AB is —

FIG. 274.

$$t = t' + \frac{BC}{AC - BC}(t' - t'').$$

Denoting by l the distance AB, the acceleration of gravity (g) may now be calculated by the ordinary formula —

$$g = \frac{\pi^2 l}{t^2}.$$

[1] For a half-seconds pendulum, the following dimensions are suggested: extreme length of the shaft (DE), 45 *cm.*, breadth 3½ *cm.*, thickness 1 *cm.*; ends sharpened to an angle of about 70°; triangular knife-edges (steel better than cast iron) 2 *cm.* long, sides 1 *cm.* broad; distance of each knife-edge from nearest extremity, 10 *cm.*; holes 1 × 2 *cm.*; disc 14 *cm.* in diameter, 2 *cm.* thick; centre of disc 24 *cm.* from one knife-edge, 1 *cm.* from the other. This pendulum should weigh about 3 kilograms. The centre of gravity should be about 5 *cm.* from one knife-edge, and 20 *cm.* from the other. In observations of its time of oscillation, the knife-edges may rest upon the upper surface of a short steel rod, 7 *mm.* square, driven horizontally into the wall.

¶ 250. **Coefficient of Viscosity.** — A liquid contained in a Mariotte's bottle (a, Fig. 275) is fed through a rubber tube (bc) into a capillary tube (cd), and collected in a small vessel (e). The weight (w) which passes through the tube in a given length of time (t) is found, and the height (h) of the inlet (b) above the orifice (d) is determined. The length (l) of the tube (cd) is measured, and its radius (r) is found (see ¶ 170). Then if d is the density of the liquid

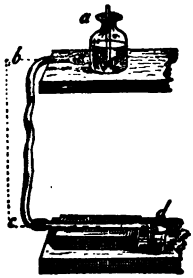

FIG. 275.

(Exp. 14), and g the acceleration of gravity (Exp. 58), the coefficient of viscosity of the liquid is given by the formula, —

$$\eta = \frac{\pi g d^2 h r^4 t}{8 \, wl}.$$

This coefficient of viscosity is the force in dynes necessary to maintain a difference of velocity equal to 1 *cm. per sec.* between two opposite faces of a centimetre cube.

The ordinary coefficient of liquid friction (see ¶ 172) depends upon the *square* of the velocity, and has no relation to the coefficient of viscosity.

¶ 251. **Electro-chemical Equivalents.** — If, in Experiment 81, I is the reduction factor of the galvanometer, determined as in Experiment 83, w the weight of copper deposited by the current C in the time t, and a the average angle of deflection, we have for the electro-chemical equivalent (q) of copper —

$$q = \frac{w}{Ct} = \frac{w}{t \, I \tan a}.$$

By the same formula we may find the electro-chemical equivalent of any other substance acted upon by the current C, whether that action be to deposit the substance in question, or to cause it to go into solution. In the case of a gas set free at one of the electrodes of a voltameter (C or D, Fig. 276), we find the weight indirectly from the volumes collected in graduated tubes (A and B), originally filled with the liquid (E) which is decomposed by the current. A battery of two or three Bunsen cells should be used with a gas voltameter.

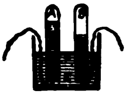

Fig. 276.

If w', w'', w''', &c., are the weights of different substances acted upon by a given current traversing a series of voltameters for a given time, the electro-chemical equivalents q', q'', q''', &c., may be found (if any one is known) from the proportion —

$$w' : q' :: w'' : q'' :: w''' : q''', \text{ \&c.}$$

¶ 252. **Correction of Rheostats.** — An arrangement of a set of resistances, convenient for the purposes of correction, is represented in Fig. 277. The outer horse-shoe (ad) contains 18 coils capable of being combined so as to give any resistance under 4,000 ohms, within one tenth of an ohm. The inner horse-shoe ($befc$) contains resistances arranged in pairs of 1, 10, 100, and 1000 ohms each. Opposite a and c are two extra blocks. These are permanently

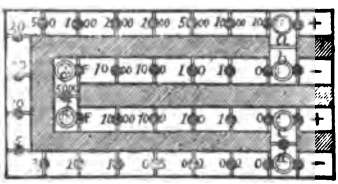

Fig. 277.

connected together, underneath, by a thick copper rod. One of them is joined to the positive pole of a battery. Two blocks opposite *b* and *d* are similarly joined together, and one of them is connected with the negative pole of the battery.

One terminal of the galvanometer is now carried to *e* (or to *f*). The other terminal is to be connected with one of the blocks in the outer line of resistances *between two coils*, or sets of coils, which are to be compared. A pair of resistances about as great as the coils in question is now introduced into the inner horse-shoe. When the battery is connected with *a* and *d*, the rheostat assumes the form of a Wheatstone's Bridge (§ 141). The inner horse-shoe furnishes two of the arms *be* and *cf*. The connections of these arms may be interchanged by breaking the battery connections at *a* and *d*, and making them at *b* and *c*. The arrangement of blocks furnishes in fact a commutator within the box of coils. By the use of this commutator, errors due to inequality in a given pair of resistances may be eliminated (§ 44).

The 1-ohm coil is first to be tested against the smaller coils, together equal to 1 ohm; then joined in series with the smaller coils, and tested against each of the 2-ohm coils; then the 5-ohm coil, the 10-ohm coil, &c., are to be tested each against its equivalent in terms of the coils below it in the line of resistances. If differences are observed, the sensitiveness of the galvanometer to a change of 1 ohm (or 0.1 ohms) in the outer line of resistances must be determined. The differences in question may then be estimated by

interpolation (see ¶ 216). The results are to be reduced as in ¶ 217. When the ratios of the different coils in the outer series have been found, that of any pair of coils in the inner horse-shoe may be determined by comparison.

¶ 253. **Resistance of Electrolytes.** — We may substitute in Exp. 87 an alternating current for a common battery current; in this case the galvanometer must be replaced by some instrument like the dynamometer, sensitive to alternating currents. A telephone is sometimes found to give satisfactory results with a rapidly alternating current. Usually a loud note is heard in the telephone; but when the Wheatstone's bridge is in adjustment, the sound either completely ceases or reaches a minimum.

The advantage of using alternating currents is that, in the short time during which they last, the effects of polarization are so small as to be almost inappreciable. The method is especially valuable in the determination of the resistances of batteries and electrolytes. It is not, however, always successful, on account of various causes tending to destroy the minima of sound. To obtain satisfactory results, the resistance to be measured should be not less than 10 or 15 ohms. The electrodes should consist of platinum strips, at least 10 *sq. cm.* in area, and freshly coated with platinum through electrolytic action (Kohlrausch, 6th ed. 72 II.),

¶ 254. **Measurement of Electrical Capacity.** — A "condenser" consists of two sets of thin metallic plates, arranged alternately, as in Fig. 278, so that

although the plates are very close together, there is no metallic connection between the two sets. The plates are generally separated by thin layers of glass, mica, or paper dipped in paraffine. The plates of one set are all connected with one binding-post (A); those of the other set with another binding-post (B).

FIG. 278.

A condenser is charged by connecting A and B each with one pole of a battery. It may then be disconnected from the battery, and discharged through a galvanometer by carrying the terminals to A and B. Care must be taken not to touch both terminals at the same time.

The capacity of a condenser is defined as the quantity of electricity which can thus be stored in it by a battery having an electromotive force equal to 1 unit *in absolute measure*. The "farad" is a thousand millionth part of the electro-magnetic unit of capacity. The distance between the plates of a condenser is usually very small in comparison with the area of the separate plates. To calculate the electrical capacity of such a condenser, we measure the thickness (t) and total area (A) of the insulating layers, then if s is the "specific inductive capacity" of the insulating material (¶ 256), the capacity (C) of the condenser is given in electrostatic units by the equation —

$$C = \frac{As}{4\pi t}, \qquad \text{I.}$$

or, since it has been found by experiment that 1 microfarad is equivalent to about 900,000 electrostatic

units,[1] the capacity (c) in microparads may be calculated by the formula —

$$c = \frac{As}{36{,}000{,}000\,\pi t} \text{ microfarads (nearly).} \qquad \text{II.}$$

The specific inductive capacity (s) of the insulating material must in general be found as in ¶ 256; but when the plates of a condenser are separated by air spaces, since the specific inductive capacity of air is taken as 1, the capacity of a condenser may be calculated from direct measurements of the area and thickness of the insulating material.

The capacity of any condenser may be determined by measuring the quantity of electricity stored in it by a battery of known electromotive force. With the aid of clockwork, a condenser is to be charged by a battery and discharged through a galvanometer n times a second; the deflection of the galvanometer being noted. Then if R is the resistance in ohms through which the same battery produces the same deflection (see Exp. 95, II.) we have —

$$c = \frac{1{,}000{,}000}{nR} \text{ microfarads.} \qquad \text{III.}$$

In practice we must employ a very sensitive galvanometer capable of measuring currents at least in millionths of an ampère. The time of oscillation of the needle should be 10 seconds or more, in order that the intermittent discharge through the instrument may produce a sensibly constant effect. An ordinary condenser, of 1 microfarad capacity cannot

[1] Everett, Units and Physical Constants, Arts. 177, 185.

be charged and discharged satisfactorily more than 10 or 100 times per second.[1] To avoid large errors due to this cause, the speed of the mechanism should be reduced until an approximate agreement is obtained between two or more results.

The experiment may be performed with an ordinary astatic galvanometer, but only by the use of a condenser of great capacity and a battery of high electromotive force.

¶ 255. **Comparison of Condensers.** — The capacities of two condensers may be compared by charging them, successively, by a given battery, then discharging them successively through a ballistic galvanometer (see ¶ 187). The capacities will then be approximately as the chords of the throws (§ 109).

The capacities of two condensers may be compared with great precision by including the condensers in two adjacent arms of a Wheatstone's bridge (see Exp. 87). One pole of the battery must be applied between the two condensers. The resistances in the other two arms of the bridge should be great, and adjusted so that a sudden *reversal* of the battery current causes no sudden deflection of the galvanometer.[2] If C_1 and C_2 are the capacities of the two

[1] Owing to effects of "electrical absorption" and "residual charge," the quantity of electricity stored in or obtained from a condenser depends somewhat upon the time during which connections are made. See Ganot's Physics, § 773. When a condenser is rapidly charged and discharged, these phenomena almost entirely disappear; but the resistance of the various conductors may reduce the quantity of electricity which can flow in and out of the condenser to an indefinitely small amount.

[2] See Glazebrook and Shaw, Practical Physics, §§ 81, 82.

condensers, R_1 and R_2 the resistances adjacent to them, respectively, we have —

$$C_1 : C_2 :: R_1 : R_2.$$

We have seen (¶ 254) that the capacity of a condenser with air spaces between its plates may be measured. The capacity of such condensers is generally so small that comparisons cannot be made by ordinary methods. By substituting an alternating current for the battery and a telephone for the galvanometer (see ¶ 253) in the combination described above, comparisons of these and even smaller capacities should be possible.

¶ 256. **Specific Inductive Capacity.** — When two condensers are similar in every respect except the nature of the insulating materials used in their construction, their capacities (c and c') are to each other as the "specific inductive capacities" (s and s') of these materials. Since the specific inductive capacity of air may be taken as 1, we have in general, from ¶ 254, I.,—

$$s = \frac{4\pi cd}{A}.$$

The specific inductive capacity of a given insulating material may accordingly be found by constructing a condenser with that material between its plates, measuring the area of and distance between these plates, and determining as in ¶ 254 or as in ¶ 255 the capacity of the condenser.

Winkelmann's method for testing specific inductive capacities consists in the use of three parallel plates,

A, *B*, and *C* (Figs. 279 and 280), equal in area, and 15 or 20 *cm.* in diameter. *A* and *B* are separated by an air space of the thickness *a*, while *B* and *C* are separated by an air space of the thickness *b*, and by a thickness *c* of the material whose specific inductive capacity is to be determined. The outer plates *A* and *C* are connected either through a telephone (*T*, Fig. 279) with each other, or through a differential telephone (*DT*, Fig. 270), and through a metallic conductor (*G*) with the ground. The central plate

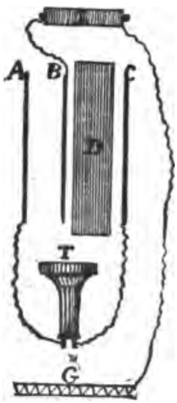

Fig. 279.

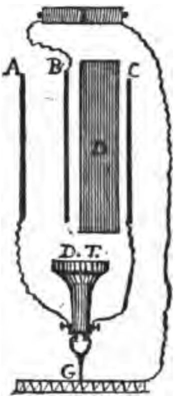

Fig. 280.

(*B*) is joined to one pole of an induction coil, the other pole of which is connected through *G* with the ground. The distances *a* and *b* are then adjusted so that the sound heard in the telephone is reduced to a minimum. The specific inductive capacity (*s*) is then given by the formula —

$$s = \frac{c}{a - b}.$$

In Winkelmann's method we may consider that the plates A and B form one condenser, while the plates B and C form another condenser. When the capacities of these two condensers are equal, a given charge of electricity on B must raise A and C to the same potential; hence *if the effect be simultaneous* no current will flow through the telephone. In practice, most dielectrics cause a slight retardation in the charging of a condenser, so that although the telephone gives a minimum of sound, it never becomes perfectly silent.

¶ 257. **Comparison of Electromotive Forces by means of a Condenser.** — The pole cups of a condenser (A and B, Fig. 278) are to be connected as in ¶ 254 with the poles of a battery, then disconnected from the battery, and connected with the terminals of a ballistic galvanometer, the throw of which is to be observed. The experiment is to be repeated with a second battery. If a' and a'' are the throws, E' and E'' the electromotive forces, we have (see § 109), if the angles are small, —

$$\frac{E'}{E''} = \frac{chord\ a'}{chord\ a''} = \frac{a'}{a''}, \text{ nearly.}$$

In this experiment it is important that the duration of charging, discharging, and changing connections should be exactly the same in the two cases.

¶ 258. **Electrostatic System.** — Two gilt pith-balls (b and c, Fig. 281), of equal weight (w) and diameter (d) are both to be suspended from an insulated point a, by fine cotton threads of equal length (l).

The threads may be blackened with a lead-pencil to make sure that they will conduct electricity. One pole of a battery (*de*), of several hundred volts, is to be connected with the point (*a*) of suspension; the other pole with the ground.

The balls *b* and *c*, being similarly charged, will now repel each other. A considerable divergence should be observed. The distance (*s*) between the centres of the two balls is to be found by a sextant placed at a fixed distance (see ¶ 124). The electro-

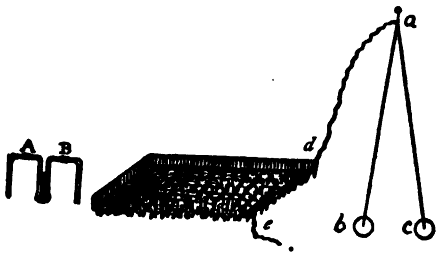

Fig. 281.

motive force (*e*) of the battery in electrostatic units is then (roughly) —

$$e = \sqrt{\frac{2\,wgs^3}{ld^2}}.$$

The pith-balls should be about 1 *cm.* in diameter, and not over .05 *g.* in weight. The cords *ab* and *ac* should be at least 100 *cm.* long, but not over 0.01 *g.* in weight. All electrical conductors should be removed as far as possible from the neighborhood of the balls *b* and *c*.

A water battery (*de*, Fig. 281) will be found convenient for this experiment. It may be constructed

of alternate strips of zinc and copper soldered together in pairs and attached with pitch to the under side of a board so that drops of water or dilute sulphuric acid may be taken up between adjacent pairs (as A and B).

It has been found by experiment that one unit of electromotive force in the electrostatic system is equal to about 300 volts, or 30 thousand million absolute units in the electromagnetic system. It is an interesting fact that the ratio between the absolute units of the two systems is equal, within the limits of errors of observation, to the velocity of light (see § 93).

INSTRUMENTS OF PRECISION.

The apparatus employed in the course of experiments which has been described is of the simplest possible form. The most accurate results can be obtained only by the use of instruments especially designed for a given purpose. The following sections

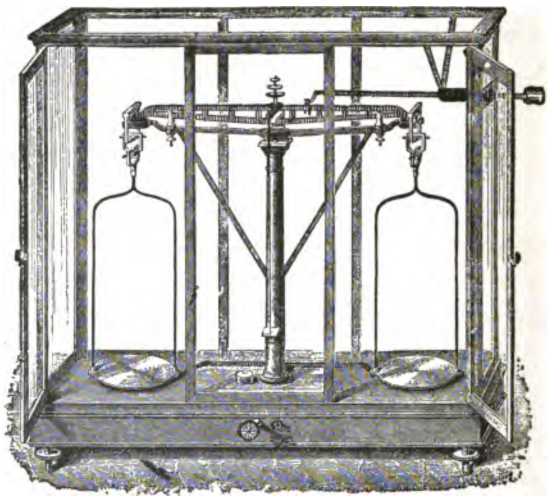

FIG 282.

contain a brief description of the construction and adjustments of certain instruments of precision, which though unsuitable for an elementary class of students, might be advantageously employed by advanced students in place of the ordinary apparatus.

¶ 259. **Analytical Balances.** — The adjustments of an analytical balance (Fig. 282) and the precautions in using it are essentially the same as those described in Experiment 6. In addition to the mechanism, operated from outside the case, by which in a fine balance *all* weight may be removed from the knife-edges, there is often a pan-arrester, which has to be moved before two weights can be exactly balanced. A preliminary adjustment of the weights should be carried as far as centigrams on an ordinary balance. The weights may then be transferred to the analytical balance, and a finer adjustment made by means of a rider (*e*, Fig. 283) made of platinum wire. The rider can be placed

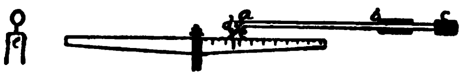

Fig. 283.

at any point (*e*) of a graduated scale on the balance-beam by means of a hook (*d*) attached to a rod (*ac*) passing through a tube (*b*) in the side of the balance-case. The necessary motion is given to the hook by pushing, pulling, or twisting the rod (*ac*).

The indication of the pointer is always found while it is in oscillation (¶ 20); but since the weights may be adjusted by means of the rider with any degree of precision, the method of interpolation (¶ 20), though generally quicker, need not be employed.

In finding the position of the rider necessary for an exact balance, the same method of approximation should be employed, at first, as in the adjustment of weights; that is, the rider should be placed midway

between two distances on the scale, one too great the other too small, until the deflection of the pointer and the sensitiveness of the balance indicate directly where it should be placed. When finally observations of the swings of the pointer show that it would come to rest at its zero-position, the position of the rider is noted.

The accuracy of the rider is tested by weighing a small weight with it. To obtain results accurate to a tenth of a milligram, the set of weights employed (even the best) should be most carefully tested (¶ 25).

The advantage of weighing with a rider is that the final adjustment of two weights may be made with the balance-case closed. The air within the case should always be kept perfectly dry with chloride of calcium (or with concentrated sulphuric acid), which must be renewed from time to time. Neither arm of the balance should be exposed to the heat of a fire or lamp, or to the cold glass of a window. The method of double weighings should if possible be employed. If it is not employed, care must be taken that the pans are equal in weight, and that in the zero-position, the balance-beam is horizontal and the pointer vertical.[1]

[1] When the greatest accuracy is desired, arrangements must be made to carry on the ordinary processes of weighing from a distance. Thus at the International Bureau of Weights and Measures at St. Cloud, not only the suspension of weights from the balance-beam, but also the interchange of the contents of the scale-pans is accomplished by a series of shafts leading from each instrument nearly to the centre of a large room in which the finest balances are contained. Mechanical contrivances are also employed for the final adjustment of weights *in vacuo*.

¶ 260. **Comparators.** — A simple form of comparator is represented in Fig. 284. It consists of two reading microscopes (*A* and *B*) mounted on supports (*E* and *F*) which slide along a rail (*GH*). The sliding supports may be clamped at any point of the rail by thumb-screws (*C* and *D*). A small scale of tenths of millimetres (*b* and *b′*, Fig. 284) is placed in the tube of each microscope at a distance from the object glass (*c*) equal to twice its focal length. The eye-

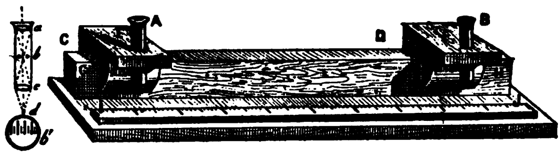

Fig. 284.

piece (*a*) is first focussed upon this scale, then raised or lowered until a given object is in focus. Let us suppose that the two microscopes are thus set, one upon each end of a scale. It is obvious that if a standard scale be now substituted any difference between the two will be not only readily detected, but easily measured in tenths of a millimetre and such fractions of a tenth as may be estimated by the eye (§ 26).

Care must be taken to have the *upper* surfaces of the two scales on the same level, so that both scales may be in focus, and to have the microscopes firmly clamped, and not subjected to any strain between observations.

¶ 261. **The Dividing-Engine.** — A dividing-engine (Fig. 285) consists essentially of a micrometer (*c*) with

a long screw (DG) fixed in position, so that when the micrometer is turned, a nut (EF) gives a slow motion to a slide (B) to which a reading microscope (A) is usually attached. The length of an object parallel to the screw is determined by the number of turns of the micrometer necessary to make the microscope travel from one end of the object to the other. The microscope is of course provided with cross-hairs, so that it may be set exactly on a given point. The screw is always to be turned in a given direction in measuring a given distance; otherwise an error due to looseness of the screw ("backlash") may be made. The pitch of the screw in different parts is

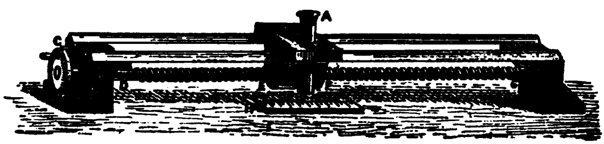

FIG. 285.

found by measuring with it a standard scale of known length (see ¶ 52). If the nut is long and fits equally well in all parts of the screw, no *great* variations of pitch can occur.

The dividing-engine is especially useful in measuring distances between the lines of a scale, or lengths of columns of mercury in the calibration of a tube (see ¶ 71). The results may be more precise than those obtained with any other instrument for the measurement of length.

¶ 262. **The Cathetometer.** — (*κατὰ*, down, *τίθημι*, to

place, and $\mu\acute{\varepsilon}\tau\rho o\nu$, measure) is an instrument for measuring vertical distances (Fig. 286). It consists of a horizontal telescope or reading microscope (*b*) sliding on a vertical shaft (*ah*), which is capable of rotating about its own axis. Sometimes the shaft is graduated, the carriage to which the telescope is attached being provided with a vernier, so that the height of the telescope may be read. Slow motion may also be given by a micrometer screw (*ef*). The cathetometer may then be used for measuring small vertical distances, just as the dividing-engine (¶ 261) is used for horizontal distances. The micrometer is useful in measuring precisely, for instance, the distance through which a wire is stretched (Exp. 65). For ordinary purposes, neither the micrometer nor the vernier is required. The shaft is first adjusted by the eye so as to be as nearly perpendicular as possible, by means of the levelling-screws (*h*, *i*, and *l*) at the base of the instrument, then the telescope is made horizontal according to a spirit-level (*c*) with which it is provided. Then the shaft is rotated about its axis. If the axis is not vertical, the bubble in the spirit-level will tend to move in a given direction. The

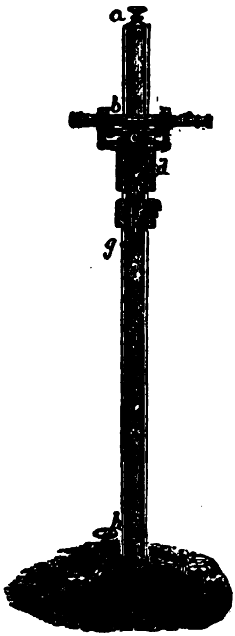

FIG. 286.

top of the shaft is to be inclined slightly in this direction. After a series of trials the axis may in this way be made perfectly vertical.

The object to be measured is to be set up with the aid of a plumb-line, beside a vertical scale, so as to be at the same distance from the cathetometer as the scale is, both at the top and at the bottom. The telescope of the cathetometer, accurately levelled, is to be focussed by means of the cross-hairs upon one end of the object (¶ 116, 3), then rotated so as to bear upon the scale, and the reading of the scale noted. If the spirit-level is disturbed, the cathetometer must be readjusted and the reading redetermined. The reading of the lower end of the object is to be found in the same way. By putting a graduated scale in place of the cross-hairs, the divisions of a scale may be divided into very small parts. This method is not so precise as that depending upon the use of a vernier or micrometer attached to the cathetometer, but may, in unskilled hands, give fully as accurate results.

¶ 263. **Micrometer Eye-Pieces.** — Instead of moving a telescope or a reading microscope bodily, as in ¶¶ 261 and 262, it is sometimes convenient to mount the cross-hairs upon a small slide within the eye-piece of an instrument, and to give a slow motion to the slide by means of a micrometer screw. The value of the micrometer divisions must be found for each instrument. A micrometer eye-piece gives indications much more precise than a fixed scale; but care must be taken not to alter the setting of an instru-

ment by pressure upon the eye-piece in adjusting the micrometer, and, as in the dividing-engine (¶ 261), to turn the instrument always in a given direction up to a setting. If the micrometer is turned too far, it must be turned backward a considerable way, then forward to the desired point.[1]

In the best optical circles two microscopes with micrometer eye-pieces are usually provided. These are placed on opposite sides of the circle, in order that errors due to excentricity may be avoided.

¶ 264. **Regulators.** — For experiments involving the accurate measurement of time, a clock with a compensating pendulum, or a chronometer with a compensating balance is indispensable. The clock or chronometer should be provided with an electric break-circuit, and must be rated by observations with either a sextant (¶ 243) or a transit (see Pickering's Physical Manipulation, § 178), or by comparison with time signals from some observatory.

In the Physical Laboratory of Harvard College, the regulator employed is a common seconds-clock with a wooden pendulum-rod controlled by an electrical time circuit. The control consists simply of a fine spiral spring connecting the pendulum with the armature of a telegraph instrument in the circuit. Electrical signals, sent from the Astronomical Observatory at intervals of two seconds, are thus made to act mechanically upon the pendulum. When the latter

[1] The "backlash" should be taken up, in so far as possible, by the action of a spring. Errors due to "backlash" may be thus greatly diminished, but not completely eliminated.

has been carefully rated without the control, very small impulses are sufficient to prevent it from gaining or losing.

¶ 265. **Kater's Pendulum** (Fig. 287). — In Kater's form of reversible pendulum (see 249) the rod (*de*) is usually made of brass, a little over a metre long, 2 or 3 *cm.* wide and about 5 *mm.* thick. Two steel knife-edges, *bc* and *fg*, are attached firmly to this rod with a distance of about 1 metre between them. They are supported when the pendulum is in use, by agate planes, *b* and *c*. The bob (*h*) is a brass cylinder, weighing 1 or 2 kilograms. Movable counterpoises, *d* and *e*, serve to adjust the centre of oscillation. Two light and firm metallic pointers (*a* and *i*) may be used to magnify the oscillations.

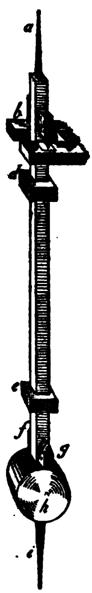

Fig. 287.

In addition to these adjustments, clamps with tangent-screws may be employed to obtain a slow motion of the counterpoises. The knife-edges *bc* and *fg* are sometimes made movable (one or both of them). In this case, verniers are usually attached, so that the distance between the knife-edges may be read by a scale on the shaft *de*. The zero-reading of the vernier is found by bringing the knife-edges together against a pressure equal to the whole weight of the pendulum. The accuracy of the main scale is tested by a comparator (¶ 260) at the ordinary temperature of the experiments, and under a strain equal to the average weight which the shaft sustains.

¶ 266. **Chronographs.** — A chronograph consists generally of a cylindrical drum (*A*, Fig. 288) rotated uniformly by clock-work. The surface of the drum is coated with lampblack, so that a style (*B*), attached to the armature (*c*) of a telegraph instrument may make a mark upon it. The line *AB* represents the trace caused by an ordinary seconds break-circuit. At the point *D* there is an extra break due to a signal given by hand. If the drum revolves uniformly, the exact time of such a break can evidently be determined by measuring the distance from it to the nearest second-mark, and comparing this with the distance between two second-marks.

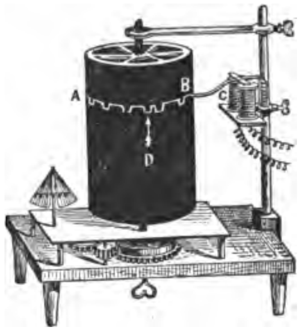

FIG. 288.

The pitch of a tuning-fork may be determined very exactly by the trace made on the surface of a chronograph (see ¶ 139).

It may be said in general that the chronograph is valuable as a means of determining precisely the interval of time between any two phenomena which, with or without the agency of electricity, are capable of affecting the motion of a style.

¶ 267. **The Siren.** — The siren (Fig. 289) is an instrument for producing a musical note of any pitch, and at the same time registering the number of vibrations constituting that note. It is operated by a constant air pressure from a bellows, specially con-

structed for this purpose. The air enters the wind-chest of the instrument at (F), issues obliquely from a series of holes (of which E is one) in the top of the wind-chest, and strikes obliquely against the sides of a series of holes (of which D is one) in a disc (C), which is thereby set in motion. When the two series of holes come opposite, the air escapes freely from the wind-chest; when they are not opposite, the current of air is nearly cut off. The irregular flow of the air sets the atmosphere in vibration. The num-

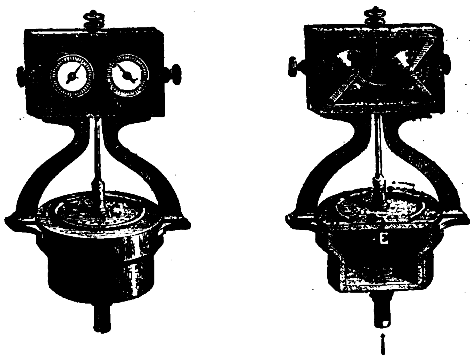

FIG. 289.

ber of vibrations in a given length of time is indicated by the dials A and B.

In practice the speed of the siren is regulated by pressure on the top of the bellows used to drive it. The note is slowly raised until it agrees with one whose pitch is to be determined. When the two notes are nearly in unison beats will be heard (¶ 140). By a slight change of air pressure, perfect unison may generally be obtained. This will be shown by a

cessation of beats. The unison is maintained for a given length of time during which the number of vibrations made by the siren is registered. In some instruments the dials may be thrown in and out of gear at a given moment. This facilitates the observations of the dials, but care must be taken that the speed of the siren is not affected.

It must be remembered that beats occur not only when two notes are in unison, but also when they are nearly an octave apart, and to a somewhat less extent, when they are separated by any other musical interval (¶ 134). A musical ear is therefore almost a necessity in the adjustment of a siren. The chief advantage of the siren is that it enables us to find the pitch of notes not easily determined (as is Exps. 52, 54, and 55), by either optical or graphical methods.

¶ 268. **Mirror Galvanometers.** — A very sensitive galvanometer is made by suspending a small mirror (*F*, Fig. 290) in the middle of a coil *E* of insulated wire, by means of a single fibre of cocoon silk (*DE*). Small bits of "hair-spring" (used in watches) highly magnetized, all in the same manner, are fastened with the smallest possible quantity of wax to the back of the mirror. A large curved magnet (*BC*) capable of sliding up and down the tube (*A*) or turning round it, is adjusted so as to nearly neutralize the effect of the earth's magnetism on the magnets attached to the

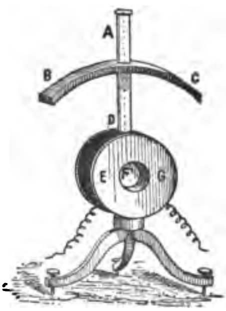

Fig. 290.

mirror. The sensitiveness of this instrument when accurately adjusted, though less permanent than that of an astatic combination, is for the time being fully as great.

In some galvanometers a converging mirror is used, so that a spot of light may be projected on a transparent screen. The existence of a current is indicated by the motion of the spot of light with respect to a scale graduated on the screen.

In other instruments a plane mirror is employed, with a long-focus lens mounted permanently in front

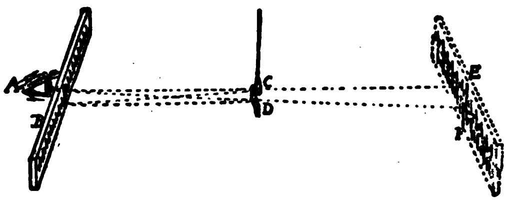

FIG. 291.

of it. The deflection of the mirror is frequently observed by means of the reflection (E, Fig. 291) of a scale (B) in the mirror (C), seen from a point (A), where either the eye or a telescope may be placed.[1]

¶ 269. **Electrical Standards.** — Copies of "standard ohms" may be obtained from most dealers in electrical apparatus. The terminals should be thick copper

[1] Prof. B. O. Peirce has shown that excellent results may be obtained without any telescope (A), by placing beneath the mirror C a fixed mirror D, so that the two reflections (E and F) of the scale (B) very nearly coincide. When the two mirrors are parallel, the zeros of the two scales are opposite, *no matter where the eye may be placed.* The slightest deflection of the mirror causes an apparent motion of the scale reflected in it.

rods, capable of being amalgamated with mercury and connected by mercury cups with a Wheatstone's Bridge Apparatus. Unless special care be taken in making these connections, the most accurate standards of resistance may lead to very erroneous results.

Standard cells of Latimer Clark's pattern may easily be obtained. Their electromotive force is about 1·435 volts at 15°. The decrease is about ·00077 volts for a rise of temperature of 1° Centigrade. The uses of a constant cell have been alluded to in ¶¶ 228, 230.

"Standard ampères" are now being made by some dealers. When the attraction of a coil of wire for

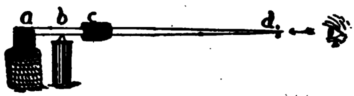

Fig. 292.

a piece of soft iron is balanced by gravity (Fig. 292), an allowance must be made for variations in gravity when the instrument is transported from one latitude to another. A standard ampère depending upon the action of a spring, though subject to many theoretical objections, would be practically useful as a check upon results obtained by other methods. Let us suppose that such an instrument is connected in series with a rheostat and a tangent galvanometer, that a current, sent through both, is increased until the instrument indicates 1 ampère, and that the galvanometer is then read. The reciprocal of the tangent

of the angle of deflection should agree closely with the reduction factor already found (Exp. 83).

¶ 270. **Electrometers.** — Various forms of quadrant electrometer may now be obtained from manufacturers. The theory of these instruments is exceedingly complicated, and the results are more or less uncertain. The principal use of the instrument is in the case of inconstant cells, to confirm results obtained by the use of a condenser. Such instruments in gen-

FIG 293.

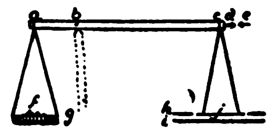

FIG. 294.

eral have to be calibrated by means of cells of known electromotive force.

Thomson's absolute electrometer (Figs. 293 and 294) depends upon the attraction between two plates *j* and *i*, when charged oppositely with electricity. The plate *j* is suspended from one end (*c*) of a balance-beam (*ac*). The force exerted upon it is counterpoised by weights in a pan (*f*) suspended from the other end of the beam (*a*.) The deflection of the beam is observed by means of a sight (*d*) and a lens (*e*). The plate *i* is very much larger than *j*, which is surrounded by a ring (*h*) charged to the same potential

as the movable disk (j), to equalize the distribution of electricity upon the latter.

If w is the weight required to balance the attraction of the two plates, d the distance between them, and a the area of the suspended plate (j), then the difference of potential (e) between the plates is given in electrostatic measure by the formula—

$$e = d\sqrt{\frac{8\pi g w}{a}}.$$

It is said that an absolute electrometer may be made sensitive to the difference in potential between the two poles of a Daniell cell. It is especially valuable for the calibration of other forms of electrometer better suited for actual use, and for determinations of the fundamental relations between the electrostatic and electro-magnetic systems.

END OF PART II.